IJSES
Volume 9, Issue 11, pp. 161-166, 2025.
International Journal of Scientific Engineering and Science
ISSN (Online): 2456-7361

Challenges in Visualizing Fractions Among Grade 4 Students

Krisha Nova L. Baquit[1], Amelia T. Buan[2], Joneil B. Medina[3]
[1,2,3]Department of Science and Mathematics, Mindanao State University, Iligan City, Philippines, 9200

Abstract*—Visualization plays a critical role in making complex mathematical ideas comprehensible to elementary students. However, the literature consistently identifies fraction as one of the least understood concepts due to the abstract nature of the part-whole relationship. Evidence suggests that students have difficulty visualizing, relating, and solving problems involving fractions in real-life situations. Hence, in developing relevant instructional designs to address the issue, an understanding of students' contextualized, nuanced challenges is imperative. Drawing on an empirical school-based Lesson Study report, a teacher interview, and a diagnostic test, this article presents a multi-method analysis of grade 4 students' challenges with fraction visualization. Qualitative data sources revealed errors and difficulties in constructing equal partitions, representing consistent wholes, and interpreting visual models. These lead to confusion in distinguishing numerators from denominators and erroneous conceptualization of fraction equivalence. The diagnostic test results further confirmed the findings that the reduction of fractions, comparison, and ordering of dissimilar fractions are not mastered. Surprisingly, finding equivalent fractions reached moderate mastery. The multi-method analysis provided triangulated, nuanced, and specific areas of difficulty that inform instructional design decision-making. This article strongly recommends applying guided, student-centered learning principles to address the identified challenges.*

***Keywords*—** *Elementary mathematics: fractions: mathematical learning difficulties: misconceptions: visualization: visual models,*

I. Introduction

Learning fractions is essential in elementary mathematics for various meaningful reasons. Bruce, Chang, Flynn, and Yearly (2013) noted that a weak understanding of fractions can prevent students from progressing to higher-level mathematics. According to Wilkins & Norton (2018), as students learn fraction concepts that go beyond seeing them only as parts of a whole, they build a strong foundation for learning other key mathematics topics like algebra, ratios, and proportions. Fractions are important not only because they serve as a foundation for understanding more complex and abstract mathematical ideas, but also because they have practical applications in daily life across a lifetime (Kamberi et al., 2022), such as measuring ingredients in cooking, buying things from the store, and comparing quantities. Therefore, understanding what fractions are and the relationships between different fractions helps students make decisions and solve everyday problems.

Although fractions are introduced as early as Grade 1 in the Philippine K to 12 Curriculum, many students continue to struggle with understanding fractions. International assessment frameworks such as TIMSS outline competencies that Filipino learners are expected to master by Grade 4, including recognizing fractions as parts of a whole or on a number line, identifying equivalent fractions, and comparing, ordering, and adding and subtracting simple fractions (Gronmo, Lindquist, Arora, & Mullis, 2013). These competencies are fully aligned with the Grade 4 Mathematics curriculum, which indicates that Filipino students should be developmentally prepared to handle such tasks (Balagtas et al., 2019). However, research consistently shows that fractions remain one of the most difficult areas for students. Studies report that items involving fractions are frequently answered incorrectly (Wijaya, 2017).

One of the important ways to build a meaningful understanding of fractions is through visualization. The use of visual models in representing fractions is important for students to understand fractions engagingly and effectively (Kamberi et al., 2022). This allows students to see the relationships for themselves and make sound judgments about fractions. According to Singh (2020), as discussed by Dienes and Perner (1999), mathematical concepts become easier for pupils to understand when they are introduced through concrete or hands-on examples. Grade 4 students' difficulties in visualizing fractions pose a significant barrier to their conceptual understanding and problem-solving skills in mathematics. Identifying the common challenges that students face in visualizing fractions is critical, as it provides evidence-based insights that educators and researchers. This insight can be used to design effective instructional strategies, interventions, and learning tools that directly address students' misconceptions and improve fraction learning outcomes.

II. Literature

In the Philippine basic education curriculum, particularly the Grade 4 level, students are expected to develop foundational skills in understanding and working with fractions. However, the Philippines has long struggled with low mathematics proficiency, as shown by poor rankings in international assessments such as Trends in International Mathematics and Science Study (TIMSS), Southeast Asia Primary Learning Metrics (SEA-PLM), and Programme for International Student Assessment (PISA). Despite efforts to improve, a significant portion of Filipino students continues to fall below the expected proficiency levels, indicating ongoing challenges in mathematics education. Among many factors contributing to these performances is the difficulty in understanding foundational concepts, including fractions.

The topic of fractions is considered a difficult subject to learn and teach. Petit et al. (2022) argue that there are several reasons why learning and teaching fractions are difficult. The core reason is that the properties of whole numbers do not necessarily apply to fractions. Experience with whole numbers can mistakenly lead to thinking that multiplication always

IJSES
Volume 9, Issue 11, pp. 161-166, 2025.
International Journal of Scientific Engineering and Science
ISSN (Online): 2456-7361

makes the number bigger and division always makes them smaller (Petit et al., 2022). However, these assumptions do not hold true when working with fractions. Also, comparing fractions, students often focus only on the numerator or only on the denominator, leading to misconceptions. For instance, when comparing 1/2 and 1/3, some students think that 1/3 is greater simply because 3 is bigger than 2. For Namkung et al. (2018), this phenomenon is called the whole-number bias, where students overgeneralize whole-number knowledge to fractions. In addition, Alkhateeb (2019) cites that students' over-reliance on rule-based procedures is hindering their ability to solve fraction comparison, operations, and interpretation. This reliance often leads to misapplication and misconceptions about procedures due to a lack of conceptual understanding (Pesen, 2008). These challenges highlight the need for instruction that deepens students' conceptual understanding rather than reliance on rules alone.

One way to address these difficulties is through visualization. Visualization is the skill and process of creating, interpreting, and using images or diagrams in the mind, on paper, or with technology to express information, explore ideas, and deepen understanding. The use of graphs, pictures, and symbols is a helpful tool to make learning easier (Kamberi et al., 2022). This is especially true for those at the concrete cognitive stage (Sirajuddin et al., 2025). Visual tools such as fraction bars make important information easier to see (Barbosa & Vale, 2021). By seeing both the part and whole in fraction models, students see the relational nature of fractions. Understanding the relational nature of fractions is important in learning fractions (McNamara & Shaughnessy, 2010)

Building strong foundations in visualizing fractions at the elementary level is important. Students at this level are still developing their ability to think abstractly. Surprisingly, despite curriculum standards indicating that fraction concepts are introduced as early as grade 1 (DepEd, 2016), many Grade 4 students still struggle to understand fractions. One possible reason for this issue is often associated with teacher practice.

Despite efforts to adapt student-centered approaches in teaching mathematics in the Philippines, there are still some teachers who retained conventional practices (Buan et al., 2021). In the conventional way of teaching, teachers teach for problem solving and overlook visualization leading to serious learning gaps. Sugiyama (2008) explained three different levels of teaching mathematics. Teachers "tell" in Level 1. They demonstrate essential mathematics (facts, concepts, procedures, and practices). In Level 2, teachers "explain". They explain the meaning of the essential mathematics. These 2 levels are lecture method approaches to teaching, which do not develop mathematical conceptual understanding among students. Meanwhile, teachers in Level 3 guide students to become independent students who discover essential mathematics. They let their "students do" the problem solving (Sugiyama, 2008). When teachers provide opportunities for students to engage in genuine problem-solving, they facilitate students' acquisition of both procedural and conceptual understanding (Selmer & Kale, 2013).

In addition, Singh (2020) suggested that students may fail to fully understand fractions because learning materials are not effectively used in the lower primary level. Even the best material has limited value if teachers are unable to effectively use them to design meaningful tasks. Additionally, they are ineffective if students also cannot engage with those tasks (Cohen et al., 2003). Hence, investigating students' understanding of fractions at earlier grades and what issues they encounter in using visualization effectively can help teachers address the issue of learning fractions.

III. METHODS

A. Research Design

This study employed Qualitative descriptive design to identify and interpret the challenges that Grade 4 students experience in visualizing fractions. The purpose of this research design is to gather information about the students' difficulties without manipulating any variables. This study will identify specific misconceptions, patterns of thinking, and common errors of students in visualizing and understanding fractions through classroom observation transcript, observational notes from the lesson study report, teacher interview, and diagnostic test results to inform decision-making process in developing relevant instructional designs.

B. Context of the Study

This study began from a school-based lesson study in an elementary school in the Philippines. A lesson study is a teacher-driven professional development model where educators collaboratively plan a "research lesson", implement it in an actual classroom, then reflect and discuss the observations (Buan et al., 2021). According to Bautista and Baniqued (2021), this cycle helps leverage classroom practices and enriches teachers' content knowledge, process skills, and beliefs. The lesson study aimed to help students recover from learning loss and strengthen their reasoning and communication skills in mathematics. The lesson study team focused on teaching operations on similar fractions to grade 4 students through problem-solving. During the lesson study implementation, it became clear that many students struggled to visualize fractions using paper strips. This affected their ability to understand and solve problems. Thus, this present study was conducted to examine more closely the challenges students face in visualizing fractions.

C. Data Sources and Research Instruments

To capture a comprehensive view of the challenges that students face in challenging fractions, the following instruments were used:

TABLE I. Research Instruments

Data Source	Description	Purpose
Lesson Study Report and Transcript	An adapted instrument that documented the collaborative planning, observation, and reflection from a lesson study	To provide evidence of the difficulties that students face in a mathematics classroom
Instrument	**Description**	**Purpose**
Teacher Interview Guide	A semi-structured interview questionnaire administered to the Grade 4 mathematics teacher.	To obtain the teacher's perspective on students' conceptual challenges, procedural errors, and difficulties in visualizing fractions.
Diagnostic Test	A 10-item diagnostic test on fractions focusing on equivalent fractions, comparing, ordering and reducing fractions.	To identify which fraction skills are most difficult and how these relate to challenges in visual representation.

International Journal of Scientific Engineering and Science
Volume 9, Issue 11, pp. 161-166, 2025. ISSN (Online): 2456-7361

D. Research Participants

The respondents of the study who took the diagnostic test were 23 Grade 4 students for pilot testing and 70 Grade 4 students for the implementation at a public elementary school in Iligan City, Philippines. The test was conducted in the 4th quarter grading period of the school year 2024-2025. Part of the participants of this study is the Grade 4 mathematics teacher at the school, who was interviewed to gain insights into her classroom practice in teaching fractions, and to identify the issues and challenges in teaching fractions.

E. Data Analysis

The data analysis in this study involved a combination of qualitative and quantitative methods. In this study, qualitative data was used to gain insights into the challenges students face in visualizing fractions through thematic analysis of data from the lesson study report and transcript, teacher interview, and diagnostic test. A structured six-step process was implemented to analyze data: familiarization, coding, generating themes, reviewing themes, defining and naming themes, and reporting the findings. Quantitative data was also used to complement and support qualitative insights by providing measurable evidence. The use of both data ensured a comprehensive understanding of the challenges in visualizing fractions.

IV. Results and Discussion

This section presents and discusses the findings of the study which explored the challenges that Grade 4 students face in visualizing fractions. The data were obtained through a thematic analysis from the lesson study report and transcript, teacher interview, and outputs from the diagnostic test results. Descriptive statistics from the diagnostic test was also administered.

A. Thematic Analysis on the Qualitative Sources

The analysis of the lesson study report, teacher interview, and students written outputs in the diagnostic test provided a fuller understanding of the challenges in learning and teaching fractions. Across three data sources, the following themes are as follows:

Theme 1: Conceptual Misunderstandings

The lesson study showed specific misconceptions that students have about fractions. One student believed that the numerator represented the shaded part, while the denominator represented the unshaded part, which showed confusion in identifying fraction elements. Although this was observed only once, it reflects underlying gaps in students' conceptual knowledge that may contribute to broader difficulties in visualization and reasoning. The results of the present study are aligned with the findings of Bermejo & Yu (2023). In their study, students were to name the shaded squares from the whole. The correct answer was 5/9; however, there were students who assumed that the fraction in the model is 5/4, seeing that the number of shaded squares is 5, while the number of unshaded squares is 4.

Interview data included the teacher's statement that students "*are hard to understand the different fractions or part and whole relationships" (MT1)* with equivalence as a specific example. The teacher reported that students are confused in recognizing that 2/4 is equivalent to ½, with students saying that 2/4 is bigger, showing the lack of recognition in fraction equivalence. Such a challenge is also observed in students' answers in the diagnostic test. In the diagnostic test, visual models representing 6/9 and 2/3 are given, and yet only 44% (n=70) was able to interpret that the two models are equal. Lesson study observers confirmed that some students failed to connect fractions with visual models, showing a weak understanding of the part-whole relationship.

Theme 2: Lack of Representational Understanding

Another common error that occurred across three sources are unequal wholes and uneven partitions in representing fractions using visual models. In the lesson study report, when students used paper strips to model fractions during addition tasks, instead of folding each strip into the same number of equal parts, they tore off or cut excess paper to force the strip to fit the required numerator and denominator. This led to inconsistent "wholes" across different fractions, revealing that students did not fully understand that all fractions being compared or combined must refer to the same whole. Although folding and coloring activities were generally engaging, several students produced uneven partitions, particularly with odd denominators, further reflecting conceptual gaps in visualizing fractions accurately. This is supported by some of the students' responses in the diagnostic test, as shown in Figure I.

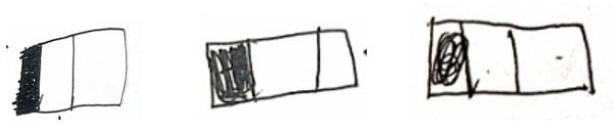

Fig. I. Representational Errors

In representing 1/3, each student partitioned the whole into three parts; however, the parts were not equal in size. This uneven partitioning results in a flawed visuals as they do not accurately communicate the fraction's value. These students failed to recognize that fractions represent equal shares of a whole, which lead to students being confused in ordering the fractions correctly. The lesson study report and transcript showed that students often give answers by applying memorized rules without visual basis, the lesson study observer recorded that answers "came out suddenly" and were not derived from visualization. Observers also noted that *"No reasoning on the side of the students"* and noted that visualization was not used as a basis for reasoning in many student responses. These observations were documented as evidence of a disconnection between students' visual work and their verbal or written explanations. The teacher also explicitly described students as *"more on memorized rules"* in the interview. Together, these results indicate that students' errors are not only procedural but also conceptual and require targeted instruction to clarify foundational ideas.

Theme 3: Accuracy and appropriateness of chosen visual models, instructional choices, and task design implications

Attached in the lesson study report are the initial and final versions of the lesson plan used for classroom implementation. As observed in the development of the lesson plan, there is a switch from circle (pie) representations to rectangle representations. The lesson study team justified the change by

International Journal of Scientific Engineering and Science
Volume 9, Issue 11, pp. 161-166, 2025. ISSN (Online): 2456-7361

noting that rectangles are easier to partition into equal parts than circles, improving representational accuracy. The lesson plan modification as an instructional choice was motivated by the potential struggle that students will face with partitioning circles equally.

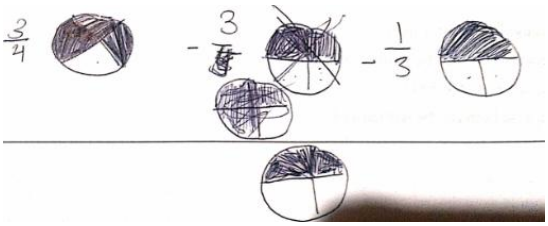

Fig. II. Accuracy and Appropriateness of Visual Models

Figure II shows the student's output in representing a fraction using circles. As observed, the parts of the whole are not equal, predominantly visible in their representation of 1/3. Instead of dividing the circle into three equal parts, the students first divided the circle into two halves, then divided one half into two again. Thus, creating unequal parts, with one larger section and 2 smaller sections rather than three equal thirds. The same issue is observed in the lesson plan from the lesson study report, where one part of the whole is bigger than the others, which indicates unequal partitioning. As cited in the study of Ubah (2021), the use of circles in representing fractions can be problematic for students, especially with odd denominators.

The findings from the two sources show that the challenges that students face in learning fractions stem from conceptual misunderstandings, difficulties in visualizing fractions, and the accuracy and appropriateness of visual models. According to NCTM (2020), to build students' understanding of fractions, they need to learn the conventional models or representations used for fractions and the meaning these visuals illustrate.

B. Descriptive Analysis of the Diagnostic Test Results

The analysis of student performance on the 10-item diagnostic test focused on fraction skills among 70 grade 4 students. The test assessed various competencies in visualizing, comparing, and interpreting fractions represented in multiple forms such as number lines, fraction bars, pictorial models, and word problems.

To evaluate the overall performance of the students, the raw scores were compiled and analyzed using descriptive statistics. To evaluate the overall performance of the students, the raw scores were compiled and analyzed using descriptive statistics. The results are shown in Table II.

TABLE II. Diagnostic Test Results.

Statistics	Mean	Standard Deviation	Variance	Minimum Score	Maximum Score	Range
Value	3.67	1.75	3.05	1	9	8

The result of the diagnostic test shows that the students performed below the expected proficiency level, with an overall mean average (M = 3.16 , SD = 1.75). This indicates that most grade 4 students were able to answer only a few items correctly. This reflects limited understanding of the fraction concepts being assessed, particularly on equivalent fractions, reducing fractions to lowest terms, comparing dissimilar fractions, and ordering dissimilar fractions highlighting the role of visualization. The poorest performance was reflected in a minimum score of 1, while the maximum score was 9, which yields a range of 8. This wide range signifies notable diversity in performance levels, and the low clustering of scores around the mean also highlights significant learning gaps.

These results point toward a consistent challenge among learners in visualizing fractions, especially in making meaning from visual representations of fraction. Given the role of visual representation in deepening conceptual understanding in mathematics, these results highlight the need for targeted instructional interventions. Students should be encouraged to use a variety of problem-solving strategies, particularly using visual methods, as using visual models greatly helps in tackling different types of mathematical problems (Barbosa & Vale, 2021).

Although the summary statistics offered a general overview of students' overall performance, they did not indicate which aspects of fraction understanding were most challenging. To obtain more detailed information about specific fraction skills where students struggled or excelled, a skill-based item analysis was conducted using the diagnostic test results. The percentage of correct responses for each fraction skill was calculated to determine areas of proficiency and difficulty in visualizing fractions. This analysis facilitated the identification of the particular skills that presented the greatest challenges for students, consistent with the study's objective of identifying difficulties in visualizing fractions.

TABLE III. Diagnostic Test Results by Skill Area

Skill Area	Items	Percentage of Students with Correct Responses (%)	Description of Mastery
Equivalent fractions	4, 5, 7	50.0 %	Moderate mastery
Reducing fractions to lowest terms	6	15.7 %	Very low mastery
Comparing dissimilar fractions	2, 3, 10	20. 9 %	Very low mastery
Ordering dissimilar fractions	1, 8, 9	47.1 %	Low mastery

Mastery level scale:
80% – 100% = High mastery | 60% - 79% = Average mastery
40% - 59% = Low mastery | Blow 40% = Very low mastery

The results of the skill-based item analysis reveal that students demonstrated the lowest performance in reducing fractions to lowest terms (15.7%) and comparing dissimilar fractions (20.9%), which indicates very low mastery in these areas. Meanwhile, ordering dissimilar fractions (47.1%) and equivalent fractions (50.0%) showed relatively better but still limited understanding, as they fall within the low to moderate mastery levels. This suggests that students experience greater challenges in visualizing fractions and manipulating dissimilar fractions.

The three sources (Lesson study report, teacher interview, and diagnostic test) consistently point to weak visualization of fractions among Grade 4 students. While students can perform rule-based steps, they lack the conceptual understanding needed to reason visually about fraction concepts.

V. Conclusion and Recommendations

This study examined the challenges that Grade 4 students face in visualizing fractions by synthesizing the evidence from the lesson study report, teacher interview, and diagnostic tests. Across all data sources, it showed that students show weak conceptual understanding of fractions and lack of representational understanding in constructing accurate visual models and interpreting part-whole relationships.

Students demonstrated persistent difficulties in the key ideas in the construct of fractions. They also demonstrated issues with partitioning the whole into equal parts, and connecting visual models to symbolic procedures. There are also misconceptions such as misunderstanding the meaning of numerators and denominators, confusing fraction equivalence and using inconsistent wholes in showing fractions. These misconceptions revealed that students rely heavily on procedural steps without understanding the underlying concepts. As cited in Singh (2020), Orhoun (2007) argued that using visual models to teach fractions enhances students' reasoning skills more effectively than rules-based teaching, which tends to have only short-term benefits.

The diagnostic test further confirmed these challenges, as shown by very low to moderately low scores. These scores indicate that there exists a substantial learning gap, variability in prior knowledge, and a need for stronger foundations in understanding fractions using visual models.

Together, the findings show that students' struggles arise from a combination of conceptual gaps and limited visualization skills. To improve students' understanding of fractions, it requires a shift toward meaningful visual strategies and concept-building approaches rather than procedural teaching alone. Based on the identified challenges, the following recommendations are proposed:

1. Teachers should provide structured opportunities for students to construct, interpret, and compare visual fraction models (e.g., strips, area models, and number lines). Instruction must emphasize equal partitions, consistent wholes, and show connections between visual and symbolic representations.
2. Instruction should incorporate guided discussions, error analysis, and reflective questioning that specifically target common misconceptions.
4. Given the low mastery in reducing and comparing dissimilar fractions, teachers should implement targeted remediation sessions that integrate visualization and problem-solving strategies.
6. Student-centered and problem-solving approaches facilitate students' transition from mechanical procedures to deeper reasoning, supported by visual representations.
7. Regular formative assessments should be conducted to monitor student progress and identify persistent errors.
8. Future studies should evaluate the effectiveness of targeted instructional interventions utilizing visual representations to determine which strategies significantly enhance students' ability to visualize and understand fractions.

Acknowledgment

The authors wish to acknowledge Mindanao State University - Iligan Institute of Technology and the Department of Science and Technology, Science Education Institute, through the Lesson Study Project 2023, which provided the authors with the opportunity to work with the teachers at the local school and facilitated the conceptualization of this research. Thank you to all those who supported and contributed to the completion of this study. Special thanks to the co-authors, mentors, and participants of this study. All glory and thanks be to God.

References

[1] Alkhateeb, M. A. (2019). Common errors in fractions and the thinking strategies that accompany them. *International Journal of Instruction*, *12*(2), 399–416.
[2] Balagtas, M. U., Garcia, D. C. B., & Ngo, D. C. (2019). Looking through Philippine's K to 12 curriculum in mathematics and science vis-a-vis TIMSS 2015 assessment framework. *EURASIA Journal of Mathematics, Science and Technology Education*, *15*(12), em1788.
[3] Barbosa, A., & Vale, I. (2021). A visual approach for solving problems with fractions. *Education Sciences*, *11*(11), 727.
[4] Bautista, R. G., & Baniqued, W. B. (2021). From competition to collaboration: Unraveling teachers' lesson study experiences. *International Journal of Evaluation and Research in Education*, *10*(3), 921–929.
[5] Bermejo, J. L. A., & Vistro-Yu, C. P. (2023). "It's not just the language": Grade 4 pupils' understanding of fractions. [This reference is incomplete; it is missing publication information, so it is presented as a standalone work].
[6] Buan, A. T., Medina, J. B., & Liwanag, G. P. (2021). Capacity building in teaching mathematics through problem solving. In *Journal of Physics: Conference Series* (Vol. 1835, No. 1, p. 012090). IOP Publishing.
[7] Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, *25*(2), 119–142.
[8] Department of Education. (2016). *K to 12 curriculum guide: Mathematics (Grade 1 to Grade 10)*. https://depedbohol.org/v2/wp-content/uploads/2016/03/Math-CG_with-tagged-math-equipment.pdf
[9] Gronmo, L. S., Lindquist, M., Arora, A., & Mullis, I. V. S. (2013). TIMSS 2015 mathematics framework. In I. V. S. Mullis & M. Martin (Eds.), *TIMSS 2015 assessment frameworks* (pp. 11–24). TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College, and International Association for the Evaluation of Educational Achievement (IEA).
[10] Ismail, S. A. S., Maat, S. M., & Khalid, F. (2024). 35 years of fraction learning: Integrating systematic review and bibliometric analysis on a global scale. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(12), em2543.
[11] Kamberi, S., Latifi, I., Rexhepi, S., & Iseni, E. (2022). The influence of practical illustrations on the meaning and operation of fractions in sixth grade students, Kosovo-curricula. *International Electronic Journal of Mathematics Education*, *17*(4), em0717.
[12] Kor, L. K., Teoh, S. H., Mohamed, S. S. E. B., & Singh, P. (2018). Learning to make sense of fractions: Some insights from the Malaysian primary 4 pupils. *International Electronic Journal of Mathematics Education*, *14*(1), 169–182.
[13] McNamara, J., & Shaughnessy, M. (2010). *Beyond pizzas & pies*. Math Solutions.
[14] Medina, J. B., Luib, J. T., & Liwanag, G. P. (2024). *Lesson study report for Grade 4*. [This reference is incomplete; it is missing publication information, so it is presented as a standalone work].
[15] Namkung, J. M., Fuchs, L. S., & Koziol, N. (2018). Does initial learning about the meaning of fractions present similar challenges for students with and without adequate whole-number skill? *Learning and Individual Differences*, *61*, 151–157.
[16] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Author.
[17] Pesen, C. (2008). Students' learning difficulties and misconceptions in pointing the fractions on the number line. *Journal of Inonu University Faculty of Education*, *9*(15), 157–168.
[18] Petit, M. M., Laird, R. E., Ebby, C. B., & Marsden, E. L. (2022). *A focus on fractions: Bringing mathematics education research to the classroom*. Routledge.
[19] Selmer, S., & Kale, U. (2013). Teaching mathematics through problem solving. *Innovación Educativa (México, DF)*, *13*(62), 45–59.

[20] Singh, K. K. (2020). *Effectiveness of virtual manipulative teaching method in enhancing year four pupils' understanding of fractions* [Master's thesis, University of Malaya (Malaysia)]. University of Malaya Repository.

[21] Sirajuddin, S., Akib, I. L., & Nasrun, N. (2025). Visualizing fractions: Enhancing problem-solving performance through diagrammatic reasoning in elementary mathematics. *Jurnal Penelitian dan Pengkajian Ilmu Pendidikan: e-Saintika*, *9*(2), 512–524. https://doi.org/10.36312/e-saintika.v9i2.3139

[22] Sugiyama, Y. (2008). *Introduction to elementary mathematics education*. Toyokan Publishing Co.

[23] Wijaya, A. (2017). The difficulties of Indonesian fourth graders in learning fractions: An early exploration of TIMSS 2015 results. *American Institute of Physics*. https://doi.org/10.1063/1.4995154

[24] Wilkins, J. L., & Norton, A. (2018). Learning progression toward a measurement concept of fractions. *International Journal of STEM Education*, *5*(1), 27.