

Numerical Simulation of Steady Incompressible Flow Through a Stenosed Channel Using a Weak Galerkin Finite Element Method

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Abstract—We evaluate a stabilized weak Galerkin (WG) discretization for steady incompressible flow in a smooth two-dimensional stenosed channel. The method uses element-interior P1 velocity, edgewise P0 velocity, elementwise P0 pressure, standard WG stabilization, skew-symmetric convection, and a centered natural-outlet correction, with no upwind term. A manufactured solution recovers the expected rates, and a Poiseuille test verifies the outlet treatment and mass conservation. A four-level mesh study adopts the finest available $N_y = 24$ grid. For $\delta = 0.4$, sampled $Re = 20$ – 200 cases show decreasing maximum speed and pressure difference; recirculation is detected only at $Re = 150$ and 200 , so the sampled onset lies in $100 < Re \leq 150$. At $Re = 100$, increasing δ from 0.2 to 0.5 raises speed, pressure difference, and wall shear; recirculation is detected only at $\delta = 0.5$, giving $0.4 < \delta \leq 0.5$. The work is a numerical verification and sampled parameter study, not a critical-parameter or external-benchmark determination.

Keywords—Stenosed channel; incompressible flow; Navier–Stokes equations; skew-symmetric form; weak Galerkin method.

I. INTRODUCTION

Smooth stenoses occur in ducts, microchannels, filters, and idealized arterial models. They accelerate the flow, alter pressure and wall shear, and may cause downstream separation; these effects depend on geometry and Reynolds number [7]–[9].

WG methods replace classical derivatives by discrete weak operators and use independent interior and boundary velocity unknowns, supporting discontinuous spaces and unstructured meshes [1]–[5]. Related developments include divergence-free, pressure-robust, and superconvergent WG schemes [10]–[12].

Zhang and Lin [4] analyzed a stabilized WG method for stationary Navier–Stokes equations, proving stability, conditional uniqueness, and optimal velocity-energy and pressure estimates. Their skew-symmetric convection form gives exact cancellation when its last two arguments coincide. This study extends that framework to a smooth open channel.

We assess the existing skew-symmetric, non-upwind WG implementation using a manufactured solution, a Poiseuille outlet test, and sampled channel cases with $Re \in \{20, 50, 100, 150, 200\}$ and $\delta \in \{0.2, 0.3, 0.4, 0.5\}$. Reported quantities are speed, section-averaged pressure difference, wall shear, conservation, symmetry, and detected reverse flow.

Section II defines the problem, Section III the WG formulation, Section IV the solver and diagnostics, Section V verification and mesh sensitivity, Section VI results, and Section VII conclusions.

II. GOVERNING EQUATIONS AND FLOW CONFIGURATION

A. Stationary incompressible Navier–Stokes equations

Let Ω be a bounded two-dimensional flow region with boundary $\partial\Omega$. The nondimensional stationary incompressible Navier–Stokes equations are

$$-\nu \Delta u + (u \cdot \nabla)u + \nabla p = f \quad \text{in } \Omega \quad (1)$$

$$\nabla \cdot u = 0 \quad \text{in } \Omega \quad (2)$$

Here $u = (u_1, u_2)^T$, p is pressure, f is body force, and $\nu > 0$ is kinematic viscosity. The natural outlet fixes the pressure reference in channel cases; the manufactured test uses zero-mean pressure. For the stenosed channel, $f = 0$ and $\nu = Re^{-1}$, based on unit unobstructed height and mean inlet speed.

B. Stenosed-channel geometry and boundary conditions

The channel has $L = 12$ and height $H\delta(x)$. A smooth stenosis centered at $x^c = 4$ with length $L_s = 2$ is defined by $H\delta(x) = 1 - (\delta/2)[1 + \cos(2\pi(x-x^c)/L_s)]$ for $|x-x^c| \leq L_s/2$ and $H\delta(x) = 1$ otherwise. Thus the throat height is $1-\delta$; $\delta = 0.2, 0.3, 0.4$, and 0.5 .

A fully developed parabolic profile with unit mean velocity is prescribed at the inlet,

$$u_1(0, y) = 6\left(\frac{1}{4} - y^2\right), \quad u_2(0, y) = 0, \quad -\frac{1}{2} < y < \frac{1}{2} \quad (3)$$

No slip is imposed on both walls and $v(\partial u / \partial n) - p n = 0$ at $x = 12$. The seven-height downstream segment limits outlet influence near the stenosis. The $\delta = 0$ channel provides the Poiseuille test.

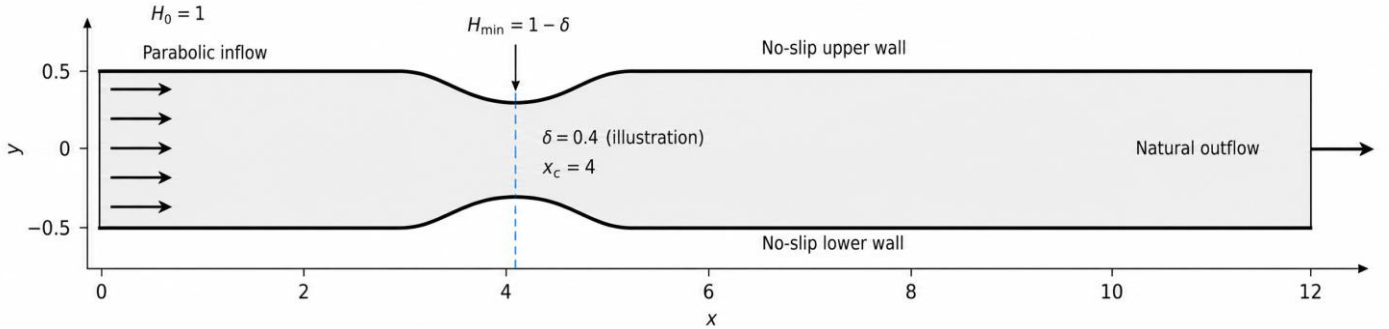


Fig. 1. Computational domain, smooth symmetric constriction, and boundary conditions for the stenosed-channel flow.

III. WEAK GALERKIN FINITE ELEMENT METHOD

A. Discrete weak functions and differential operators

Let T_h be a shape-regular triangular or polygonal mesh. A weak function is $v = \{v_0, v_b\}$, with independent interior and edge components. For $k \geq 1$, the local space is

$$W_{h,k}(T) = \{v = \{v_0, v_b\} : v_0 \in P_k(T), v_b|_e \in P_{k-1}(e)\} \quad (4)$$

The discrete weak gradient $\nabla_w v \in [P_{k-1}(T)]^2$ is defined by

$$(\nabla_w v, q)_T = -(v_0, \nabla \cdot q)_T + \langle v_b, q \cdot n \rangle_{\partial T} \quad (5)$$

For a vector weak function $v = \{v_0, v_b\}$, the discrete weak divergence $\text{div}_w v \in P_{k-1}(T)$ satisfies

$$(\nabla_w \cdot v, q)_T = -(v_0, \nabla q)_T + \langle v_b \cdot n, q \rangle_{\partial T} \quad (6)$$

B. Velocity-pressure spaces and stabilization

The edge component is single valued on each mesh edge. X_h^0 has zero edge values on Dirichlet boundaries, and X_h^g uses the projected inflow and wall data. The pressure spaces are

$$M_h^0 = \{q \in L_0^2(\Omega) : q|_T \in P_{k-1}(T), T \in T_h\}, \text{ manufactured/full-Dirichlet case} \quad (7a)$$

$$M_h^{\text{out}} = \{q \in L^2(\Omega) : q|_T \in P_{k-1}(T), T \in T_h\}, \text{ open-channel case} \quad (7b)$$

The diffusion form contains the discrete weak gradient and the standard weak Galerkin stabilization, We use $k = 1$: $u_0|_T \in [P_1(T)]^2$, $u_b|_e \in [P_0(e)]^2$, and $p_h|_T \in P_0(T)$; weak gradients and divergences are elementwise constant.

$$a_h(u, v) = (\nabla_w u, \nabla_w v)_h + s_h(u, v) \quad (8)$$

$$s_h(u, v) = \sum_T h_T^{-1} \langle Q_b u_0 - u_b, Q_b v_0 - v_b \rangle_{\partial T} \quad (9)$$

Term (9) is the standard WG consistency stabilization, not an upwind or convection-dependent term.

C. Skew-symmetric convective form

For $w, u, v \in X_h$, the nonlinear contribution is discretized by the skew-symmetric trilinear form

$$b_h(w; u, v) = \frac{1}{2} \langle (w_0 \cdot \nabla_w) u, v_0 \rangle_h - \frac{1}{2} \langle (w_0 \cdot \nabla_w) v, u_0 \rangle_h \quad (10)$$

The identity $b_h(w; v, v) = 0$ gives the closed-boundary energy cancellation [6].

D. Skew-consistent natural-outlet correction

At an open outlet, the implementation adds the centered correction

$$c_{\text{out}}(w; u, v) = \frac{1}{2} \langle (w_b \cdot n) u_b, v_b \rangle_{\Gamma_{\text{out}}} = \frac{1}{2} \sum_{e \in \Gamma_{\text{out}}} \sum_q w_q (w_b(x_q) \cdot n_e) (u_b(x_q) \cdot v_b(x_q)) \quad (10a)$$

$$c_h(w; u, v) = b_h(w; u, v) + c_{\text{out}}(w; u, v) \quad (10b)$$

Here n_e is the outward outlet normal; u_b , v_b , and w_b are edge traces, and w_q includes edge length. The coefficient and sign match the code. The term is centered and contains no upwinding, artificial viscosity, edge penalty, or one-sided flux.

E. Discrete formulation

The open-channel solution $(u_h, p_h) \in X_h^g \times M_h^{\text{out}}$ satisfies

$$a_h(u_h, v) + c_h(u_h; u_h, v) - (p_h, \text{div}_w v) = (f, v_0) \quad (11)$$

$$(\text{div}_w u_h, q_h) = 0 \quad (12)$$

For homogeneous Dirichlet data, the velocity–pressure pair in [4] satisfies a mesh-independent discrete inf–sup condition. Under the regularity and small-data hypotheses stated there, the discrete problem has a stable solution and the solution is unique when $\text{CON0}\|f\|/v^2 < 1$. If $u \in [H^{k+1}(\Omega)]^2$ and $p \in H^k(\Omega)$, the theoretical estimate has the form

$$\|\nabla u - \nabla_w u_h\|_h + \|p - p_h\|_0 \leq Ch^k (\|u\|_{k+1}^2 + \|p\|_k) \quad (13)$$

Inflow and no-slip data are imposed through edge unknowns, while outlet edges remain free. The natural outlet provides the pressure reference without a mean-pressure multiplier; the manufactured test uses M_h^0 . A mixed-boundary theory is not developed here.

IV. NONLINEAR SOLVER AND IMPLEMENTATION

A. Newton linearization

At iterate (u_h^m, p_h^m) , Newton uses $c_h(u_h^m; \delta u_h^m, v) + c_h(\delta u_h^m; u_h^m, v)$, including both outlet-trace derivatives. The update is $X^{m+1} = X^m + \lambda_m \delta X^m$, with backtracking only when the residual decrease test fails.

The monitored quantities are $r_{\text{abs}}^m = \|R(X^m)\|_2$, $r_{\text{rel}}^m = \|R(X^m)\|_2 / \max(\|R(X^0)\|_2, \text{eps})$, and $s_{\text{upd}}^m = \|\lambda_m \delta X^m\|_2 / \max(\|X^m\|_2, 1)$. The stopping test is (14); final relative residuals are below 3×10^{-13} .

$$\begin{aligned} (r_{\text{abs}}^m \leq 10^{-10} \text{ or } r_{\text{rel}}^m \leq 10^{-8}) \\ s_{\text{upd}}^m \leq 10^{-10}. \end{aligned} \quad (14)$$

For channel cases, the natural outlet fixes the pressure reference and no zero-mean constraint is imposed. The manufactured test uses zero-mean pressure. Dirichlet edge velocities are eliminated strongly, and the existing quadrature rules are retained.

B. Reynolds-number continuation

Stokes flow initializes continuation. Each converged solution seeds the next Reynolds number or stenosis ratio; only the initial guess changes.

C. Postprocessing quantities

The reported speed is $|u|_{\max} = \max_{\Omega} \text{hypot}(u_1, u_2)$, evaluated from the interior P1 field with eighth-order triangular quadrature. The global maximum is replaced by the $x \leq L-1$ maximum only when an outlet-layer value exceeds it by more than 2%. Section-averaged pressure gives $\Delta p = \bar{p}(2) - \bar{p}(8)$, which includes both background viscous and constriction effects. Signed wall shear is $\tau_w = \nu t \cdot (\nabla u + \nabla u^T) n$, and mass imbalance is $|Q_{\text{in}} - Q_{\text{out}}| / |Q_{\text{in}}|$.

Recirculation is searched on 1191 streamwise and 101 wall-normal samples. A downstream wall interval is accepted only when at least three consecutive points fall below $-\max(10^{-8}, 10^{-5} \max|u_1|)$, its length exceeds $\max(2\Delta x, 0.5 \min h_i)$, and the wall-referenced streamfunction excursion exceeds $\max(10^{-10}, 10^{-5} \max|\psi_{\text{upper}} - \psi_{\text{lower}}|)$. Separation and reattachment are linearly interpolated, with $L_r = x_r - x_s$ evaluated on each wall. Thus reported onset values are sampled detections, not critical thresholds.

V. NUMERICAL VERIFICATION

A. Manufactured solution

On $\Omega = (0, 1)^2$, the divergence-free manufactured solution is

$$\begin{aligned} u_1(x, y) &= x^2(1-x)^2(2y-6y^2+4y^3), \\ u_2(x, y) &= -(2x-6x^2+4x^3)y^2(1-y)^2. \end{aligned} \quad (15)$$

$$p(x, y) = x^3 - y^3. \quad (16)$$

The velocity generated by $\psi = x^2(1-x)^2y^2(1-y)^2$ is divergence free and vanishes on $\partial\Omega$; $p = x^3 - y^3$ has zero mean. With $f = -\nu\Delta u + (u \cdot \nabla)u + \nabla p$, Table I gives approximately second-order interior-velocity L^2 convergence and first-order energy and pressure L^2 convergence.

TABLE I. ERRORS AND CONVERGENCE RATES FOR THE MANUFACTURED SOLUTION.

h	$\ Q_{\text{out}} - u_0\ _0$	rate	$\ Q_{\text{hu}} - u_h\ $	rate	$\ Q_p - p_h\ _0$	rate
1/10	5.187920×10^{-2}	—	4.148413×10^{-1}	—	3.853013×10^{-2}	—
1/20	1.313725×10^{-2}	1.98	2.133209×10^{-1}	0.96	1.911708×10^{-2}	1.01
1/40	3.298409×10^{-3}	1.99	1.076315×10^{-1}	0.99	9.524825×10^{-3}	1.01
1/80	8.257108×10^{-4}	2.00	5.396893×10^{-2}	1.00	4.755182×10^{-3}	1.00

B. Mesh-sensitivity study

A four-level mesh-sensitivity study at $Re = 100$ and $\delta = 0.4$ uses $N_y = 12, 16, 20$, and 24 . The reported h is the maximum element diameter in $3 \leq x \leq 5$. All cases converge with near-machine-precision absolute residuals, final relative residuals below 3×10^{-13} , and machine-precision flow-rate balance.

TABLE II. MESH-SENSITIVITY RESULTS AT $Re = 100$ AND $\delta = 0.4$.

Mesh	N_y	h near throat	Elements	Velocity DOFs	Pressure DOFs	$ u _{\max}$	Δp	Mass error
Coarse	12	0.110304	3456	31416	3456	2.344514	1.458711	0
Medium-1	16	0.082961	6208	56292	6208	2.250624	1.407365	2.22×10^{-16}
Medium-2	20	0.066479	9680	87644	9680	2.231702	1.365793	2.22×10^{-16}
Fine	24	0.055483	13824	125040	13824	2.220541	1.340062	4.44×10^{-16}

From $N_y = 20$ to 24 , $|u|_{\max}$ and maximum wall shear change by 0.503% and 0.983% , while Δp changes by 1.920% . Strict mesh independence is therefore not claimed. The finest available $N_y = 24$, $N_x = 288$ mesh (13,824 elements) is used for all sampled parameter cases.

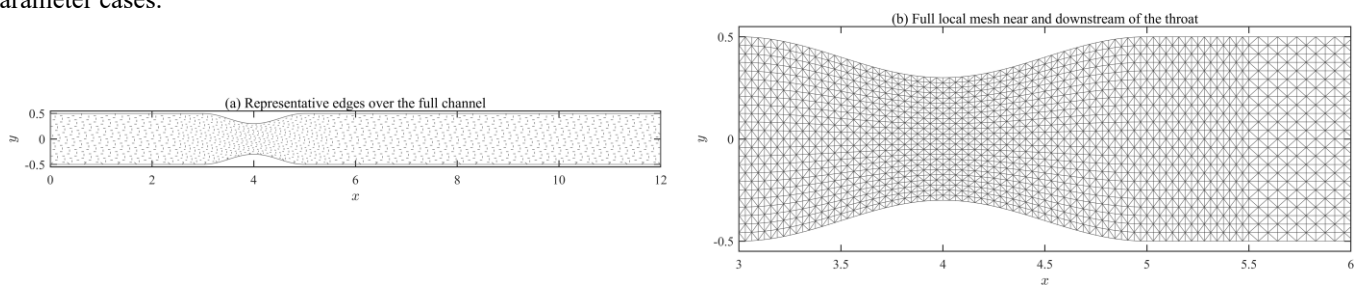


Fig. 2. Full-domain representative edges and throat-region triangulation for $N_y = 24$ and $\delta = 0.4$.

VI. STENOSED-CHANNEL RESULTS

A. Straight-channel Poiseuille verification

The $\delta = 0$ channel verifies the inflow, centered outlet correction, pressure reconstruction, mass conservation, and removal of the outlet speed artifact. For unit mean inflow, $u_1 = 6(1/4 - y^2)$, $u_2 = 0$, and $dp/dx = -0.12$.

On $N_y = 16$, the final residual is 1.81×10^{-15} and the flow-rate imbalance is 2.22×10^{-16} . The computed maximum speed is 1.55833 versus 1.5 , a 3.89% error. Velocity L^2 , energy, and pressure L^2 errors are 0.10703 , 3.19955 , and 0.19440 . The section pressure difference is 0.770219 versus 0.72 , and the fitted slope is -0.12857 .

From $N_y = 12$ to 16 , the observed velocity L^2 , energy, and pressure L^2 rates are 1.125 , 0.392 , and 0.568 ; this short sequence is not used to claim asymptotic optimality.

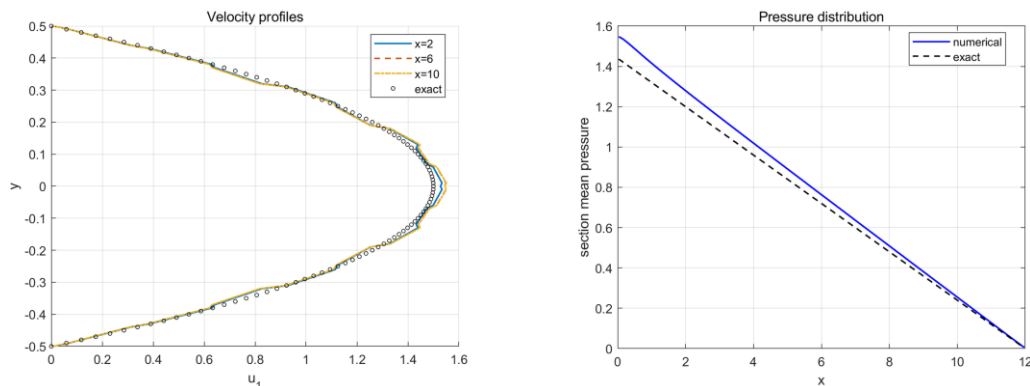


Fig. 3. Poiseuille verification on $N_y = 16$: velocity profiles and section-averaged pressure.

B. Effect of Reynolds number

The Reynolds study uses $\delta = 0.4$ and the $N_y = 24$, $N_x = 288$ mesh at $Re = 20, 50, 100, 150$, and 200 . Mean inflow remains one and $v = 1/Re$. Final relative residuals are below 3×10^{-13} , divergence errors are $O(10^{-14})$, and mass errors are at most 4.44×10^{-16} .

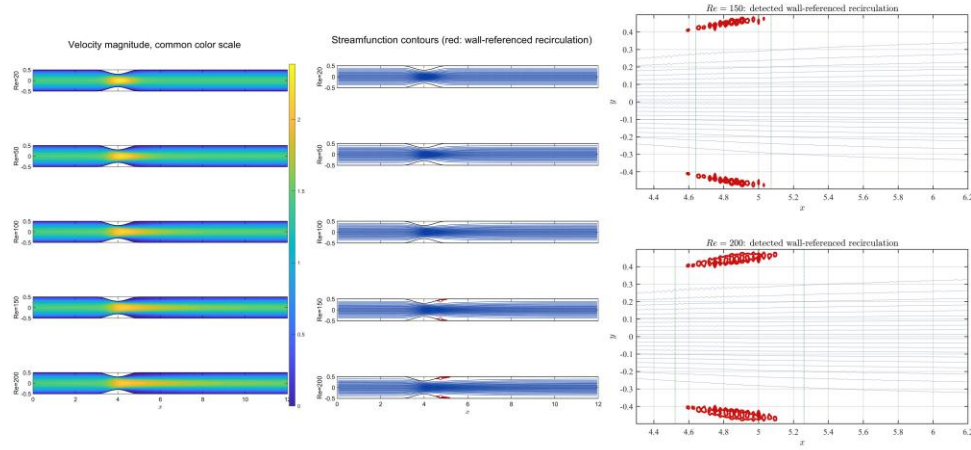


Fig. 4. Velocity magnitude, streamfunction contours, and downstream recirculation zooms for the sampled Reynolds numbers at $\delta = 0.4$.

TABLE III. EFFECT OF THE REYNOLDS NUMBER AT $\delta = 0.4$.

Re	$ u _{\max}$	$x(u _{\max})$	Δp	$\max \tau_w $	Recirc.	L_r	Mass error
20	2.398369	4.094370	5.455812	1.028254	No	—	2.22×10^{-16}
50	2.305614	4.218142	2.324627	0.485419	No	—	4.44×10^{-16}
100	2.220541	4.218142	1.340062	0.290782	No	—	4.44×10^{-16}
150	2.179453	4.280642	1.055159	0.217154	Yes	0.432035	2.22×10^{-16}
200	2.154561	4.280642	0.936064	0.176815	Yes	0.738588	4.44×10^{-16}

As Re increases, v decreases. Across the samples, $|u|_{\max}$ falls from 2.398369 to 2.154561, Δp from 5.455812 to 0.936064, and $\max|\tau_w|$ from 1.028254 to 0.176815. Recirculation is detected only at Re = 150 and 200, with $L_r = 0.432035$ and 0.738588. Hence the sampled onset is bracketed by $100 < \text{Re} \leq 150$.

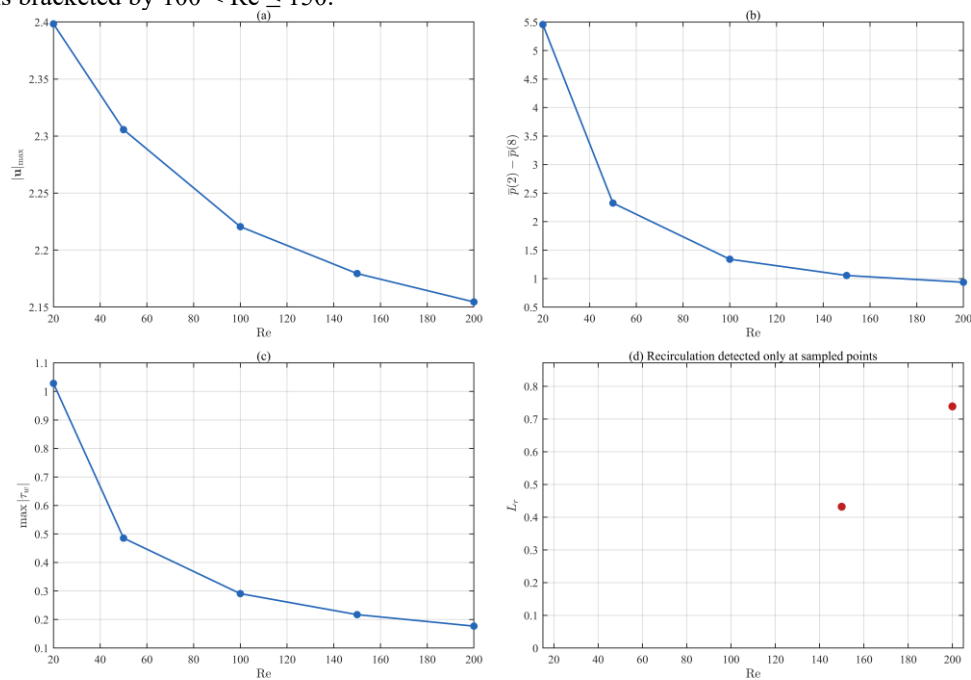


Fig. 5. Sampled Reynolds-number dependence of $|u|_{\max}$, Δp , $\max|\tau_w|$, and detected L_r at $\delta = 0.4$.

C. Effect of stenosis ratio

At $Re = 100$, $\delta = 0.2, 0.3, 0.4$, and 0.5 are computed on the $N_y = 24$ mesh. In all cases $Q_{in} = Q_{throat} = Q_{out} = 1$, and mean throat speeds agree with $1/(1-\delta)$ to about 3.4×10^{-14} .

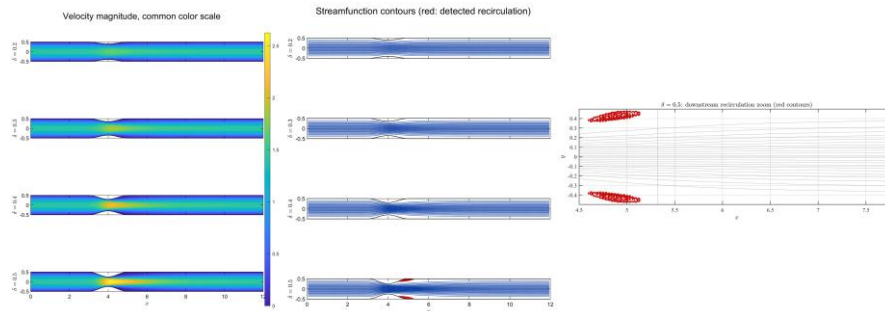


Fig. 6. Velocity magnitude, streamfunction contours, and the $\delta = 0.5$ downstream zoom at $Re = 100$.

TABLE IV. EFFECT OF THE STENOSIS RATIO AT $Re = 100$.

δ	H_{min}	$ u _{max}$	$x(u _{max})$	Δp	$\max \tau_w $	Recirc.	L_r	Mass error
0.2	0.8	1.758866	4.280642	0.898947	0.134389	No	—	2.22×10^{-16}
0.3	0.7	1.947073	4.280642	1.054602	0.196297	No	—	4.44×10^{-16}
0.4	0.6	2.220541	4.218142	1.340062	0.290782	No	—	4.44×10^{-16}
0.5	0.5	2.629923	4.219370	1.901624	0.443899	Yes	0.861143	0

Across the sampled δ values, $|u|_{max}$ rises from 1.758866 to 2.629923, Δp from 0.898947 to 1.901624, and $\max|\tau_w|$ from 0.134389 to 0.443899. The pressure difference includes background viscous and constriction effects. Recirculation is detected only at $\delta = 0.5$, where $L_r = 0.861143$; the sampled onset is therefore bracketed by $0.4 < \delta \leq 0.5$.

Pressure and wall-shear symmetry errors are $O(10^{-16})$ and $O(10^{-15})$, while velocity symmetry errors are 1.16×10^{-4} – 1.67×10^{-4} .

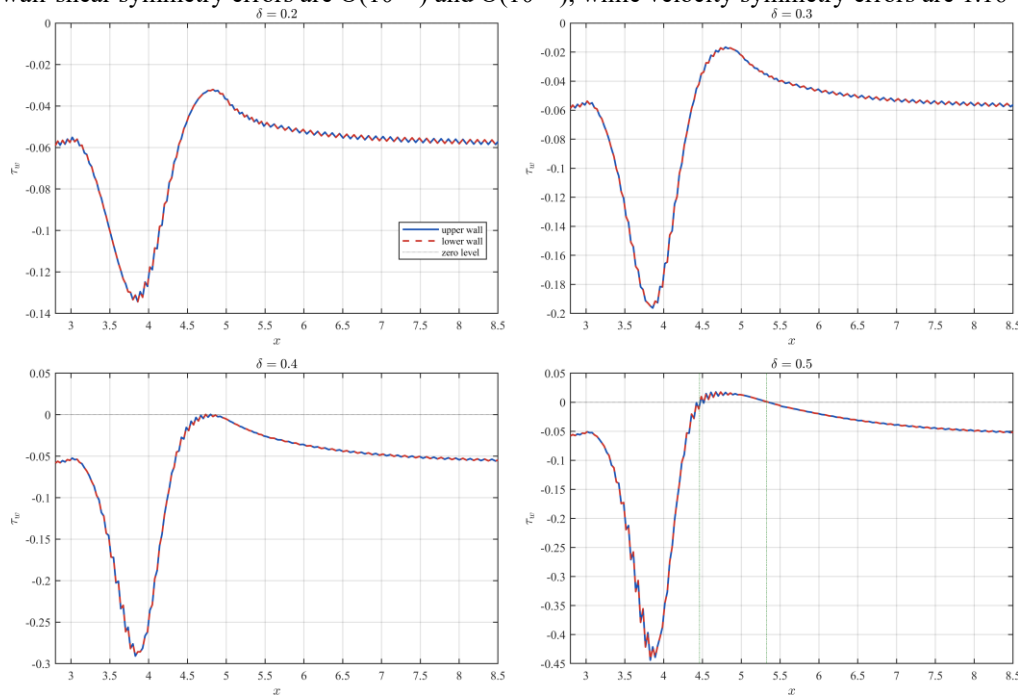


Fig. 7. Signed wall shear for the sampled stenosis ratios at $Re = 100$; $\delta = 0.5$ separation and reattachment are marked.

D. Reference-case diagnostics

Table V lists numerical diagnostics for $Re = 100$, $\delta = 0.4$, and $N_y = 24$; physical quantities are already in Tables III and IV.

TABLE V. NUMERICAL DIAGNOSTICS AT $Re = 100$ AND $\delta = 0.4$.

Diagnostic quantity	WG result	Purpose
Relative flow-rate imbalance	4.44×10^{-16}	Mass conservation
Discrete divergence error	2.71×10^{-14}	Incompressibility
Final absolute Newton residual	2.413×10^{-15}	Nonlinear convergence
Final relative Newton residual	9.967×10^{-14}	Scaled convergence
Maximum wall velocity	0	No-slip enforcement
Velocity symmetry error	1.44×10^{-4}	Field symmetry
Pressure symmetry error	4.07×10^{-16}	Pressure symmetry
Wall-shear symmetry error	1.96×10^{-15}	Wall diagnostic symmetry

E. Discussion

The manufactured and Poiseuille tests verify convergence, boundary treatment, pressure reconstruction, and conservation. The mesh study shows that Δp is more refinement-sensitive than speed or wall shear; $N_y = 24$ is the finest available common mesh, not a mesh-independent limit.

With unit mean inflow, increasing Re lowers v , Δp , and wall-shear magnitude, while recirculation is detected only at sampled $Re \geq 150$. At $Re = 100$, speed, Δp , and wall shear increase with δ , and recirculation is detected only at $\delta = 0.5$. These sampled brackets are not critical-parameter estimates.

VII. CONCLUSION

A skew-symmetric WG method with a centered natural-outlet correction has been tested for steady flow through a smooth stenosed channel. Manufactured and Poiseuille cases verify convergence, outlet treatment, and conservation; the finest available common mesh is $N_y = 24$, $N_x = 288$.

All sampled cases converge with final relative residuals below 3×10^{-13} and mass errors below 5×10^{-16} . Recirculation is detected at $Re = 150$ and 200 for $\delta = 0.4$, and only at $\delta = 0.5$ for $Re = 100$, giving sampled onset brackets $100 < Re \leq 150$ and $0.4 < \delta \leq 0.5$. The reported Δp includes background viscous and constriction effects. These results are a numerical verification and sampled parameter study, not a critical-parameter or external-benchmark validation.

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