Teaching Exploration of Comprehensive Evaluation Methods in Mathematical Modeling Courses

Xunru Yin[1], Weiyan Liu[1], Shan Jiang[1]

[1]School of Mathematics and Statistics, Taishan University, Tai'an 271000, China

Abstract—*Comprehensive evaluation is a core module in mathematical modeling courses, yet teaching often falls into the dilemma of emphasizing methods over underlying ideas. This paper takes urban livability evaluation as a case study to demonstrate the application of two models: the Linear Weighted Sum Method and the TOPSIS Method. Through deliberately designed data that produce inconsistent ranking results between the two methods, cognitive conflict is created among students. This guides students to gain a deeper understanding of the distinction between the "compensatory" philosophy of the Linear Weighted Sum Method and the "closeness to the ideal solution" philosophy of the TOPSIS Method. On this basis, students' ability to rationally select comprehensive evaluation models according to practical problem contexts is cultivated, thereby achieving an advancement in teaching objectives..*

Keywords— *Mathematical Modeling, Comprehensive Evaluation, Linear Weighted Sum Method, TOPSIS.*

I. INTRODUCTION

Mathematical modeling is an important practice-oriented course for majors such as Mathematics and Applied Mathematics, Statistics, and related engineering disciplines in higher education institutions. It aims to cultivate students' comprehensive ability to describe real-world problems using mathematical language, establish mathematical models, and solve and analyze them [1]. In recent years, with the Ministry of Education's continuous promotion of "New Engineering" construction and the cultivation of innovative talents, teaching reform in mathematical modeling courses has attracted widespread attention.

From the perspective of domestic and international teaching reform practices, exploratory efforts have mainly concentrated on the following aspects. First, at the level of teaching modes, new pedagogical formats such as case-driven teaching, project-based learning (PBL), and flipped classrooms have been widely introduced to enhance students' active participation and practical experience [2]. Second, at the level of technical tools, computational instruments such as MATLAB, Python, and R have been integrated into the classroom, helping students transition from "manual calculation" to "programming-based problem solving" [3]. Third, at the level of course content, some studies have conducted in-depth explorations of instructional design for specific topics such as optimization models, prediction models, and evaluation models [4].

However, existing teaching reform achievements mostly focus on the pedagogical optimization of a single method. Research remains insufficient regarding the cultivation of the critical competency of "systematic comparison and rational selection among multiple methods for the same type of problem." Particularly in the core module of comprehensive evaluation, individual chapters tend to be relatively independent, lacking in-depth horizontal comparison based on the same case study. This results in students not knowing which method to choose when facing practical modeling problems, nor how to interpret and make trade-offs when results are inconsistent.

Based on the above issues, this paper selects "urban livability evaluation" as its case study, taking four candidate cities as evaluation objects. Through the construction of an evaluation indicator system, the Linear Weighted Sum Method and the TOPSIS Method are respectively applied for comprehensive evaluation and ranking. This serves as an entry point to guide students in exploring the root causes of differences between the methods.

II. CASE BACKGROUND AND DATA PREPARATION

An easy way to comply with the conference paper formatting requirements is to use this document as a template and simply type your text into it.

2.1 Formulation of the Evaluation Problem

"Urban livability" has increasingly become a hot topic of social concern. A comprehensive evaluation and ranking of the livability of four cities — City A, City B, City C, and City D — is required. An evaluation indicator system for urban livability is first constructed from five dimensions. The meanings and attributes of each indicator are shown in Table 1.

TABLE I. Urban Livability Evaluation Indicator System.

Indicator Code	Indicator Name	Indicator Attribute
X_1	Per capita GDP (10,000 yuan)	the larger the better
X_2	Days of good air quality (days/year)	the larger the better
X_3	Teachers per 10,000 people (persons)	the larger the better
X_4	Hospital beds per 10,000 people (beds)	the larger the better
X_5	Average commuting time (minutes)	the smaller the better

2.2 Raw Data

For the convenience of teaching demonstration, simulated data are assigned to each indicator, as shown in Table II.

TABLE II. Raw Evaluation Data for Four Cities.

City	X_1	X_2	X_3	X_4	X_5
A	12.5	310	85	62	35
B	9.8	280	78	75	28
C	15.2	260	92	58	45
D	11.0	330	70	68	30

dataset has been carefully designed so that each city has its own strengths and weaknesses. City A performs well in economy and environment but has mediocre medical resources; City B excels in medical care and transportation convenience but is economically weak; City C is strongest in economy and education but poorest in transportation and environment; City D has the best environment but relatively weak educational resources. This characteristic of "no absolutely dominant alternative" is a typical scenario in comprehensive evaluation problems.

2.3 Data Preprocessing

Before conducting the comprehensive evaluation, two preprocessing steps are required for the raw data: indicator directionalization and non-dimensionalization.

(1) Directionalization of the cost-type indicator

For the cost-type indicator X_5, the reciprocal transformation method is adopted to convert it into a benefit-type indicator. After directionalization, a larger value indicates more convenient transportation.

(2) Standardization (normalization) of data

To eliminate the effects of different dimensions and orders of magnitude among indicators, the range standardization (Min-Max standardization) method is applied to map all indicator values to the [0, 1] interval.

The data after directionalization and standardization are shown in Table III.

TABLE III. Standardized Evaluation Data

City	Z_1	Z_2	Z_3	Z_4	Z_5
A	0.500	0.714	0.682	0.235	0.588
B	0.000	0.286	0.364	1.000	1.000
C	1.000	0.000	1.000	0.000	0.000
D	0.222	1.000	0.000	0.588	0.882

III. COMPARISON OF EVALUATION MODELS

3.1 Linear Weighted Sum Method

The Linear Weighted Sum Method is one of the most classic and widely applied comprehensive evaluation methods. Its basic idea is to assign a weight reflecting the relative importance to each evaluation indicator, and then linearly aggregate the standardized values of each indicator according to these weights to obtain a composite score for each evaluation object.

The determination of weights is the core step of the Linear Weighted Sum Method. Common approaches include subjective weighting methods and objective weighting methods. To reduce the influence of subjective factors and to maintain comparability with the TOPSIS Method, this paper adopts the entropy weight method to determine the weights.

After calculation (the detailed process is omitted; in teaching, students can be guided to implement it using Excel or Python), the entropy weights for the five indicators are obtained, as shown in Table IV.

TABLE IV. Results of the Entropy Weight Method.

Item	Z_1	Z_2	Z_3	Z_4	Z_5
Information entropy	0.862	0.837	0.796	0.810	0.819
Weight	0.158	0.187	0.234	0.218	0.203

the weights and standardized data into the composite score formula, the linear weighted composite scores for each city are calculated. For example, the score for City A:

$$S_A = 0.158 \times 0.500 + 0.187 \times 0.714 + 0.234 \times 0.682 + 0.218 \times 0.235 + 0.203 \times 0.588 = 0.553$$

The final ranking result of the Linear Weighted Sum Method is:

B (0.560) > A (0.553) > D (0.530) > C (0.392).

3.2 TOPSIS Method

The TOPSIS Method (Technique for Order Preference by Similarity to Ideal Solution) has the following core idea: among all possible alternatives, a "positive ideal solution" and a "negative ideal solution" are determined, and then the ranking is performed by calculating the distance of each actual alternative to the positive and negative ideal solutions.

Regarding the implementation procedure, the weighted standardized matrix is first constructed. Then the positive ideal solution V^+ and negative ideal solution V^- are determined. Next, the Euclidean distances from each city to the positive and negative ideal solutions are calculated. Finally, the relative closeness coefficient is computed. The results are shown in Table V.

TABLE V. Distances to Positive and Negative Ideal Solutions and Closeness Coefficients for Each City.

City	Closeness Coefficient	Ranking
A	0.525	1
B	0.521	2
C	0.454	4
D	0.500	3

IV. COMPARATIVE ANALYSIS OF RESULTS

The ranking results of the two methods are presented side by side, as shown in Table VI.

TABLE VI. Comparison of Results.

City	Linear Weighted Sum Ranking	TOPSIS Ranking
A	2	1
B	1	2
C	4	4
D	3	3

The rankings of City A and City B are reversed between the two methods. The rankings of Cities C and D remain consistent.

Students are guided to conduct a systematic comparison of the two methods, as shown in Table VII.

TABLE VII. Comprehensive Comparison.

Comparison Dimension	Linear Weighted Sum Method	TOPSIS Method
Core idea	Weighted summation with linear compensation	Closeness to ideal solution with distance measurement
Compensability	Full compensation (indicators can compensate for one another)	Limited compensation (imbalanced performance is penalized)
Evaluation type	Absolute scoring	Relative ranking
Sensitivity to extreme values	Low sensitivity (linear aggregation)	High sensitivity (squared distances amplify extreme differences)
Main advantage	Simple, intuitive, easy to understand and interpret	Clear geometric meaning; can capture the balance of alternatives
Main limitation	Results are highly dependent on weight settings	Sensitive to data distribution and extreme values
Applicable scenarios	Scenarios where inter-indicator compensation is reasonable	Scenarios focused on the balance of alternatives

After completing the comparative analysis, the key decision points for model selection are distilled, and a model selection decision tree is constructed. At the same time, students are reminded that there is no single "best" evaluation method — only the method that is most suitable for a specific problem context.

V.   CONCLUSION

Based on a comparative teaching approach centered on a single case study, this paper helps students develop a holistic and critical understanding of comprehensive evaluation methods. It achieves an advancement of teaching objectives in mathematical modeling courses — from "teaching methods" to "teaching thinking."

ACKNOWLEDGMENT

REFERENCES

[1]   Si S K, Sun Z L. Algorithms and Applications of Mathematical Modeling (3rd Edition) . Beijing: National Defense Industry Press, 2021.
[2]   Ouyang Z D. Exploration of flipped classroom teaching mode in the "Mathematical Modeling" course. *Journal of Hunan First Normal University,* 2018, 18(05): 77–79.
[3]   Wang L, Hu S Q, Zhou H B, et al. Exploration of teaching reform of the "MATLAB and Its Applications" course based on mathematical modeling competitions. *Education and Teaching Forum*, 2025, (13): 97–100.
[4]   Qi J, Sun L P. Design and practice of blended teaching in mathematical modeling courses. *Education Informatization Forum*, 2022, (06): 30–32.