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# The Mediating Role of Geometric Reasoning in Geometric Thinking and Proof Construction

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***Abstract*— Geometric proof construction represents a critical yet persistent challenge in secondary mathematics education. This study investigated the interrelationships among students' geometric thinking levels, geometric reasoning, and proof construction performance, with particular emphasis on the mediating role of geometric reasoning in the relationship between geometric thinking and proof construction. Using a descriptive-correlational design with qualitative integration, data were collected from 50 Grade 8 students through the Van Hiele Geometry Test, the CDASSG Proof Test, and a researcher-developed Geometric Reasoning Rubric. Quantitative data were analyzed using descriptive statistics, correlation analysis, and mediation analysis (PROCESS Model 4), while students' written proofs were subjected to thematic analysis to examine reasoning patterns across Van Hiele levels. Results revealed that most students were functioning at the lower Van Hiele levels, with significant differences in geometric reasoning and proof construction across levels. Strong positive correlations were found among geometric thinking, geometric reasoning, and proof construction. Mediation analysis demonstrated that geometric reasoning fully mediated the effect of geometric thinking on proof construction, indicating that students' thinking levels influenced proof performance primarily through their reasoning processes. Qualitative findings corroborate these results, showing a progression from visually driven and fragmented reasoning at lower levels to more structured, theorem-based, and coherent reasoning at higher levels.*

***Keywords*— Geometric reasoning, geometric thinking, proof construction, Van Hiele model, mediation analysis, mathematics education.*

## I. INTRODUCTION

### *1. 1 Background of the Study*

Geometry is a central domain of mathematics that develops essential cognitive abilities such as spatial visualization, logical thinking, problem solving, and deductive reasoning. It serves as a foundation for learning in science, technology, engineering, and mathematics (Seah, 2015). Beyond recognizing shapes and properties, the ultimate instructional goal of geometry is to enable students to construct formal, logically valid proofs (Seah & Horne, 2020). Proof construction represents the highest level of mathematical reasoning, where students must organize definitions, theorems, and relationships into coherent deductive arguments.

However, research consistently shows that students experience significant difficulty in transitioning from understanding geometric concepts to producing formal proofs. International studies and classroom-based research reveal that many learners can identify properties and recall theorems but struggle to justify relationships, link statements logically, and construct complete deductive arguments (Miyazaki et al., 2017; Ramirez-Ucles et al., 2022). These difficulties are often associated with weak logical sequencing, misinterpretation of geometric relationships, and reliance on visual or empirical justification rather than formal deduction (Mwadzaangati, 2019). As a result, students frequently fail to move beyond partial arguments or procedural imitation toward rigorous proof construction.

The Van Hiele Model of Geometric Thinking provides a well-established framework for understanding this progression. The model describes hierarchical levels of geometric understanding, from visualization (Level 0) and analysis (Level 1), through informal deduction (Level 2), to formal deduction (Level 3) and rigor (Level 4). Formal proof construction is theoretically situated at Van Hiele Level 3. However, numerous studies indicate that a large proportion of students remain at Levels 0–2 even when formal proof is already introduced in the curriculum (Margaretha et al., 2019; Amidu & Nyarko, 2019). This mismatch between students' geometric thinking levels and instructional demands contributes to persistent failure in proof tasks.

While Van Hiele levels describe what students understand about geometry, they do not fully explain how students use this understanding during proof construction. This gap highlights the importance of geometric reasoning. Geometric reasoning refers to the cognitive processes involved in selecting relevant information, identifying relationships, planning solution paths, and justifying statements logically (Ramirez-Ucles et al., 2022). It is the active mechanism through which students transform their geometric knowledge into structured arguments. In other words, geometric reasoning is the functional bridge between geometric thinking and proof construction.

From this perspective, geometric thinking provides the conceptual foundation, geometric reasoning governs the quality of cognitive processing, and proof construction is the observable outcome. A student's Van Hiele level shapes the type of reasoning they can perform, while the quality of that reasoning directly determines their ability to construct valid proofs. Yet, despite its theoretical importance, the mediating role of geometric reasoning between geometric thinking and proof construction remains empirically underexplored.

This study seeks to address this gap by investigating how geometric reasoning mediates the relationship between students' geometric thinking levels and their proof construction performance. By examining this pathway, the study aims to clarify not only whether geometric thinking influences proof construction, but how this influence operates through reasoning processes. Understanding this mechanism is crucial for designing instruction that does not merely teach properties and



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theorems but deliberately develops the reasoning skills necessary for successful proof construction.

*1.2 Statement of the Problem*

Proof construction is a central objective of geometry education and a critical indicator of advanced mathematical thinking. Despite prolonged exposure to geometric concepts, many students struggle to construct logically coherent and complete proofs. These difficulties are commonly manifested in fragmented arguments, incorrect application of theorems, weak logical sequencing, and reliance on visual or empirical justification rather than deductive reasoning. Such patterns indicate that possessing geometric knowledge alone does not guarantee the ability to engage in formal proof.

The Van Hiele Model of Geometric Thinking offers a structured explanation of how students' understanding of geometry develops, progressing from visualization to formal deduction. Although numerous studies have established a relationship between students' Van Hiele levels and their performance in proof-related tasks, this relationship remains largely descriptive. There is limited empirical evidence explaining **how** students' geometric thinking levels are transformed into actual proof construction ability. In particular, the cognitive processes that enable or constrain this transformation remain insufficiently examined.

Geometric reasoning, which involves selecting relevant properties, identifying relationships, planning solution pathways, and justifying statements logically, is hypothesized to be the key mechanism through which geometric thinking influences proof construction. However, existing literature has not adequately investigated geometric reasoning as a mediating variable in this relationship. As a result, instructional practices often focus on advancing students' geometric knowledge without directly addressing the reasoning processes required to convert that knowledge into valid proofs.

The problem addressed in this study is the lack of empirical clarity regarding the mediating role of geometric reasoning in the relationship between students' geometric thinking levels and their proof construction performance. Specifically, this study seeks to determine whether students' Van Hiele levels influence their quality of geometric reasoning, and whether this reasoning, in turn, significantly predicts their ability to construct proofs. By establishing this mediating pathway, the study aims to provide a deeper understanding of the cognitive structure underlying proof construction, thereby informing more targeted, effective, and developmentally appropriate approaches to teaching geometry and proof.

*1.3 Objectives of the Study*

The study will focus on the following objectives:

1. Determine students' geometric thinking levels based on the Van Hiele model.
2. Investigate the intercorrelation among geometric thinking levels, geometric reasoning, and proof construction.
3. Examine the mediating role of geometric reasoning on the relationship between geometric thinking level and proof construction.

*1.4 Significance of the Study*

This study holds significant importance by clarifying the underlying mechanisms that govern the development of advanced mathematical skills. Specifically, by empirically establishing the mediating role of geometric reasoning in the relationship between geometric thinking levels (Van Hiele) and proof construction ability, this study contributes a critical piece of knowledge to the field of mathematics education. The resulting data will move beyond simple correlation, providing evidence-based insights into *how* students translate conceptual understanding into formal proof competence. The findings of this research are anticipated to benefit the following stakeholders:

Students. The outcomes will help students recognize that success in proving is not just about memorizing theorems but about mastering the underlying geometric reasoning process. By highlighting the specific points where reasoning breaks down across the Van Hiele levels, the study can encourage students to focus their self-directed efforts on developing stronger logical deduction and structured argumentation skills.

Teachers and Educators. Mathematics educators will gain concrete, actionable evidence regarding the specific challenges students face at each cognitive stage. This information will be invaluable for creating targeted instructional strategies and interventions that are precisely aligned with students' current geometric thinking levels, thereby facilitating a more efficient and effective progression through the Van Hiele levels toward formal deduction.

Curriculum Developers. The results will provide the necessary evidence to advocate for adjustments in geometry curricula. Curriculum designers can use the data to ensure that the content and sequencing of lessons are better aligned with the observed developmental sequence of geometric reasoning, making the transition from informal deduction to formal proof less abrupt and more systematic.

Researchers. This study offers a new theoretical model by testing the specific mediation hypothesis involving three key constructs. This framework can serve as a strong foundation for future research, including studies that explore the effectiveness of new instructional technologies, pedagogical interventions, or comparative analyses across different educational systems.

Educational Policymakers. The findings offer a compelling case for policy decisions related to teacher training and assessment standards in geometry. By emphasizing the critical link that geometric reasoning provides, policymakers can support initiatives that prioritize the development of deductive reasoning skills in teacher preparation programs and national learning objectives.

This study directly contributes to bridging the observed gap in proof competency by providing robust, evidence-based recommendations aimed at enhancing the teaching and learning of rigorous mathematical proof construction.

*1.5 Scope and Limitations of the Study*

This study focused on examining the influence of students' geometric thinking on their geometric reasoning in proof construction. Specifically, it aimed to determine students' geometric thinking levels based on the Van Hiele model and analyzed how these levels affected their ability to apply logical


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deduction, utilize geometric theorems, and ensure accuracy in constructing proofs.

The study was conducted among Grade 8 students from a selected school. The participants were assessed to determine their geometric thinking levels, and their reasoning skills in proof construction were evaluated based on structured problem-solving tasks. The study focused on Euclidean geometry, particularly on fundamental geometric concepts and theorems commonly taught in Grade 8.

However, this study had certain limitations. First, the sample size was limited to a single school and a specific number of Grade 8 students, which may have affected the generalizability of the findings to a broader student population. Second, the study primarily assessed students' geometric reasoning through written assessments and problem-solving tasks, which may not have fully captured other factors influencing their proof construction abilities, such as motivation, prior knowledge, and classroom learning experiences. Third, the study did not examine the impact of instructional interventions but instead focused on analyzing the existing geometric thinking levels of students and their reasoning abilities. Lastly, although the geometric reasoning rubric demonstrated excellent inter-rater reliability, it had not undergone large-scale construct validation. Future studies may further establish its validity through factor analysis or criterion-related validity.

Despite these limitations, the study aimed to provide meaningful insights into the relationship between students' geometric thinking, geometric reasoning, and their proof construction skills. The findings could serve as a foundation for further research and instructional improvements in teaching geometry.

### *1.6 Theoretical Framework*

This study is supported by Van Hiele's Theory of Geometric Thought, this model describes how students develop geometric understanding through a series of hierarchical levels. Developed by Pierre and Dina van Hiele, this model provides a framework for understanding how learners progress from recognizing shapes visually to reasoning about them in a formal, abstract manner. The model consists of five levels, and progression through these levels is influenced by the quality of instruction and learning experiences rather than age alone. Each level represents a distinct way of thinking about geometric concepts, with higher levels building upon the understanding developed in the previous stages (Yalley et al., 2021; Tao & Fu, 2024; Nahdi et al., 2024).

At Level 0: Visualization (Recognition), students recognize geometric shapes and objects based on their overall appearance rather than their defining attributes (Nahdi et al., 2024; MdYunus et al., 2019). They can identify and name common figures such as triangles, squares, and circles but do not yet understand their formal properties. For example, a student at this stage might say a shape is a triangle simply because it "looks like one," without considering that a triangle must have exactly three sides. Their understanding is largely intuitive and visual, meaning that differences in orientation, size, or proportions may lead to confusion. Students at this level often rely on prototypes—idealized images of shapes—rather than recognizing geometric figures based on their essential properties.

As students advance to Level 1: Analysis (Descriptive), they begin to understand and describe the properties of shapes in a more structured manner (Nahdi et al., 2024; MdYunus et al., 2019). At this stage, they can identify characteristics such as the number of sides and angles, symmetry, and parallel or perpendicular relationships. They also begin to classify shapes based on their properties rather than their overall appearance. For example, they may recognize that all squares have four equal sides and right angles, but they may not yet understand how different properties relate to one another. While they can list attributes, they do not yet grasp the logical connections between these properties, such as why all rectangles with equal-length sides must also be squares.

At Level 2: Informal Deduction (Relational), students develop a deeper understanding of geometric properties and their interrelationships. They can logically order these properties and use them to make informal arguments about geometric concepts (Tao & Fu, 2024; Nahdi et al., 2024). For instance, they might recognize that a square is a special type of rectangle because it possesses all the defining properties of a rectangle while also having equal sides. At this level, students begin to understand why certain properties hold true and can explain their reasoning, but they do not yet engage in formal, structured proofs. Instead, they rely on informal justifications and reasoning based on observed patterns and logical intuition. This level marks a significant transition from recognizing and describing shapes to developing an ability to reason about them more abstractly.

By Level 3: Formal Deduction, students are capable of constructing formal geometric proofs using axioms, theorems, and deductive reasoning. They understand the structure of mathematical arguments and can follow logical sequences in proofs (Tao & Fu, 2024; Armah & Kissi, 2019). At this stage, students engage with more abstract reasoning and begin to see geometry as a system based on definitions and logical rules. They can prove theorems, derive new conclusions from known facts, and justify their reasoning systematically. This level is typically expected in high school geometry courses, where students are introduced to two-column proofs, direct and indirect reasoning, and logical arguments supported by postulates and previously established theorems.

Finally, at Level 4: Rigor, students reach the highest stage of geometric thinking, where they can understand geometry in a fully abstract and formal mathematical system (Tao & Fu, 2024). They can compare and analyze different axiomatic systems, critically evaluate the foundations of geometry, and work with more complex and generalized mathematical structures. At this stage, students move beyond simply proving theorems to understanding why geometric systems are structured the way they are. They recognize that theorems are derived from axioms and can appreciate different approaches to geometric reasoning. This level is typically reached by advanced mathematics students, such as those studying university-level geometry or engaging in research in mathematical logic and proof theory.


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The Van Hiele model explains how students progress through levels of geometric thinking, influencing their ability to construct proofs. Students advance sequentially, moving from recognizing shapes (Level 0) to understanding properties (Levels 1 and 2) and eventually engaging in formal deductive reasoning (Levels 3 and 4) (Margaretha et al., 2019; Matematika, 2024). Without mastering the concepts at lower levels, students struggle with higher-order proof construction. Many face difficulties transitioning from informal deduction (Level 2) to formal proof-writing (Level 3) due to gaps in logical reasoning and theorem application (Tao & Fu, 2024; Mason, 2009).

Students at Level 3 can follow logical arguments and apply theorems, while those at Level 4 develop an axiomatic understanding and explore multiple proof methods (Senk, 1989). However, lower-level students may still improve with structured instruction, reinforcing the need for effective teaching strategies. Research shows that teachers' own Van Hiele levels influence student learning, as those with higher levels provide better scaffolding for proof-writing (Lumbre et al., 2023).

Geometric reasoning is a progressive cognitive process that develops as students advance through the Van Hiele levels of geometric thinking. The ability to construct logical arguments, apply geometric theorems, and ensure accuracy in proof construction is directly influenced by a student's current Van Hiele level. As students progress from recognizing shapes (Level 0) to understanding their properties (Levels 1 and 2) and eventually engaging in formal deductive reasoning (Levels 3 and 4), their geometric reasoning skills evolve accordingly. This section explores how students' Van Hiele levels influence their logical deduction, theorem application, and proof construction, which are key components of geometric reasoning.

Logical deduction is a fundamental aspect of geometric reasoning, enabling students to derive valid conclusions from given information. At Level 2 (Informal Deduction), students begin to recognize logical connections between geometric properties, but they may struggle to justify their reasoning formally. They can follow simple argument structures but often rely on intuitive reasoning rather than fully structured logic. As students reach Level 3 (Deduction), they develop a deeper understanding of proof structures, allowing them to use definitions, postulates, and theorems to justify conclusions systematically. At this stage, students are expected to construct well-organized proofs, ensuring that each step follows logically from the previous one. Those who remain at lower Van Hiele levels may find it difficult to differentiate between assumptions and proven facts, leading to flawed reasoning and incomplete proofs. The transition to Level 4 (Rigor) enables students to evaluate and construct multiple proof strategies, demonstrating higher-order logical deduction skills.

The ability to apply geometric theorems effectively is another key indicator of geometric reasoning. At Level 1 (Analysis), students can identify basic geometric properties but may lack the ability to apply theorems systematically in problem-solving. As they advance to Level 2 (Informal Deduction), they start recognizing how different theorems relate to each other, allowing them to justify geometric relationships conceptually. However, they may still rely on memorization rather than deep understanding when applying theorems. By the time students reach Level 3 (Deduction), they are capable of correctly selecting and applying theorems within structured proofs. They not only recognize when a theorem is applicable but also justify its use in a logical sequence. At Level 4 (Rigor), students gain an even deeper understanding, allowing them to explore alternative theorems and proof methods, demonstrating a more flexible approach to theorem application.

Proof construction is the culmination of geometric reasoning, requiring students to integrate logical deduction, theorem application, and structured argumentation. At Level 2, students may attempt proofs but often include gaps in reasoning or unjustified assumptions. They may be able to provide partial justifications but struggle to present a fully developed logical argument. As students reach Level 3 (Deduction), their ability to construct coherent, step-by-step proofs improves significantly. They understand the importance of logical sequencing and justification, ensuring that all necessary steps are included. Those who reach Level 4 (Rigor) can analyze, critique, and refine proofs, identifying alternative proof structures and ensuring logical accuracy.

The Van Hiele levels provide a structured progression for the development of geometric reasoning, influencing students' abilities in logical deduction, theorem application, and proof accuracy. Students at higher Van Hiele levels (Levels 3 and 4) are more proficient in proof construction, as they can logically sequence arguments, apply theorems correctly, and ensure deductive accuracy. However, many students struggle to transition from informal deduction (Level 2) to formal deduction (Level 3), leading to challenges in proof-writing and logical justification. Understanding how students' geometric thinking influences their reasoning skills can help educators design instructional strategies that support students in advancing through the Van Hiele hierarchy, ultimately improving their proof-writing and overall mathematical reasoning abilities.

The Van Hiele model provides a structured framework for assessing students' geometric thinking levels, helping to identify where they are in their reasoning development and what challenges they face in advancing to higher levels. Understanding how students' geometric thinking levels relate to logical deduction, theorem application, and proof accuracy allows educators to design more effective instructional interventions tailored to their needs. This study will apply the Van Hiele Model to analyze students' geometric reasoning development, identify gaps in their proof-writing abilities, and propose evidence-based strategies to enhance their logical deduction and proof construction skills.

### *1.7 Conceptual Framework*

The study anchored in the relationship between geometric thinking levels, geometric reasoning, and proof construction abilities, where geometric reasoning serves as a mediating factor. In this framework, geometric thinking levels are identified as the independent variable (IV), representing students' cognitive development in geometry as described by




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the Van Hiele model. These levels determine how students understand and process geometric concepts, which in turn influence their reasoning skills.

Geometric reasoning functions as the mediating variable (MV), bridging the connection between geometric thinking and proof construction. It involves the ability to apply logical deduction, recognize patterns, utilize geometric theorems, and construct justifications in solving geometric problems. Since reasoning is embedded in proof-writing, students' ability to logically structure proofs and avoid misconceptions provides insight into their reasoning skills.

The dependent variable (DV) in this study is proof construction abilities, which refer to students' capacity to develop mathematically sound and logically coherent proofs. Effective proof construction requires not only an understanding of geometric concepts but also the ability to apply reasoning skills systematically.

This framework suggests that geometric thinking levels influence proof construction abilities through the mediating role of geometric reasoning. Understanding this interrelationship provides deeper insights into how students develop proof-writing skills and highlights the role of reasoning in bridging geometric thought and formal proof construction.

*Figure 1*

Conceptual Framework

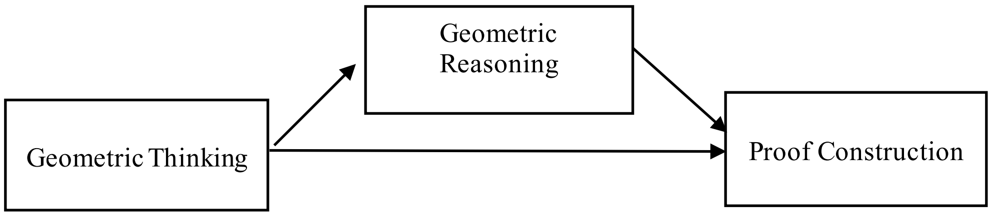


Figure 1. Conceptual Framework

### *1.8 Operational Definition of Terms*

To ensure clarity and consistency in this study, key terms related to reasoning skills in geometric proofs are defined based on their operational meaning within the research. These definitions provide a clear understanding of how each concept is measured and analyzed, ensuring that interpretations remain precise and aligned with the study's objectives.

Accuracy in Proof Construction – is defined as the extent to which a student's proof is mathematically valid, free from errors, and correctly structured. This includes proper notation, correct logical flow, and the absence of misconceptions or invalid reasoning. It is measured using a scoring rubric that assesses the precision and correctness of the provided proof.

Application of Geometric Theorems – refers to students' ability to correctly identify, select, and apply relevant geometric theorems, postulates, and definitions in the process of proof construction. It is evaluated based on the appropriateness, accuracy, and justification of theorem usage within their proofs.

Logical Deduction – refers to students' ability to move from one statement to the next in a structured and justified manner when constructing geometric proofs. It is assessed based on the correctness and coherence of reasoning steps, including the proper use of conditional statements, valid inferences, and logical sequencing of ideas.

Geometric Thinking – refers to the level of students' understanding and reasoning in geometry based on the Van Hiele Model. It is measured through their ability to recognize, analyze, and logically deduce geometric properties, relationships, and structures.

Geometric Proofs – are structured logical arguments used to verify the truth of geometric statements. In this study, they are assessed based on students' ability to use definitions, axioms, and theorems to construct step-by-step explanations in proving geometric relationships.

Geometric Reasoning – refers to students' measurable ability to analyze geometric relationships, make logical inferences, and construct justifications in proof writing. It is assessed based on the correctness, coherence, and logical structure of their reasoning within proofs.

Proof Construction – refers to the process by which students develop a formal, step-by-step justification of a geometric statement or theorem. It involves logical deduction, the application of geometric theorems, and accuracy in reasoning to establish a valid mathematical argument. This ability is assessed based on completeness, coherence, and correctness of the proof.

## II. Review or Related Literature and Studies

This chapter explores the relationship between geometric thinking and geometric reasoning in proof construction. It provides an in-depth discussion of the Van Hiele model, which describes students' progression through different levels of geometric understanding. Studies highlight how logical deduction, application of geometric theorems, and accuracy in proof construction are key factors in students' ability to write valid proofs. Furthermore, research identifies instructional strategies, such as technology integration and structured interventions, that support students' progression in geometric reasoning.

### *2.1 Related Literature*

#### *2.1.1 Geometric Thinking and the Van Hiele Model*

The Van Hiele Theory of Geometric Thinking, developed by Pierre and Dina Van Hiele in the 1950s, outlines five hierarchical levels of geometric understanding: Visualization,



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Analysis, Informal Deduction, Deduction, and Rigor (Naufal et al., 2021). These levels represent a progression of geometric thinking, with each level building upon the prior one. Importantly, students must master each level before advancing, and this progression depends on prior reasoning experiences rather than age alone (Hassan et al., 2020). At the Visualization (Level 0) stage, students recognize shapes based on their appearance but do not understand their properties. In the Analysis (Level 1) stage, students begin to recognize properties of shapes but are still unable to connect them or understand their relationships. At Informal Deduction (Level 2), students start to make informal connections between properties and justify their reasoning. Deduction (Level 3) introduces formal proof construction using axioms and theorems, while Rigor (Level 4) involves abstract reasoning and advanced proof strategies.

The transition between these levels requires structured instruction, as students often struggle to move forward without appropriate support. Instruction focused on memorization can hinder this progress, leaving students stagnating at lower levels. Conversely, discovery-based learning has been shown to help students progress more effectively by promoting deeper conceptual understanding (Rizki et al., 2018). Effective teaching strategies must align with students' current levels and promote reasoning-based learning. For example, structured reasoning tasks can help students at the Informal Deduction stage practice making informal justifications, while tasks that require formal proof construction can challenge those at the Deduction stage. Technology tools, such as GeoGebra and Augmented Reality (AR), offer dynamic, interactive experiences that make abstract geometric concepts more tangible and accessible, facilitating progression to higher reasoning levels (Palacio, 2021; Faizah et al., 2024). Additionally, tasks like mental rotation and 3D modeling help develop spatial reasoning and foster the move from analysis to deduction (Dimla & Soriano, 2019).

Curricula aligned with the Van Hiele model support gradual, concept-based development, emphasizing reasoning over rote memorization (Yalley et al., 2021; MdYunus et al., 2019). However, an overemphasis on procedural knowledge can limit students' ability to construct logical arguments and proofs, particularly at the Informal Deduction level. In this regard, interactive and exploratory learning experiences are essential to help students develop advanced geometric reasoning skills.

Empirical research underscores the importance of aligning instruction with the Van Hiele model. Studies show that many students, particularly those at lower levels, struggle to move beyond Informal Deduction due to a lack of structured interventions and strategies (Mayberry, 1983). The National Council of Teachers of Mathematics (NCTM, 2000) advocates for instructional practices that incorporate dynamic geometry software, guided discovery, and hands-on activities to facilitate students' progression through these levels effectively. In line with the recommendation, dynamic geometry software such as Geometer's Sketchpad enhances students' geometric understanding by allowing them to interactively explore geometric properties, making abstract concepts more accessible and fostering higher-level thinking (Moyer, 2021). Furthermore, inquiry-based learning and real-world applications can support the development of higher-order reasoning, helping students see the relevance of geometry in practical contexts. Curricula that integrate geometry with problem-solving, rather than isolating it, promote better comprehension and reasoning skills (Carroll, 1998).

Reflective thinking and the development of flexible mental models are crucial for improving students' ability to solve geometric problems (Damastuti et al., 2021; Chinnappan, 1998). Moreover, the language of instruction plays a key role in geometric understanding, with students often performing better when taught in their first language (Bermejo et al., 2021). Technology tools like Google SketchUp, which support spatial reasoning and engagement, can further enhance students' learning experiences (MdYunus et al., 2019).

The Van Hiele Model provides a well-researched theoretical framework for understanding geometric thinking and guiding instructional strategies. By recognizing how students progress through these levels, educators can design targeted interventions, such as structured explorations, interactive technology, and guided discussions, to enhance students' geometric reasoning. This framework is not only instrumental in developing students' ability to construct proofs but also in fostering a deeper appreciation of geometric concepts, ultimately preparing them for higher-level mathematical thinking and real-world problem-solving.

#### *2.1.2 Influence of Geometric Thinking on Geometric Reasoning*

Geometric thinking significantly shapes students' reasoning and proof-writing abilities. According to the Van Hiele model, students progress through hierarchical levels of understanding, from basic shape recognition to formal deductive reasoning. Research shows that students at Level 3 (Informal Deduction) and above demonstrate stronger proof-writing skills due to their ability to understand geometric relationships and construct logical arguments (Tao & Fu, 2024; Mason, 2009; Senk, 1989). However, even students at lower levels can develop proof skills when provided with structured support. The progression from recognizing shapes (Visualization) to understanding their properties (Analysis) and organizing these properties logically (Informal Deduction) is essential for reaching formal deductive reasoning (Deduction) (Mason, 2009; Senk, 1989).

One of the main challenges in transitioning from geometric thinking to formal reasoning is the integration of visual, logical, and algebraic reasoning. Many students struggle with this shift due to instructional gaps and limited guidance (Tao & Fu, 2024; Ramlan, 2016). Models like Knisley's learning framework, which emphasize adaptive reasoning, can help students progress through the Van Hiele levels and improve their ability to construct proofs (Rizki et al., 2018). However, for many students, the gap between informal and formal deduction remains significant. At lower Van Hiele levels, students often have difficulty translating geometric problems into mathematical models, a critical skill in proof construction (Sulistiowati et al., 2019). Furthermore, a lack of structured argumentation and limited application of geometric theorems restrict their ability to construct coherent proofs (Ramlan, 2016; Sulistiowati et al., 2019).



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Traditional teaching methods often fail to support students' transition from informal deduction to formal deduction, which is critical in proof structuring (Tao & Fu, 2024; Armah et al., 2018). As a result, students at lower levels may produce incomplete or poorly structured proofs. Instruction that includes specific scaffolding, such as guiding students to apply axioms systematically or encouraging peer discussions on logical reasoning, can help mitigate these difficulties. Moreover, formative assessments designed to identify gaps in logical reasoning can help tailor instruction and provide the necessary guidance for students to advance.

The Van Hiele model offers valuable insights into the relationship between geometric thinking and reasoning. While higher Van Hiele levels foster more sophisticated logical deduction and theorem application, many students continue to face challenges in moving between levels, especially in constructing formal proofs. Addressing these challenges through targeted instructional practices is essential for improving students' geometric reasoning abilities.

*2.2 Related Studies*

*2.2.1 Geometric Reasoning in Proof Construction*

Geometric reasoning involves the process of analyzing geometric objects, properties, and relationships through logical and mathematical methods. It requires students to critically assess axiomatic properties, construct logical arguments, and establish new connections between geometric concepts to develop valid proofs (Seah & Horne, 2019). Proofs serve as structured justifications that verify mathematical statements based on established principles, promoting logical thinking and mathematical rigor. Beyond memorization, proof construction fosters a deep understanding of why geometric theorems hold, advancing students' problem-solving and analytical reasoning skills.

Several key aspects contribute to geometric reasoning in proof construction. Logical analysis is fundamental, as students must engage in structured argumentation using axioms, postulates, and theorems to justify their statements (Seah & Horne, 2019). Problem-solving also plays a crucial role, requiring students to apply geometric concepts to different types of problems, such as constraint problems (finding equivalent definitions) and display problems (describing an object's properties) (Arnon, 1988). Additionally, integrating multiple geometric concepts over time enables students to transition from intuitive reasoning to formal deductive thinking (Van Hiele, 1986).

Students develop proof-writing skills through progressive stages aligned with the Van Hiele model of geometric thinking. Initially, they engage with geometric concepts through visualization and pattern recognition, forming informal conjectures. As they advance, they begin to justify statements using logical arguments, though their reasoning may still be case-specific rather than generalized. At the next stage, students apply formal geometric definitions, axioms, and theorems in structured proofs, such as using triangle congruence criteria to establish relationships (Tao & Fu, 2024). Eventually, they construct rigorous proofs using various formats like two-column, paragraph, or flowchart proofs, ensuring each step is logically substantiated (Mason, 2009; Rizki et al., 2018). At the most advanced level, students engage in abstract, axiomatic reasoning, allowing them to analyze geometric systems, including Euclidean and non-Euclidean geometries (Seah & Horne, 2019). These reasoning skills extend beyond mathematics, providing a strong foundation for analytical reasoning in various academic and professional fields.

*2.2.2 Key Aspects of Geometric Reasoning in Proof Construction*

Geometric reasoning in proofs involves multiple interconnected skills that are crucial for constructing strong and valid arguments. Logical deduction enables students to progress systematically from one statement to another, ensuring consistency in their reasoning. The application of geometric theorems is equally important, requiring students to accurately select and use theorems to justify their conclusions. Additionally, precision in proof construction is vital, as clear organization and careful formulation of statements help prevent logical errors.

Despite its importance, proof construction presents significant challenges for many students. Difficulties in logical sequencing often lead to gaps in reasoning, making their arguments unclear. Justifying each step of a proof is another common struggle, as students may find it difficult to explain why a theorem or property applies in a given context. Moreover, structuring proofs in a coherent and formally acceptable manner remains a challenge, frequently resulting in incomplete or incorrect arguments.

*2.2.2.1 Logical Deduction*

Logical deduction is essential in geometric reasoning, enabling students to derive new relationships from axioms and theorems through structured argumentation. It ensures coherence in proof construction by eliminating reasoning gaps and reinforcing formal proof writing (Seah & Horne, 2019; Aisyah et al., 2023). However, developing this skill is challenging, as students often struggle with logical sequencing, misapplying rules, and making unsupported generalizations (Tao & Fu, 2024; Hamami & Morris, 2023). Many students fail to recognize the necessity of each logical step in a proof, leading to reasoning gaps that compromise the validity of their arguments. Furthermore, students at lower Van Hiele levels often rely on empirical verification (e.g., measuring angles or drawing diagrams) rather than formal deductive reasoning, which limits their ability to construct rigorous mathematical proofs. (Margaretha et al., 2019; Fitriyani et al., 2018).

The transition from informal to formal deduction is particularly challenging. Research indicates that traditional teaching methods often focus on memorization rather than reasoning, hindering students' ability to engage in deductive proof writing (Cesaria et al., 2021). Studies reveal that few students reach the deduction level, with some findings indicating that no students achieved deduction or rigor, while only those with high mathematical disposition advanced beyond analysis (Miatun et al., 2021). Factors such as inadequate exposure to logical argumentation, lack of emphasis on proof strategies, and cognitive overload contribute to these difficulties (Tao & Fu, 2024).



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Studies have shown that students need explicit instruction in proof techniques, scaffolded deductive reasoning tasks, and structured opportunities to construct and critique proofs in order to develop strong logical deduction skills (Mason, 2009; Senk, 1989). Without these instructional supports, many students remain at lower Van Hiele levels and struggle to construct coherent mathematical arguments.

*2.2.2.2 Application of Geometric Theorems*

The application of geometric theorems is a critical component of constructing valid proofs, as it provides the necessary tools for justification and argumentation. Mastery of geometric theorems enables students to establish the validity of their reasoning by referencing established results. This process extends beyond mere memorization; it requires students to recognize when and how to apply theorems effectively (Weber, 2001). This process involves the interaction of multiple cognitive skills, including logical deduction, algebraic reasoning, and visual interpretation.

Students find it difficult to determine which theorems are relevant to a given problem and how to apply them correctly. Research indicates that students frequently misinterpret diagrams, overlook necessary conditions, and fail to connect geometric relationships logically (Karpuz et al., 2020). These difficulties are often attributed to a lack of conceptual understanding and inadequate exposure to proof-based instruction (Ramirez-Ucles et al., 2022).

One approach that has been found effective in improving theorem application is the three-column proof method, which helps students structure their reasoning by explicitly linking given statements, theorem justifications, and conclusions (Hawkins, 2007). Additionally, problem-based learning and dynamic geometry software such as GeoGebra have been shown to enhance students' ability to apply theorems by allowing them to explore geometric relationships through interactive visualizations (Lin & Yang, 2007). However, despite these advancements, students continue to struggle with distinguishing between necessary and sufficient conditions, which leads to errors in theorem application and logical inconsistencies.

*2.2.2.3 Accuracy of Proof Construction*

Ensuring accuracy in proof construction is fundamental to maintaining the deductive validity of an argument. A well-constructed proof must be both logically correct and universally understandable, meaning that different individuals should be able to verify and arrive at the same conclusions when following the given steps. Accuracy in proof writing involves organizing knowledge systematically, comparing different viewpoints, and employing appropriate representations to support the reasoning process. By reinforcing accuracy in proof construction, students develop a stronger foundation in deductive reasoning and improve their ability to communicate mathematical arguments effectively (Ramirez-Ucles et al., 2022; Hohol et al., 2019; Hawkins, 2007).

Geometric reasoning in proofs is essential for constructing logical mathematical arguments. It requires understanding proof structures, factors affecting accuracy, and strategies for improvement. A well-structured proof follows a logical sequence, incorporates definitions and theorems, and maintains clarity to ensure coherence (Aphrodite & Rita, 2021; Scristia et al., 2020; Weber, 2015). However, research suggests that many students fail to connect hypotheses with conclusions, leading to flawed or incomplete proofs (Carroll, 1977; Daguplo, 2014; Haj-Yahya, 2019). Misconceptions, such as believing that auxiliary constructions are unnecessary or assuming properties that are not given, frequently result in proof errors.

To enhance proof accuracy, educators can implement structured proof exercises, guided discovery learning, and proof-reading techniques (Weber, 2015). The flow-proof method, for instance, encourages students to break proofs into logical steps and verify each component before moving forward (Scristia et al., 2020; Anwar et al., 2023). Additionally, integrating empirical validation techniques, such as using dynamic geometry software to test conjectures before formalizing them into proofs, has been shown to improve accuracy and logical coherence (Komatsu, 2017; Marrades et al., 2000). By reinforcing accuracy in proof construction, students develop a stronger foundation in deductive reasoning and improve their ability to communicate mathematical arguments effectively.

*2.2.2.4 Interrelation of Domains*

These three domains, logical deduction, theorem application, and accuracy of proof construction, are interdependent and collectively shape the development of geometric reasoning. Logical deduction provides the framework for constructing valid arguments, while the application of geometric theorems supplies the necessary tools for justification. Accuracy in proof construction ensures that these arguments are logically sound and free from inconsistencies. Together, these elements create a cohesive approach to geometric reasoning that is not only critical for educational contexts but also serves as a foundation for higher mathematical reasoning. Without a proper balance of these domains, students may encounter difficulties in constructing valid proofs, leading to incomplete or flawed reasoning (Ramirez-Ucles, 2022; Seah & Horne, 2019; Aisyah et al., 2023).

In conclusion, logical deduction, the application of geometric theorems, and accuracy in proof construction work together to develop strong geometric reasoning. Logical deduction ensures the coherence of arguments, the application of theorems provides justification for claims, and accuracy in proof construction guarantees logical correctness and clarity. The interrelation of these domains highlights their significance in fostering deep mathematical understanding and enhancing students' ability to construct rigorous and reliable proofs. By developing proficiency in these domains, students can engage in formal mathematical reasoning and successfully navigate complex geometric problems, ultimately strengthening their ability to construct rigorous and reliable proofs.

*2.2.3 Relationship Between Geometric Thinking and Proof Performance*

The relationship between geometric thinking and proof performance is a critical area of research in mathematics education, particularly in understanding how students develop formal reasoning skills. The Van Hiele levels of geometric thought provide a structured framework for analyzing this




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progression, showing how students advance from basic shape recognition to formal deductive reasoning. In the context of proof construction, the Van Hiele model helps illustrate how students' geometric thinking evolves and influences their ability to write valid geometric proofs. As students progress through the Van Hiele levels, they develop increasingly sophisticated skills in logical deduction, theorem application, and proof construction. Instructional strategies based on Van Hiele theory have been shown to significantly enhance students' reasoning and proof-writing skills, demonstrating the importance of targeted teaching approaches in geometry education (Senk, 1989; Polat et al., 2019).

International research further highlights the relationship between geometric thinking and proof construction. Chinnappan et al. (2012) examined how knowledge components such as geometry content knowledge, problem-solving skills, and geometric reasoning that collectively influence proof-writing performance among Sri Lankan students. Their findings reinforce the idea that successful proof construction depends on both conceptual understanding and logical reasoning abilities. Similarly, Anwar et al. (2023) investigated the learning trajectory of geometry proof construction among Indonesian prospective mathematics teachers. By introducing a flow-chart proof format, the study demonstrated that structured interventions significantly improved students' ability to understand logical connections within proofs. Additionally, Ramirez-Ucles et al. (2022) analyzed reasoning styles and representation methods used by eighth-grade students in solving geometric proof problems. Their findings suggested that students who employed harmonic and analytical reasoning were more successful in generalizing arguments compared to those who relied primarily on visual reasoning. Moreover, the study emphasized that transitioning from perceptual to abstract reasoning is essential for developing strong proof-writing abilities.

Several local studies have explored strategies emphasizing the role of technology to enhance students' Van Hiele levels and proof-writing abilities. Coronado (2017) examined the impact of Geometer's Sketchpad on students' geometric understanding and proof-writing skills, revealing that students progressed from abstract to deductive reasoning after using the software. Similarly, De Las Penas et al. (2023) developed the mobile app *Two Column Proof*, which provided students with a structured framework for writing proofs using visual representations, thereby strengthening their logical reasoning. Alicbusan et al. (2017) introduced *Geometry Proof Tutor*, a learning environment that allowed students to practice proof construction through multiple representations, such as proof trees. Their study indicated that while students benefited from structured feedback, challenges such as complex user interfaces hindered seamless learning. Additionally, Daguplo (2014) identified key factors affecting students' proof-writing performance, including foundational knowledge, interest, and reasoning skills. His findings emphasized the need for more structured classroom activities and increased practice opportunities to address these challenges. Collectively, these studies underscore the importance of integrating technological tools and structured learning environments to improve students' geometric reasoning and proof-writing skills.

### *2.2.4 Key Assessment Tool*

To assess the van hiele geometric thinking, educators and researchers use various tools, each with their strengths and limitations. One of the most widely recognized methods for evaluating Van Hiele levels is the Van Hiele Geometry Test. This tool provide valuable insights into students' reasoning abilities, helping educators tailor instruction to support their development in geometric thinking.

The Van Hiele Geometry Test (VHGT) is a multiple-choice assessment tool developed in the early 1980s by Zalman Usiskin and colleagues as part of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project. The test is designed to measure students' levels of geometric thinking based on the Van Hiele model, a theory that describes how students progress in geometric understanding through hierarchical levels. Since its inception, the test has been widely used in research studies across various countries to evaluate students' geometric reasoning, the effectiveness of instructional methods, and the relationship between geometric thinking and proof-writing skills. The VHGT consists of 25 multiple-choice questions, with five questions targeting each of these five levels. The original goal of the test was to investigate whether students' geometric thinking aligns with the Van Hiele model and to determine if these levels predict success in proof-writing and geometry coursework.

Numerous studies have utilized the Van Hiele Geometric Test (VHGT) to assess students' geometric thinking levels in different contexts. One common approach is classifying students according to the Van Hiele levels to understand their reasoning processes in geometry. Burger and Shaughnessy (1986) conducted clinical interviews to examine how students transition through these levels, providing insights into their cognitive development in geometric thinking. Similarly, de Villiers and Njisane (1987) employed a written test similar to the VHGT to categorize students' reasoning abilities, highlighting variations in their understanding of geometric concepts. More recently, Kim (2016) applied the VHGT to South Korean students across multiple grade levels, analyzing how their geometric comprehension evolved over time. These studies collectively demonstrate the effectiveness of the VHGT in identifying students' geometric thinking levels and tracking their cognitive progression in geometry.

Several studies have employed the Van Hiele Geometric Test (VHGT) to evaluate the effectiveness of various instructional methods in enhancing students' geometric thinking. By comparing pre- and post-test results, researchers have been able to measure the impact of specific teaching approaches on students' reasoning abilities. For instance, Nasiru, Abdullah, and Norulhuda (2019) designed instructional strategies tailored for Nigerian students, aligning their lessons with the Van Hiele learning phases. Their study demonstrated that using structured, sequential instruction significantly improved students' geometric understanding. Similarly, Yilmaz and Koparan (2016) investigated the effect of a university-level geometry teaching course on prospective teachers' geometric


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reasoning. Their findings indicated that exposure to targeted instruction helped future educators develop higher-order geometric thinking skills. These studies highlight the VHGT's role in assessing the effectiveness of teaching methods and shaping instructional strategies to enhance geometric learning.

To further assess students' Van Hiele levels and their transition to formal deductive reasoning, the CDASSG Proof Test builds on the insights provided by the VHGT. The CDASSG Proof Test was developed in the early 1980s as part of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project. It was designed to assess students' proof-writing skills and logical reasoning abilities in geometry. The test consists of proof-related tasks that require students to justify statements, construct valid arguments, and demonstrate logical progression in mathematical reasoning. By evaluating students' deductive reasoning skills, the test serves as a valuable tool for measuring their readiness for advanced geometry courses. Additionally, it aligns with the Van Hiele model, particularly focusing on students' transition from informal to formal deductive reasoning.

Several studies have employed the CDASSG Proof Test to examine students' proof construction abilities and their geometric reasoning. Senk (1985, 1989) conducted one of the earliest investigations using the test, revealing that students at higher Van Hiele levels demonstrated stronger proof-writing skills. Similarly, Usiskin (1982) explored the connection between students' cognitive development and their success in geometry, providing further evidence of the link between reasoning ability and proof construction. Mason (1998) examined the impact of different instructional methods on students' proof skills, utilizing the CDASSG Proof Test as a primary assessment tool. These studies collectively highlight the strong correlation between students' geometric reasoning, as classified by the Van Hiele model, and their ability to construct mathematical proofs. While numerous studies utilized this test in the decades following its inception, recent research explicitly employing the CDASSG Proof Test appears to be limited.

### *2.3 Synthesis*

The reviewed literature and studies emphasize the critical relationship between geometric thinking, geometric reasoning, and proof construction abilities. Research consistently demonstrates that students who struggle with logical deduction, the application of geometric theorems, and accuracy in proof construction often have difficulty progressing through the Van Hiele levels. These challenges hinder their ability to develop formal proofs, which require a structured understanding of geometric relationships and logical argumentation.

Studies on the Van Hiele model highlight that students must sequentially progress through hierarchical levels of geometric understanding, from Visualization to Rigor, to effectively engage in proof construction. However, many students stagnate at lower levels, particularly at the Informal Deduction stage, due to instructional gaps, a lack of reasoning-based tasks, and overemphasis on memorization rather than conceptual understanding. This stagnation limits their ability to construct valid proofs, as they may rely on empirical reasoning rather than formal deductive methods.

Research further suggests that geometric reasoning serves as a crucial bridge between geometric thinking and proof construction. Strong reasoning skills allow students to identify logical patterns, justify mathematical statements, and systematically apply theorems in constructing proofs. However, the transition from informal to formal reasoning remains a major obstacle for many learners. Studies indicate that students often misapply geometric theorems, struggle with sequencing logical arguments, and produce incomplete or incorrect proofs. These difficulties highlight the need for structured interventions that explicitly develop reasoning skills in proof-writing contexts.

Various instructional strategies have been explored to support students in overcoming these challenges. Studies suggest that guided discovery learning, dynamic geometry software (e.g., GeoGebra, Geometer's Sketchpad), structured proof exercises, and scaffolded reasoning tasks can enhance students' geometric thinking and proof-writing skills. These approaches encourage active exploration, visualization, and logical reasoning, fostering deeper conceptual understanding. Despite these advancements, a key gap in the literature remains: while existing studies acknowledge students' difficulties in proof construction, there is no explicit framework that directly links Van Hiele levels to specific proof-writing skills. The lack of a clear model that maps how each geometric thinking level influences specific aspects of proof construction (e.g., logical deduction, theorem application, sequencing of arguments, and justification of steps) limits the ability of educators to design targeted interventions.

Addressing this gap is essential for improving students' proof-writing performance. By establishing a clearer relationship between geometric thinking levels, reasoning abilities, and proof construction skills, future research can provide more structured teaching strategies and assessment tools. Such an approach will enable educators to align instruction with students' cognitive development in geometry, ensuring they receive the necessary support to progress through the Van Hiele levels and develop strong proof-writing skills. Ultimately, bridging this gap will contribute to a more effective learning experience in geometry, fostering students' ability to construct rigorous and logically sound mathematical proofs.

## III. Research Methodology

This chapter outlines the methodology of the study, detailing the research design, setting, participants, instruments, data gathering procedures, data analysis, and data coding and scoring. It begins with a discussion on research design, providing a framework for how the study is structured and the approach taken. The research setting and participants are described to provide context to the study, while the instruments section explains the tools used for data collection. Following that, the data gathering procedure is outlined to ensure clarity on how the information was obtained, and the data analysis section explains how the data was processed and interpreted. Finally, the data coding and scoring are discussed to explain


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how responses were categorized and analyzed to draw meaningful conclusions from the collected data.

*3.1 Research Design*

This study employed a descriptive-correlational research design with a qualitative component. This approach was selected as the most appropriate non-experimental method because it fulfills two primary research objectives (Creswell & Creswell, 2018). The descriptive aspect provides a rigorous profile of the student sample by classifying them according to their current developmental stage, thereby establishing the categorical independent variable. The correlational aspect allows for the statistical quantification of the direction and strength of the associations between the variables. While this design does not permit manipulation of the variables, it provides the essential empirical foundation needed to establish the relationships and test the hypothesized theoretical mechanism (mediation) that links the variables. The inclusion of the qualitative component serves to further substantiate the statistical findings by analyzing students' written responses to identify specific reasoning patterns and argumentation strategies across different developmental levels, offering a rich, contextualized view that supports the quantitative results.

*3.2 Research Setting*

T The research was conducted in a developmental school at a state university that offered a standard secondary geometry curriculum. The site was selected because it provided a diverse group of students who were actively learning geometric concepts and engaging in proof construction. The learning environment exposed students to both theoretical and practical aspects of geometry through regular classroom instruction.

Instruction in the setting was primarily classroom-based, with opportunities for students to participate in teacher-guided discussions, collaborative activities, and hands-on learning tasks. Classrooms were equipped with typical instructional resources such as textbooks, visual aids, and other teaching materials necessary for the teaching of geometric concepts. These conditions provided students with structured opportunities to interact with geometric ideas and develop their reasoning skills. The academic environment was therefore appropriate for examining students' geometric thinking levels, geometric reasoning processes, and proof construction performance.

Furthermore, the study was conducted within the context of a curriculum that emphasized both conceptual understanding of geometry and the development of logical reasoning skills. The instructional program was aligned with national and regional mathematics education standards, which underscored the importance of critical thinking, problem solving, and mathematical justification. These standards highlighted proof construction as a key learning outcome in geometry, requiring students to reason deductively and communicate their ideas clearly. This curricular context provided an appropriate framework for examining how students' geometric thinking levels and reasoning processes influenced their ability to construct geometric proofs.

*3.3 Research Participants*

The participants of the study were Grade 8 students enrolled in a Geometry class. These students were selected based on their active enrollment in the Geometry course, ensuring that all participants had been introduced to the basic concepts and principles of geometry necessary for engaging with geometric proofs.

The selection of Grade 8 students was based on the premise that they were at a critical stage in developing geometric reasoning and transitioning from informal to formal deductive thinking. At this level, students were expected to engage with geometric theorems, logical deduction, and proof writing, making them suitable for analyzing how Van Hiele levels influenced their reasoning skills in proof construction.

The study included participants with varying levels of geometric understanding to ensure diverse representation of geometric thinking skills. Their Van Hiele levels were assessed using a diagnostic test, and their proof-writing abilities were evaluated through structured tasks measuring logical deduction, theorem application, and proof accuracy.

In addition to the student participants, the evaluators of the proof construction tasks were mathematics teachers with expertise in geometry and experience in teaching proof. These teachers were responsible for assessing the students' proofs using structured rubric to ensure consistency, objectivity, and accuracy in scoring.

Participation in the study was voluntary, and necessary ethical considerations, such as obtaining parental consent and ensuring the confidentiality of students' responses, were strictly observed.

*3.4 Instruments Used*

This study utilized three research instruments: the Van Hiele Geometry Test, the CDASSG Proof Test, and a Geometric Reasoning Rubric. These instruments were designed to assess students' geometric thinking levels, proof-writing abilities, and reasoning strategies.

The Van Hiele Geometry Test was a diagnostic assessment tool designed to evaluate students' levels of geometric thinking based on the Van Hiele model. This model described a hierarchical progression of five levels of geometric understanding: Level 0 (Visualization), Level 1 (Analysis), Level 2 (Informal Deduction), Level 3 (Deduction), and Level 4 (Rigor). The test consisted of 25 multiple-choice questions, with five questions per level, assessing different aspects of geometric reasoning. Each question was carefully structured to determine whether a student could recognize, analyze, or logically deduce geometric properties. The test was scored using the 4-of-5 criterion, where a student was required to correctly answer a minimum of three or four questions per level to be classified at that level. The final score was calculated using a weighted sum system, assigning 1 point to Level 0 (Visualization), 2 points to Level 1 (Analysis), 4 points to Level 2 (Informal Deduction), 8 points to Level 3 (Deduction), and 16 points to Level 4 (Rigor). A student's Van Hiele level was determined by the highest level at which they met the passing criterion in sequence, ensuring that students did not skip levels. The Van Hiele Geometry Test showed moderate to high reliability, with stable student responses and a hierarchical


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structure reinforcing consistency. It also demonstrated strong construct and criterion validity, aligning with the Van Hiele theory and predicting performance in proof-based tasks.

The CDASSG (Cognitive Development and Achievement in Secondary School Geometry) Proof Test was an assessment designed to measure students' ability to construct mathematical proofs in geometry and was based on Usiskin's study. The CDASSG Proof Test consisted of six proof-based tasks that required students to apply reasoning skills to geometric problems. These tasks included various proof formats, such as fill-in-the-blank proofs, constructing given-to-prove statements, and writing full formal proofs. The study utilized all three forms of the CDASSG test, each administered on separate days to assess students' proof-writing abilities at different difficulty levels. The test was scored using Usiskin's original method, where each proof was graded on a 0–4 scale, resulting in a maximum total score of 24 points. Two key scoring metrics were used: PrfTOT (total proof score), which was the sum of all proof scores, and PrfCOR (number of correct proofs), which counted the number of proofs that scored 3 or 4 (indicating strong proficiency). The CDASSG Proof Test demonstrated high inter-rater reliability through a structured grading rubric and multiple graders, ensuring scoring consistency. It also exhibited strong construct and criterion validity, effectively measuring proof-writing skills and indicating that students with higher Van Hiele levels tended to achieve higher proof scores.

The Geometric Reasoning Rubric (GRR) was a custom-developed instrument designed specifically for this study to provide a quantifiable measure of the mediating variable: students' intermediate reasoning skills, focusing on the application of deductive logic and geometric principles. Crucially, the Geometric Reasoning Rubric was not subjected to prior external validation, nor was its internal consistency statistically confirmed through methods such as Cronbach's Alpha. As a bespoke tool, the reliability of its scores was entirely dependent on the rigor of the rating process. Therefore, to ensure the objectivity and consistency of the measurement, reliability was established solely through the computation of Inter-Rater Reliability (IRR). The scoring of the proof-based tasks was conducted independently by trained raters, and the consistency of their numerical ratings was subsequently verified using the Intraclass Correlation Coefficient (ICC). The resulting ICC value of [.984–.996] confirmed a high degree of agreement between the raters, thereby guaranteeing the procedural reliability of the GR_Total scores used as the mediator (M) in this analysis.

### *3.5 Data Gathering Procedure*

The first phase of data collection involved administering the Van Hiele Geometry Test to assess students' levels of geometric thinking. The test consisted of 25 multiple-choice questions, with five questions per Van Hiele level. Students completed the test under standard testing conditions within a single session. Their responses were scored based on the chosen classification criterion (4-of-5), and their Van Hiele levels were determined accordingly.

Following the Van Hiele Geometry Test, the CDASSG Proof Test was conducted over three consecutive days, with one form administered per day. The sequence of test administration followed a structured progression: on the first day, the easiest form was administered to help students become familiar with proof construction without undue difficulty. On the second day, the combined form, which included a mix of easy and difficult proof tasks, was given to assess students' adaptability to different proof types. On the third day, the difficult form was administered to evaluate students' ability to handle complex proof problems. Each proof test session was conducted under controlled conditions, and students' responses were scored based on Usiskin's 0–4 scale, with a maximum total score of 24 points. The PrfTOT (total proof score) and PrfCOR (number of correct proofs) were recorded for each student.

### *3.6 Data Coding and Analysis*

All data in this study were systematically coded and organized for both quantitative and qualitative analysis. Students' results from the Van Hiele Geometry Test were analyzed using the 4-of-5 criterion, which required each student to correctly answer at least four out of five items at a given level before advancing to the next. The highest level successfully achieved was recorded as the student's Van Hiele Geometric Thinking Level (VH_Level) and served as the independent variable in the quantitative analysis.

Students' geometric reasoning was measured through their responses in the CDASSG Proof Test using the Geometric Reasoning Rubric. This rubric evaluated four components: Data Interpretation and Use, Strategic Path Planning, Logical Flow and Coherence, and Justification Accuracy. Items 1 and 2, which involved simpler tasks such as completing statements and drawing figures, were scored out of five points, while the remaining items that required full proof construction were scored out of ten points. The total geometric reasoning score (GR_Total) for each student was obtained by summing all rubric-based scores and was used as the mediating variable in the analysis.

Proof construction performance was assessed using Usiskin's 0–4 scoring scale, which evaluated the accuracy, completeness, and logical validity of each proof. The total Proof Construction Score (PCTS_Tot) represented each student's overall proficiency in formal proof writing and served as the dependent variable.

To ensure reliability of scoring, all student outputs were independently evaluated by two trained raters with expertise in geometry and proof. Inter-rater agreement was established prior to finalizing the scores, and any discrepancies were resolved through discussion and consensus.

All quantitative data (VH_Level, GR_Total, and PCTS_Tot) were entered into SPSS for descriptive, correlational, and mediation analyses. Pearson correlation coefficients were computed to examine relationships among geometric thinking levels, geometric reasoning, and proof construction. Mediation analysis was then conducted to test whether geometric reasoning significantly mediated the relationship between geometric thinking and proof construction.

In addition to quantitative analysis, qualitative data from students' written proofs and responses were subjected to



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thematic analysis. This process involved careful examination of students' reasoning patterns, proof structures, and justifications to identify recurring themes related to how students at different Van Hiele levels approached proof construction. Codes were developed inductively from the data and grouped into broader themes such as reliance on visual features, fragmented reasoning, misapplication of theorems, inability to plan proof sequences, and emerging structural awareness. These themes were then aligned with Van Hiele levels and geometric reasoning components to provide deeper insight into how students' thinking and reasoning manifested in their proof attempts.

The integration of quantitative and qualitative analyses allowed for a comprehensive understanding of not only the statistical relationships among geometric thinking, geometric reasoning, and proof construction, but also the underlying cognitive patterns that explained these relationships. This mixed-method approach strengthened the interpretation of the mediation model by grounding statistical results in actual student reasoning behavior.

### *3.7 Ethical Considerations*

Prior to data collection, necessary ethical procedures were followed to ensure compliance with research standards. Ethical clearance from an institutional review board (IRB) or a research ethics committee was secured. Permission was obtained from relevant school administrators, teachers, and parents (if applicable) through formal request letters explaining the purpose and significance of the study.

To ensure the confidentiality, integrity, and security of the collected data, several measures were implemented. Informed consent was obtained from all participants, with students and their guardians (for minors) signing consent forms to confirm their understanding of the study's purpose and data privacy policies. To protect participants' identities, anonymization was applied by assigning each student a unique identification code instead of using personal information.

Data were securely stored using multiple safeguards. Physical documents, such as test papers and written proofs, were kept in a locked cabinet accessible only to the researcher, while digital files, including test scores and transcripts, were encrypted and stored on a password-protected computer. Access to the data was strictly limited to authorized personnel, specifically the researcher and academic advisor. Additionally, to prevent unauthorized access, the data were retained only for a specified period (e.g., one year) and were securely deleted or shredded after the completion of the analysis.

## IV. Results and Discussion

This chapter presents the findings on the relationships among geometric thinking levels, geometric reasoning, and proof construction. It reports the descriptive, correlational, and mediation results, interpreted within the framework of the Van Hiele Model. The analysis follows the study's objectives, examining Van Hiele level as the independent variable, geometric reasoning as the mediator, and proof construction as the dependent variable. The chapter begins with students' cognitive profiles, integrates qualitative findings, and concludes with the mediation analysis highlighting geometric reasoning as the link between geometric thinking and proof construction.

### *4.1 Students' Geometric Thinking Levels*

This section presents the distribution of students across the different Van Hiele geometric thinking levels to determine their overall stages of geometric understanding. Identifying students' levels provides a foundation for analyzing how their reasoning and proof construction abilities vary with cognitive development. Table 1 shows the frequency and percentage of students at each Van Hiele level based on their test results.

TABLE 1. *Distribution of Students Across Van Hiele Geometric Thinking Levels*

| Van Hiele Level | Level Description | Frequency (n) | Percentage (%) |
|---|---|---|---|
| 0 | Visualization | 24 | 48.0 |
| 1 | Analysis | 13 | 26.0 |
| 2 | Informal Deduction | 8 | 16.0 |
| 3 | Formal Deduction | 5 | 10.0 |
| **Total** | | **50** | **100** |

Table 1 presents the distribution of students across the Van Hiele geometric thinking levels. As shown in the table, nearly half of the students (48.0%, $n = 24$) were classified at Van Hiele Level 0 (Visualization), indicating that a substantial proportion of learners relied primarily on visual recognition of geometric figures rather than analytical reasoning. This was followed by Level 1 (Analysis), which comprised 26.0% ($n$ = 13) of the participants. Students at this level were able to identify geometric properties but had not yet demonstrated consistent logical relationships among these properties.

A smaller proportion of students reached Level 2 (Informal Deduction), accounting for 16.0% ($n$ = 8) of the sample. Learners at this level exhibited emerging abilities to connect properties and make informal logical inferences. Only 10.0% ($n$ = 5) of the students were classified at Level 3 (Formal Deduction), the level associated with the capacity to construct formal geometric proofs. No student reached Level 4 (Rigor), indicating that none had attained the most advanced stage of geometric thought. The distribution suggests that most students had not yet achieved the level of geometric thinking required for systematic and fully justified proof construction.

To further contextualize and explain these quantitative findings, students' written proof responses were examined as qualitative supporting data. The analysis of these proofs focused on identifying patterns in students' reasoning processes, use of geometric properties, logical sequencing of statements, and accuracy of justifications across the different Van Hiele levels. Rather than presenting the proofs as standalone qualitative results, selected excerpts from students' written work were used to illustrate how differences in geometric thinking levels were reflected in the structure and quality of proof construction.

Specifically, proof responses from each Van Hiele level were analyzed to demonstrate characteristic reasoning behaviors associated with that level. These qualitative descriptions explain the observed distribution in Table 1 by showing how students at lower levels relied primarily on visual



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or descriptive reasoning, while those at higher levels demonstrated increasingly structured and deductive approaches to proof construction.

*4.1.1 Proof Construction at Van Hiele Level 0 (Visualization)*

Students classified at Van Hiele Level 0 demonstrated reasoning that was predominantly grounded in visual perception and surface features of geometric figures. Their proof attempts relied heavily on the appearance, orientation, and overall shape of diagrams rather than on the interpretation of given information or the analysis of geometric properties. Instead of identifying and coordinating relationships such as midpoint, congruence, or parallelism, students tended to classify figures holistically and intuitively, treating shapes as visual objects rather than as structured systems of properties.

Although some students reproduced the external format of formal proofs and employed geometric terminology, their reasoning was characterized by procedural imitation, fragmented use of isolated facts, and absence of inferential control. Rather than exhibiting isolated mistakes, students' responses frequently reflected overlapping reliance on visual dominance, imitative use of proof structure, and inability to integrate geometric facts into coherent relational frameworks. These patterns indicate that while students at this level may recognize shapes and replicate the appearance of mathematical reasoning, they have not yet developed awareness of geometric properties as interconnected relationships that can be logically coordinated.

*Visual Dominance and Surface-Feature Reasoning*

Students operating at Van Hiele Level 0 relied primarily on the visual appearance of figures when attempting to construct proofs. Rather than interpreting diagrams through formal definitions or given information, students classified shapes and drew conclusions based on how figures looked. This theme captures reasoning dominated by perceptual cues, intuitive shape recognition, and surface features, with minimal attention to underlying geometric properties.

In several responses, students concluded that a quadrilateral was a square because it appeared "diamond-shaped" or that a triangle was isosceles because the sides seemed visually equal in the drawing. These judgments were made without reference to the given conditions or to defining properties. The visual configuration of the diagram functioned as the primary source of evidence, indicating that students perceived geometry as a study of shapes rather than as a system of relational properties.

A similar pattern was evident when students assumed perpendicularity or parallelism based solely on how lines were drawn. For example, some students asserted that an angle was a right angle because it "looked like a corner" or that two lines were parallel because they appeared not to meet. In these cases, no justification was provided from the givens or from known theorems. The reasoning was grounded in visual impression rather than in deductive inference.

In other proof attempts, students treated rotated or skewed figures as different shapes, revealing dependence on orientation. For instance, a square drawn at an angle was described as a "diamond," leading students to misclassify the figure and apply inappropriate properties. This indicates that shape recognition was tied to prototypical visual images rather than to invariant defining characteristics.

Across these responses, students rarely referred to specific geometric properties such as equal sides, right angles, or parallel lines unless those properties were visually obvious. Even when such properties were stated in the givens, students often ignored them in favor of what the diagram suggested. This demonstrates that at Level 0, perception overrides analysis: figures are interpreted holistically and intuitively rather than analytically.

These patterns reflect the Van Hiele characterization of visualization-level reasoning, where figures are recognized as wholes and properties are not yet isolated or coordinated. Students at this level do not yet perceive geometric relationships as objects of reasoning; instead, they rely on visual cues to guide their conclusions. This suggests that visualization-level reasoning constrains the development of inferential control, as students have not yet shifted from perceiving figures as visual objects to analyzing them as structured systems of properties.

*Procedural Imitation without Inferential Control*

Students at Van Hiele Level 0 frequently reproduced the external format and language of geometric proofs without demonstrating awareness of their inferential function. This theme captures instances in which students imitated the structure of two-column proofs, listed statements and reasons, and employed formal geometric terminology in a procedural manner, while the underlying reasoning remained visual, intuitive, or unsupported. The appearance of formality masked the absence of logical control.

In several responses, students wrote sequences such as "Given," "Therefore," and "In conclusion," and arranged statements into a two-column layout, yet the steps did not follow logically from one another. For example, a student wrote that a segment "bisects" another segment without identifying a point of bisection or establishing the conditions for bisection. The term *bisect* was used as a proof-like label rather than as a relational operation with specific geometric meaning. This indicates imitation of mathematical language without conceptual grounding.

A similar pattern was observed when students used terms such as "midpoint," "congruent," and "parallel" as standalone justifications. In one response, the student stated that a line was a midpoint of another line, conflating the concept of a point with that of a segment. In another, the student wrote "definition of congruence" to justify a segment equality without establishing any prior congruence relationship. These usages reflect surface familiarity with terminology but not control over their formal roles within a proof.

Some students also mimicked common proof templates, such as listing "Given → Prove → Solution," or writing "By SAS" or "By SSS" without having identified the required elements of the congruence criterion. In these cases, the structure of the proof was copied, but the logical conditions for the stated theorems were not satisfied. The students appeared to know what a proof is supposed to look like, but not how it functions as a chain of dependent inferences.



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Across these responses, reasoning was often replaced by assertion. Students wrote conclusions that visually matched the diagram or echoed the problem statement, accompanied by formal-looking justifications that were not logically connected. This suggests that proof was perceived as a procedural exercise in arranging statements rather than as an activity of establishing logical necessity.

These patterns indicate that students at Level 0 are beginning to appropriate the outward conventions of formal geometry, but their use of structure and terminology is imitative rather than inferential. Proof is treated as a format to be filled rather than as a system of reasoning to be constructed. This reinforces the characterization of Level 0 reasoning as perceptually anchored and procedurally driven rather than conceptually integrated.

*Fragmented Use of Geometric Facts without Relational Integration*

Students at Van Hiele Level 0 frequently identified isolated geometric facts but did not coordinate these facts into coherent relational structures. This theme captures proof attempts in which students listed properties, repeated given information, or cited geometric terms without establishing logical connections among them or aligning them with the proof goal. Rather than constructing a chain of reasoning, students accumulated disconnected statements that failed to advance the argument.

In several responses, students wrote down given information such as segment equalities or midpoint statements but stopped without progressing toward the conclusion. For example, a student listed that a point was the midpoint of a segment and that two sides were equal, yet did not relate these facts to any triangle structure or inferential pathway. The proof terminated after restating known information, indicating that the student did not perceive how properties could be combined to generate new conclusions.

A similar pattern was observed when students invoked congruence criteria such as SSS or SAS without identifying the required elements. In these cases, students wrote "by SSS" or "by SAS" without specifying three corresponding sides or two sides and an included angle. The criteria functioned as labels rather than as relational structures. This suggests that students recognized the names of the theorems but not the conditions under which they apply.

In other attempts, students listed multiple properties in sequence—such as midpoint, congruence, parallel lines, or angle equality—without indicating how these properties were connected. For instance, a student might state that a segment is a midpoint, that two angles are congruent, and that two lines intersect, yet never establish a triangle or a logical dependency among these statements. The result was a collection of facts rather than a structured proof.

Several students also repeated given statements verbatim as reasons for subsequent steps, creating circular or redundant reasoning. This pattern reflects difficulty in generating new information from existing facts. Instead of using properties to derive conclusions, students remained at the level of restatement, suggesting that proof was understood as recounting information rather than as producing logical consequences.

These responses demonstrate that students at Level 0 can recognize individual geometric facts but do not yet integrate them into relational systems. Properties are treated as independent pieces of information rather than as components of an interconnected structure. The absence of coordination prevents students from constructing inferential pathways from given to conclusion.

The three Level 0 themes, visual dominance, procedural imitation, and fragmented use of geometric facts, form a single interconnected pattern of reasoning. Students relied primarily on the appearance of figures, attempted to legitimize conclusions by imitating proof formats, and listed isolated facts without integrating them into relational structures. These patterns indicate that students at this level do not yet perceive geometry as a system of interconnected properties governed by logical necessity. Instead, reasoning remains perceptual, imitative, and structurally disconnected, which prevents the construction of coherent inferential pathways from given information to conclusions.

*4.1.2 Proof Construction at Van Hiele Level 1 (Analysis)*

Students classified at Van Hiele Level 1 demonstrated analytical awareness of geometric properties such as midpoint, angle congruence, parallelism, and triangle congruence structures. Their proof attempts reflected the ability to identify and name geometric relationships within figures rather than relying solely on visual appearance. However, their reasoning was characterized by limited control over the coordination, prioritization, and formal articulation of these properties. Instead of isolated errors, students' responses frequently exhibited overlapping breakdowns in semantic interpretation, structural relevance recognition, symbolic execution, and deductive sequencing. These patterns indicate that while students at this level can analyze geometric components, they have not yet developed stable control over deductive integration.

*Semantic and Representational Control Breakdown*

Students operating at the analytical level demonstrated awareness of geometric properties and relationships; however, their proof constructions frequently revealed instability in the interpretation of given information and in the formal representation of geometric ideas. This theme captures breakdowns in semantic control, where students distorted or overgeneralized the meaning of given statements, and in representational control, where students struggled to accurately encode relationships using standard geometric symbols and language. Rather than reflecting lack of knowledge, these errors indicate difficulty in controlling the formal language and symbolic grammar of proof.

In one response, the student attempted to prove that $FM = ME$ by treating the given statement "$EF$ contains $M$" as equivalent to "$M$ is the midpoint of $EF$." This semantic distortion led the student to assume segment congruence without justification. Although the student had correctly identified angle relationships and recognized a valid triangle congruence structure, the misinterpretation of the term *contains* as *midpoint* resulted in an unsupported conclusion. This illustrates how goal-directed reasoning can override accurate



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interpretation of given information when semantic control is weak.

A similar breakdown was evident in another proof where the student correctly invoked the Alternate Interior Angles Theorem but symbolically represented the relationship using the triangle symbol (Δ) instead of the angle symbol (∠). While the conceptual reasoning was appropriate and structurally necessary for establishing similarity, the incorrect symbolic encoding disrupted the formal articulation of the argument. The reasoning itself was present, but the representational instability prevented the proof from being deductively coherent.

In several responses, students also cited the "definition of congruence" to justify corresponding parts of congruent triangles, rather than using the principle of CPCTC. This pattern reflects imprecise control over formal mathematical language, where terms are used interchangeably without attention to their specific logical roles. Such symbolic imprecision did not stem from random guessing but from unstable differentiation between definitions, properties, and theorems.

These examples indicate that students at Van Hiele Level 1 possess emerging analytical awareness of geometric relationships but lack stable control over the semantic interpretation of given information and the symbolic representation of those relationships. The breakdown is not conceptual in nature; rather, it reflects difficulty in managing the formal language of geometry and in maintaining precision in the expression of deductive reasoning. This suggests that control over geometric language and symbolism is foundational to successful proof construction and that instability at this level contributes to broader breakdowns in structural and deductive reasoning. Thus, semantic and representational control breakdown should be understood as a core characteristic of analytical-level reasoning rather than as an isolated error pattern.

*Structural Misalignment and Failure to Recognize Relational Relevance*

Students at Van Hiele Level 1 frequently demonstrated the ability to identify multiple geometric relationships within a figure; however, their proof constructions revealed difficulty in discerning which of these relationships were structurally necessary for the given proof goals. This theme captures instances in which students selected correct but functionally irrelevant properties, neglected critical relationships, or failed to align chosen relationships with the structural requirements of the conclusion. Rather than reflecting lack of knowledge, these patterns indicate breakdowns in relational filtering and hierarchy recognition.

In one proof attempt, the student correctly identified that $\angle CDB \cong \angle ABD$; however, this angle pair was not structurally relevant to establishing the triangle congruence needed to justify the target conclusion. At the same time, the student failed to recognize the necessity of the vertical angle pair at the intersection point, which was critical for completing the congruence configuration. Although the identified relationship was mathematically valid, it did not contribute to the deductive pathway of the proof. This reflects inappropriate relational selection despite accurate local reasoning.

A similar misalignment was evident in a response where the student used the congruence of segments $DE \cong DF$ to justify that $\triangle ABC$ is isosceles. While the student correctly recognized a pair of equal segments, the relationship pertained to a different triangle and did not address the defining requirement of equal sides in $\triangle ABC$. The student demonstrated awareness of congruence but failed to coordinate the relationship with the structural demands of the proof goal, indicating weak goal alignment.

In another case, the student identified alternate interior angles correctly but did not connect these angles to the corresponding sides required to establish triangle congruence. Instead, the reasoning stalled after listing angle relationships, suggesting difficulty in integrating recognized properties into a coherent structural framework. The student's attention remained at the level of isolated relationships rather than at the level of relational function within the proof.

Across several responses, students also focused on visually salient features, such as midpoint locations or intersecting lines, without establishing how these features contributed to the deductive structure of the argument. For example, students often highlighted midpoint information even when the proof goal required demonstrating side equality in a different triangle. This indicates an overreliance on noticeable features of the diagram rather than on structurally necessary relationships.

These patterns demonstrate that students at the analytical level can recognize multiple properties but lack control over their structural relevance. The difficulty lies not in identifying relationships, but in filtering, prioritizing, and coordinating those relationships in service of a specific proof objective.

*Deductive Control and Proof Sequencing Breakdown*

While students at Van Hiele Level 1 demonstrated analytical awareness of geometric properties and relationships, their proof constructions frequently revealed breakdowns in deductive control and logical sequencing. This theme captures instances in which students assembled relevant components of a proof but failed to coordinate them into a logically coherent and complete deductive chain. These breakdowns manifested as premature conclusions, fabricated rules, misapplication of theorems, and skipping of essential inferential steps. Rather than indicating absence of reasoning, these patterns reflect unstable control over the deductive grammar of proof.

In several responses, students prematurely concluded triangle congruence or similarity after establishing only one angle pair or a single segment relationship. For example, one student identified vertical angles correctly and immediately concluded that the triangles were congruent or similar, without establishing additional required conditions. This indicates difficulty in evaluating the sufficiency of conditions needed for a given theorem and reflects overestimation of the deductive power of isolated relationships.

A similar pattern was evident when students attempted to use special triangle classifications, such as the 45–45–90 triangle, to justify right-angle conclusions. In these cases, students first established an isosceles relationship and then invoked the 45–45–90 triangle classification to assert that an angle was right. This reasoning is deductively circular, as the classification presupposes the presence of a right angle. The



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students recognized a structural feature but lacked control over the logical directionality of the inference.

In other responses, students fabricated informal rules from partial understanding of geometric properties. For instance, one student asserted that "whenever two lines meet at a midpoint, they always have equal length or equal angles," using this invented rule to justify segment and angle congruence. Although the student correctly identified the midpoint and angle relationships, the invented generalization indicates an attempt to bridge gaps in deductive reasoning through unsupported inference. This reflects emerging analytical awareness without stable deductive discipline.

Several students also assembled nearly complete congruence configurations but failed to articulate or sequence the reasoning explicitly. In such cases, students identified midpoint relationships, vertical angles, and equal segments but moved directly to the conclusion of triangle congruence without naming or justifying the specific congruence criterion. The deductive structure was implicitly present, but the lack of formal articulation disrupted the logical integrity of the proof. This indicates intuitive structural assembly without explicit deductive control.

Across these responses, students frequently skipped intermediate inferential steps, moved from local observations to global conclusions, or substituted assumptions for logical inferences. These patterns suggest that students at this level are in transition: they can analyze geometric components and recognize potential proof pathways, but they do not yet control the sequencing and justification required for formal deductive reasoning.

The three Level 1 themes, semantic and representational control breakdown, structural misalignment and failure to recognize relational relevance, and deductive control and proof sequencing breakdown, form an interconnected pattern of reasoning. Students demonstrated awareness of geometric properties but struggled to interpret given information accurately, select structurally relevant relationships, and coordinate these relationships into coherent deductive sequences. These overlapping difficulties indicate that while analytical awareness is present, control over deductive integration remains unstable, resulting in proofs that are conceptually informed but logically incomplete.

*4.1.3 Proof Construction at Van Hiele Level 2 (Informal Deduction)*

Students classified at Van Hiele Level 2 approached geometric proof with emerging deductive intent, demonstrating the ability to plan multi-step arguments, coordinate multiple geometric relationships, and target global structural conclusions. Unlike students at Levels 0 and 1, whose reasoning was dominated by visual perception or isolated property analysis, students at this level exhibited awareness of logical dependency and attempted to construct chains of reasoning that moved from given information toward a conclusion. Their proof attempts reflected an understanding that geometric relationships must be justified through connected inferences rather than through appearance or isolated facts.

*Emerging deduction with imprecise theorem use and correspondence control*

Students classified at Van Hiele Level 2 demonstrated emerging deductive coordination, characterized by the ability to plan multi step arguments, integrate multiple geometric relationships, and target global structural conclusions such as triangle congruence, similarity, and isosceles structure. Across the sample proofs, students correctly identified relevant properties including midpoint relationships, angle congruence, parallelism, vertical angles, and alternate interior angles, and selected appropriate deductive strategies such as SAS, ASA, CPCTC, and similarity criteria. This indicates that the underlying deductive structure was conceptually present.

However, their reasoning was consistently marked by informal articulation and unstable control over logical structure. Students frequently assumed that relationships were self-evident and therefore omitted explicit verification of theorem conditions, correspondence mapping, and logical dependencies. Theorems were often misnamed or vaguely referenced, such as using "definition of parallelogram" to justify side congruence, "corresponding angles" in place of alternate interior angles, or "opposite angles" instead of vertical angles. In several cases, students included non-essential properties, indicating difficulty filtering necessary from unnecessary information within deductive structures. Logical direction errors were also observed, such as using equal base angles to justify an isosceles triangle or stating that similar polygons have congruent angles instead of using angle congruence to establish similarity.

These patterns indicate that while students at this level are no longer operating visually or procedurally and have moved into genuine deductive reasoning, their control over formal articulation, theorem identification, correspondence alignment, and logical direction remains informal and unstable. Deduction is present in structure but not yet disciplined in expression, which is characteristic of Van Hiele Level 2 reasoning.

*4.1.4 Proof Construction at Van Hiele Level 3 (Formal Deduction)*

Students classified at Van Hiele Level 3 approached geometric proof with stabilized deductive reasoning, demonstrating the ability to plan and execute logically sequenced arguments grounded in formal geometric relationships. Unlike students at Level 2, whose deduction remained informal and compressed, students at this level exhibited control over inferential dependency and the coordination of multiple properties toward global conclusions. Their proofs reflected an understanding of geometry as a logical system governed by theorems, definitions, and postulates rather than as a collection of isolated facts. The following section examines the patterns that characterize formal deductive reasoning at this level.

*Stabilized Formal Deduction with Minor Articulation and Justification Attribution Lapses*

Across the analyzed sample proofs, students consistently constructed multi-step deductive chains that moved systematically from given information to targeted conclusions such as triangle congruence, similarity, parallelism, and isosceles structure. They appropriately used midpoint properties to establish segment congruence, identified vertical angles, selected suitable triangle congruence criteria such as SAS, ASA, and SSS, and applied CPCTC to derive



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corresponding angle or side relationships. In several samples, students also introduced transversals and used alternate interior angle relationships to justify parallelism, reflecting awareness of the necessary logical pathway for establishing line relationships.

The sample proofs further showed that students at this level were able to coordinate multiple geometric relationships within a single argument. For example, students combined midpoint reasoning with vertical angles and side congruence to construct SAS configurations, or integrated angle congruence with side relationships to justify ASA congruence. They demonstrated control over inferential dependency by correctly sequencing statements such that triangle congruence preceded the use of CPCTC, and angle congruence preceded conclusions about isosceles structure or parallelism. These patterns indicate that students no longer relied on visual appearance or isolated property recognition but instead operated within a formal deductive framework where relationships were treated as logically dependent.

However, despite this stabilized deductive structure, minor lapses in formal articulation and justification attribution were observed. In several samples, students misattributed the source of angle congruence to the SAS postulate rather than to corresponding parts of congruent triangles, reversed the logical direction of definitions by attributing segment congruence to the definition of congruence instead of to midpoint properties, or compressed multiple givens into single statements without explicit separation. Some students also applied the alternate interior angle relationship without explicitly invoking its converse when concluding parallelism, or used informal phrasing such as "same" in place of "congruent." These issues did not reflect misunderstanding of the geometric relationships or breakdowns in deductive planning, but rather localized imprecision in formal expression, theorem naming, and justification labeling.

Importantly, the errors observed at this level were not structural in nature. Students selected appropriate triangle pairs, chose correct congruence criteria, and identified relevant angle relationships. Their deductive routes were conceptually sound, and their conclusions were mathematically valid. The breakdowns occurred primarily in the articulation of inferential sources and in the precision of formal language. This indicates that while deductive reasoning is stabilized at Level 3, full mastery of formal proof conventions and rigorous justification labeling is still developing.

These patterns characterize Van Hiele Level 3 reasoning as formally deductive in structure, logically sequenced, and relationally integrated, but with residual constraints in articulation precision and justification attribution. Students at this level are capable of constructing complete deductive arguments, yet their proofs may still exhibit minor formal lapses that reflect the transitional nature of early formal reasoning.

### *4.2 Intercorrelation Among Variables*

This section explores the relationships among the three main variables: Van Hiele geometric thinking level, geometric reasoning, and proof construction. Examining their intercorrelations helps determine how closely these cognitive components are related and whether higher geometric thinking levels are associated with stronger reasoning and proof performance. The results of the correlation analysis are presented in Table 2

TABLE 2. *Descriptive Statistics and Intercorrelations Among Study Variables (N=50)*

| Variable | *M* | *SD* | 1 | 2 | 3 |
|---|---|---|---|---|---|
| 1. Geometric Thinking Level | 0.88 | 1.02 | — | | |
| 2. Geometric Reasoning | 26.20 | 16.47 | .877*** | — | |
| 3. Proof Construction | 15.92 | 7.81 | .832*** | .964*** | — |

***. *Correlation at 0.001 (2-tailed)*

The correlation analysis revealed strong, positive, and statistically significant relationships among all three variables ($p < .001$). Van Hiele geometric thinking level showed a high correlation with geometric reasoning ($r = .877$) and with proof construction ($r = .832$). Additionally, geometric reasoning and proof construction exhibited a very strong correlation ($r = .964$). These findings indicate that as students progress through higher levels of geometric thinking, their reasoning and proof-construction skills improve proportionally. The strength of these correlations supports the idea that geometric reasoning acts as a cognitive link between conceptual understanding and the ability to construct formal geometric proofs.

These results are consistent with previous research highlighting the interdependence of geometric thinking, reasoning, and proof. Senk (1985, 1989) and Usiskin (1982) both found that students at higher Van Hiele levels produce more coherent and logically structured proofs, as their reasoning becomes increasingly analytical and deductive. Seah and Horne (2019) described reasoning, theorem application, and proof construction as an integrated set of abilities, where growth in one domain naturally enhances the others. Similarly, Ramirez-Ucles et al. (2022) observed that students who employ analytical reasoning outperform those who depend only on visual reasoning, demonstrating the importance of higher-level thinking in proof tasks. The current findings reinforce this body of evidence by showing that geometric reasoning serves as the central process connecting geometric understanding to proof performance, confirming its role as the bridge between comprehension and formal deductive reasoning.

### *4.4 Mediation Analysis (Geometric Reasoning as Mediator)*

A simple mediation analysis was conducted using the PROCESS macro for SPSS (Model 4; Hayes, 2025). This analysis tested whether geometric reasoning functions as a mediating variable that explains how differences in Van Hiele levels translate into variations in students' proof construction scores. The mediation model and its corresponding estimates are presented in Table 3.

The results indicated that geometric thinking level significantly predicted geometric reasoning, $a = 14.12$. In turn, geometric reasoning significantly predicted proof construction while controlling for geometric thinking level, $b = .48$. However, the direct effect of geometric thinking level on proof construction became non-significant once geometric reasoning was included in the model, $c' = -.48$, suggesting full mediation.



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The indirect effect was statistically significant, *ab* = 6.83, with a 5,000-sample bootstrap confidence interval that did not include zero (5.23 to 8.70). This indicates that geometric reasoning carries the effect of geometric thinking level to proof construction and serves as a significant mediator in the model.

TABLE 3. *Mediation Analysis of Geometric Reasoning Between Geometric Thinking Level and Proof Construction*

| | | M (Geometric Reasoning) | | | | | Y (Proof Construction) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Predictor | | *B* | *SE* | *t* | *p* | | *B* | *SE* | *t* | *p* |
| X (Geometric Thinking) | a | 14.12 | 1.12 | 12.67 | <.001 | c' | -0.48 | .61 | -.78 | .438 |
| M (Geometric Reasoning) | | — | — | — | — | b | 0.48 | .04 | 12.67 | <.001 |
| | | $R^2$ = .77 | | | | | $R^2$ = .93 | | | |
| | | $F(1, 48) = 160.40, p < .001$ | | | | | $F(2, 47) = 315.85, p < .001$ | | | |

The mediation analysis revealed that geometric reasoning fully mediated the relationship between Van Hiele geometric thinking level and proof construction, meaning that the effect of geometric thinking on proof performance operates entirely through reasoning ability. This finding substantiates the theoretical perspective articulated in Chapter 2, which posits that the Van Hiele level primarily represents a cognitive stage, while geometric reasoning functions as the operative cognitive mechanism that enables students to perform complex deductive tasks (Senk, 1989; Mason, 1998; Polat et al., 2019). The full mediation observed in the present study confirms that improvements in proof construction depend on the enhancement of reasoning skills fostered by progression through the Van Hiele levels. Consequently, these results empirically validate the claim advanced in the reviewed literature that geometric reasoning is the critical bridge connecting conceptual understanding and formal proof performance.

## V. Summary, Conclusion, And Recommendations

This chapter presents a synthesis of the major findings of the study, followed by the conclusions drawn from the quantitative and qualitative analyses. It also outlines several recommendations for instructional practice, curriculum development, and future research. The summary highlights how students' geometric thinking levels relate to their reasoning ability and proof construction skills, while the conclusions and recommendations offer insights for improving geometric learning and advancing research in this area.

### *5.1 Summary of Findings*

*1. Student's Geometric Thinking Level in Proof Construction*

Analysis of students' geometric proof responses revealed a clear distribution across the Van Hiele levels of geometric thinking, with the majority of students operating at the lower levels of visualization and analysis. This distribution indicates that most students had not yet developed stable deductive reasoning, which is consistent with the qualitative patterns observed in their proof constructions.

At Van Hiele Level 0, students' reasoning was dominated by visual perception and surface features of figures. Proof attempts relied on appearance, procedural imitation, and isolated statements without relational integration, indicating that geometry was viewed as a collection of shapes rather than as a logical system. At Van Hiele Level 1, students demonstrated awareness of geometric properties such as midpoint, angle congruence, and parallelism, but lacked stable control over interpretation, prioritization, and deductive sequencing. Properties were often identified correctly but not coherently coordinated into logical arguments.

At Van Hiele Level 2, students showed emerging deductive intent and the ability to plan multi step arguments. However, their reasoning remained informal, with frequent misnaming of theorems, assumed conditions, inclusion of non essential properties, and unstable logical direction. At Van Hiele Level 3, students exhibited stabilized deductive reasoning with coherent sequencing and controlled application of theorems. Minor lapses in articulation and justification attribution were observed, but these did not undermine the overall deductive structure.

The findings reveal a clear progression from visual and imitative reasoning at Level 0 to stabilized formal deduction at Level 3. The concentration of students at Levels 0 and 1 indicates that many learners have not yet developed deductive geometric reasoning, highlighting the need for instructional approaches that deliberately scaffold the transition from visual recognition to formal proof.

*2. Intercorrelation of Variables*

Correlation analysis further strengthened these findings, revealing strong, positive relationships among geometric thinking level, geometric reasoning, and proof construction ($p < .001$). The highest correlation was observed between geometric reasoning and proof construction, indicating that reasoning skills serve as the core component of students' ability to develop valid geometric proofs. These correlations highlight the interdependent nature of geometric cognition: students who reason effectively are also more capable of applying theorems and constructing logically consistent arguments.

*3. Geometric Reasoning as Mediator*

The mediation analysis produced the central finding of the study. Results showed that geometric reasoning fully mediated the relationship between Van Hiele geometric thinking level and proof construction ability. This means that the effect of geometric thinking level on proof-writing skills operates entirely through reasoning ability. In practical terms, students' improvement in proof construction does not stem solely from being at a higher Van Hiele level; rather, higher levels foster deeper and more structured reasoning skills, which directly support formal proof development. The qualitative data support this result, showing that as students move from visual to formal deductive thinking, their reasoning becomes more explicit, logically connected, and theorem-driven—precisely the qualities that enable successful proof construction.


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The study confirms that students' geometric understanding progresses through a developmental sequence consistent with the Van Hiele Model of Geometric Thought and that higher levels of geometric thinking correspond with greater reasoning ability and proof competence. The combined quantitative and qualitative results underscore the importance of instructional approaches that deliberately cultivate geometric reasoning through discussion, justification, and exploration. Such experiences are essential for helping students move toward advanced levels of geometric thought and for enabling them to construct rigorous mathematical proofs.

### *5.2 Conclusions*

Based on the findings above, the subsequent conclusions are drawn from this study.

1. Students' geometric reasoning develops progressively across Van Hiele levels. The study established a clear developmental progression in geometric reasoning, from visually driven and imitative thinking at Van Hiele Level 0 to more stabilized deductive reasoning at Level 3. However, most students remained at the visualization and analysis levels, indicating limited readiness for formal proof instruction.
2. Difficulties in proof construction are primarily cognitive rather than procedural. Qualitative findings showed that students' errors were rooted in weak interpretation of geometric relationships, poor coordination of properties, and limited control over logical sequencing, rather than mere lack of familiarity with proof formats. This confirms that proof failure reflects reasoning deficiencies rather than simple procedural gaps.
3. Geometric thinking, geometric reasoning, and proof construction are strongly interrelated. The significant correlations among the three variables demonstrated that higher geometric thinking levels are associated with stronger reasoning ability and better proof performance, highlighting their interdependence in the development of mathematical understanding.
4. Geometric reasoning is the critical mechanism linking geometric thinking to proof construction. Mediation analysis revealed that geometric reasoning fully mediated the relationship between geometric thinking and proof construction. This indicates that progression in Van Hiele levels influences proof performance primarily through the quality of students' reasoning processes.
5. Geometric reasoning serves as the bridge between conceptual understanding and proof competence. The study concludes that geometric reasoning is the cognitive link that transforms geometric knowledge into valid mathematical arguments. Strengthening reasoning skills is therefore essential for enabling students to move from observation and property listing toward coherent, deductive proof construction.

This study concludes that geometric reasoning plays a central mediating role in the relationship between students' geometric thinking levels and their proof construction performance. While higher Van Hiele levels are associated with better proof outcomes, progression in geometric thinking alone is not sufficient to ensure success in proof construction. Instead, it is the quality of students' geometric reasoning, specifically their ability to interpret relationships, plan logical sequences, and justify statements, that directly enables the construction of valid proofs. The findings demonstrate that geometric reasoning serves as the cognitive bridge that transforms conceptual geometric understanding into coherent deductive arguments. Thus, the study confirms that improving proof construction requires deliberate development of students' reasoning processes, not merely advancement in geometric thinking levels.

### *5.3 Recommendations*

Based on the results and conclusions of this study, the following recommendations are proposed to improve mathematics education.

1) Instruction should therefore be aligned with students' Van Hiele levels, beginning with visual exploration and property analysis before progressing to informal and formal deduction. Teachers should explicitly scaffold the transition from recognizing properties to coordinating them within logical structures by modeling reasoning pathways and emphasizing why relationships hold. Proof instruction should prioritize structural understanding and justification control over procedural imitation, with explicit attention to theorem naming and logical sequencing. Visual and dynamic tools may be used to support conceptual understanding, but these should be consistently linked to formal reasoning

2) The curriculum should be structured to reflect the developmental progression of geometric reasoning described by the Van Hiele model. Concepts and proof tasks must be sequenced from visual exploration and property analysis toward informal and formal deduction. Curriculum materials should provide gradual scaffolding that supports students' transition from recognizing properties to coordinating them within deductive structures. Emphasis should be placed on reasoning pathways rather than premature exposure to formal proof formats.

3) Teacher education programs should strengthen preservice and in service teachers' understanding of the Van Hiele levels and their implications for instruction. Teachers need to be equipped to diagnose students' geometric thinking levels and to design learning experiences that are developmentally appropriate. Training should emphasize how to scaffold deductive reasoning, teach justification explicitly, and interpret student errors as cognitive indicators rather than procedural failures.

4) Future studies should investigate instructional interventions that effectively support students' progression from analysis to informal and formal deduction. Longitudinal research is recommended to examine how geometric reasoning develops over time and how targeted teaching strategies influence this progression. Further research may also explore the relationship between geometric reasoning, proof construction, and other cognitive factors such as spatial ability and language proficiency.

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