International Journal of Scientific Engineering and Science
Volume 9, Issue 3, pp. 15-19, 2025. ISSN (Online): 2456-7361

Difference Method for Time Fractional Order Slow Diffusion Equations

Yinghong Zhang[1], Qingzhu Wang[2]
[1,2]School of Mathematical Sciences, Guizhou Normal University, Guiyang 550025, China

Abstract— *For the time fractional order slow diffusion equation system, the time fractional derivative is approximated using the L1 scheme, and a stable and convergent difference scheme is established. Theoretical analysis is given, and the results of the theoretical analysis are verified through numerical examples.*

Keywords— *The L1 scheme, stability, convergence, numerical examples.*

I. Introduction

Fractional calculus has appeared in various scientific fields and has been successfully applied in signal processing, anomalous diffusion, physics and Engineering (see, for example, [1-5]). Fractional differential equations can describe various complex physical and mechanical behaviors, and many phenomena in life can also be described by fractional differential equations, such as blood alcohol concentration, videotape problem, world population growth, so it is of practical significance to solve its numerical solution. The exact solutions of many fractional differential equations can not be obtained accurately, so the numerical algorithm of fractional differential equations has attracted much attention. At present, the numerical algorithm research of fractional differential equations has made some progress and has been gradually applied to mechanics, viscoelasticity [6], biology [7], simulation of fluid flow [8] and other different fields.

There are not many methods for solving fractional differential equations. For example, Zhou and dai[9] constructed Legendre spectral collocation method for nonlinear fractional differential equation coupling system; Huseynov i t, ahmadova a, Fernandez a, et al. [10] considered the two-dimensional coupled system of linear fractional differential equations with Caputo derivative and the corresponding non-homogeneous system; Zhou and Xu [11] proposed a high-order scheme for the numerical solution of fodes; Nabil t. [12] established the existence and uniqueness of solutions for nonlinear coupled systems of implicit fractional differential equations containing ψ - Caputo fractional order operators under nonlocal conditions; Zaky and Ameen [13] constructed Legendre Jacobi collocation method to solve nonlinear fractional differential equations of two-point boundary value problems with fractional derivative order at most two; [14] end value problems for nonlinear systems of fractional differential equations, etc.

The outline of the paper is as follows: in Section 2, we describe the system of time fractional slow diffusion equation. Then in Section 3, we give its establishment of differential format. In Section 4 and 5, we give the stability and convergence analysis. In Section 6, we use some numerical examples to verify the feasibility of the method. In Section 7 gives some concluding remarks.

II. Preliminary knowledge

2.1 Consider the following system of time fractional slow diffusion equation

$$\begin{cases} {}_0^C D_t^{\alpha} u(x,t) = \dfrac{\partial^2 u}{\partial x^2}(x,t) + f(x,t,u(t),v(t)),\ x\in(0,L),\ x\in(0,T] \\ {}_0^C D_t^{\beta} v(x,t) = \dfrac{\partial^2 v}{\partial x^2}(x,t) + g(x,t,u(t),v(t)),\ x\in(0,L),\ x\in(0,T] \end{cases}, \quad (1)$$

where the initial boundary conditions are as follows

$$\begin{cases} u(x,0)=\varphi_1(x),\ u(0,t)=\mu_1(t),\ u(L,t)=\psi_1(t) \\ v(x,0)=\varphi_2(x),\ v(0,t)=\mu_2(t),\ v(L,t)=\psi_2(t) \end{cases}, \quad (2)$$

where $0<\alpha,\beta<1$, f, g, φ_i, μ_i, ψ_i, $i=1,2$.

2.2 $L1$ Interpolation approximation of Caputo fractional derivative

For Caputo derivative of order $\alpha(0<\alpha<1)$:

$$ {}_0^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)}\int_a^t \frac{f'(\tau)}{(t-\tau)^{\alpha}}d\tau ,$$

using the piecewise linear interpolation $L1$ approximation.

Let $\tau = T/N,\ t_k = k\tau,\ k=0,1,\dots,N$,

where is N a positive integer, the following notation is introduced:

$$a_l^{(\alpha)} = (l+1)^{1-\alpha} - l^{1-\alpha},\quad l\ge 0. \quad (3)$$

So for Caputo fractional derivative, there are

$$\begin{aligned} {}_0^C D_t^{\alpha} f(t)\big|_{t=t_n} &= \frac{1}{\Gamma(1-\alpha)}\int_a^{t_n}\frac{f'(s)}{(t_n-s)^{\alpha}}ds \\ &= \frac{1}{\Gamma(1-\alpha)}\sum_{k=1}^{n}\int_{t_{k-1}}^{t_k}\frac{f'(s)}{(t_n-s)^{\alpha}}ds \end{aligned}.$$

Performing linear interpolation on $f(s)$ in the interval $[t_{k-1},t_k]$

$$L_{1,kf}(s) = \frac{t_k - s}{\tau} f(t_{k-1}) + \frac{s - t_{k-1}}{\tau} f(t_k),$$

Volume 9, Issue 3, pp. 15-19, 2025.
ISSN (Online): 2456-7361

$$f(s)-L_{1,k}(s)=\frac{1}{2}f''(\zeta_k)(s-t_{k-1})(s-t_k),\quad s\in[t_{k-1},t_k]\quad (4)$$

where $\zeta_k=\zeta_k(s)\in(t_{k-1},t_k)$, substitute $L_{1,kf}(s)$ into ${}_0^C D_t^\alpha f(t)|_{t=t_n}$ to approximately replace $f(s)$.

$$\begin{aligned}
&{}_0^C D_t^\alpha f(t)|_{t=t_n}\\
&\approx\frac{1}{\Gamma(1-\alpha)}\sum_{k=1}^{n}\int_{t_{k-1}}^{t_k}\frac{f(t_k)-f(t_{k-1})}{\tau}\cdot\frac{1}{(t_n-s)^\alpha}ds\\
&=\frac{1}{\Gamma(1-\alpha)}\sum_{k=1}^{n}\frac{f(t_k)-f(t_{k-1})}{\tau}\int_{t_{k-1}}^{t_k}\frac{1}{(t_n-s)^\alpha}ds\\
&=\frac{1}{\Gamma(1-\alpha)}\sum_{k=1}^{n}\frac{f(t_k)-f(t_{k-1})}{\tau}\cdot\frac{1}{1-\alpha}\left[(t_n-t_{k-1})^{1-\alpha}-(t_n-t_k)^{1-\alpha}\right].\\
&=\frac{1}{\Gamma(2-\alpha)}\sum_{k=1}^{n}\left[f(t_k)-f(t_{k-1})\right]\cdot\left[(n-k+1)^{1-\alpha}-(n-k)^{1-\alpha}\right]\\
&=\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\sum_{k=1}^{n}a_{n-k}^{(\alpha)}\left[f(t_k)-f(t_{k-1})\right]\\
&=\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\left[a_0^{(\alpha)}f(t_n)-\sum_{k=1}^{n-1}(a_{n-k-1}^{(\alpha)}-a_{n-k}^{(\alpha)})f(t_k)-a_{n-1}^{(\alpha)}f(t_0)\right]
\end{aligned}$$

Thus, the approximation formula for Caputo fractional derivative at $t=t_n$ is obtained

$$D_\tau^\alpha f(t_n):=\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\left[a_0^{(\alpha)}f(t_n)-\sum_{k=1}^{n-1}(a_{n-k-1}^{(\alpha)}-a_{n-k}^{(\alpha)})f(t_k)-a_{n-1}^{(\alpha)}f(t_0)\right],\quad (5)$$

formula (5) is usually referred to as Formula $L1$.

The coefficient $a_l^{(\alpha)}$ has the following properties:

Lemma 1 assume $\alpha\in(0,1)$, $a_l^{(\alpha)}$ as defined by (3), $l=0,1,2,\ldots$, one has

(I) $1=a_0^{(\alpha)}>a_1^{(\alpha)}>a_2^{(\alpha)}>\ldots>a_l^{(\alpha)}>0;\ a_l^{(\alpha)}\to 0$, when $l\to\infty$;

(II) $(1-\alpha)l^{-\alpha}<a_{l-1}^{(\alpha)}<(1-\alpha)(l-1)^{-\alpha},\quad l\geq 1$.

Lemma 2 Set $f(t)\in C^2[t_0,t_n]$, taking $R(f(t_n))={}_0^C D_t^\alpha f(t)|_{t=t_n}-D_\tau^\alpha f(t_n)$, there are

$$R(f(t_n))\leq\frac{1}{2\Gamma(1-\alpha)}\left[\frac{1}{4}+\frac{\alpha}{(1-\alpha)(2-\alpha)}\right]\max_{t_0\leq t\leq t_n}|f''(t)|\tau^{2-\alpha}$$

Lemma 3 Let $h>0$ and c be two constants

(a) assume $g(x)\in C^4[c-h,c+h]$, where have

$$g''(c)=\frac{1}{h^2}\left[g(c+h)-2g(c)+g(c-h)\right]-\frac{h^2}{12}g^{(4)}(\varsigma_4),\quad \zeta_4\in(c-h,c+h)$$

(b) assume $g(x)\in C^6[c-h,c+h]$, one has

$$\begin{aligned}
&\frac{1}{12}\left[g''(c-h)+10g''(c)+g''(c+h)\right]\\
&=\frac{1}{h^2}\left[g(c+h)-2g(c)+g(c-h)\right]+\frac{h^4}{240}g^{(6)}(\zeta_6),\ \zeta_6\in(c-h,c+h)
\end{aligned}$$

III. ESTABLISHMENT OF DIFFERENTIAL FORMAT

Consider fractional order problems (1)-(2) at node (x_i,t_n), and obtain

$$\begin{cases}
{}_0^C D_t^\alpha u(x_i,t_n)=\dfrac{\partial^2 u}{\partial x^2}(x_i,t_n)+f(x_i,t_n,u(t_n),v(t_n)),\ 1\leq i\leq M-1,\ 1\leq n\leq N\\
{}_0^C D_t^\beta v(x_i,t_n)=\dfrac{\partial^2 v}{\partial x^2}(x_i,t_n)+g(x_i,t_n,u(t_n),v(t_n)),\ 1\leq i\leq M-1,\ 1\leq n\leq N
\end{cases}\quad (6)$$

The time fractional derivative in equation (6) is discretized using the $L1$ interpolation approximation formula, the spatial second-order derivative is discretized using the second-order center difference quotient, and then Lemma 2 and Lemma 3 are applied

$$\begin{cases}
\dfrac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\left[a_0U_i^n-\sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})U_i^k-a_{n-1}U_i^0\right]\\
=\delta_x^2U_i^n+f_i^n+(r_1)_i^n,1\leq i\leq M-1,\ 1\leq n\leq N\\
\dfrac{\tau^{-\beta}}{\Gamma(2-\beta)}\left[a_0V_i^n-\sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})V_i^k-a_{n-1}V_i^0\right]\\
=\delta_x^2V_i^n+g_i^n+(r_2)_i^n,1\leq i\leq M-1,\ 1\leq n\leq N
\end{cases},\quad (7)$$

where

$$U_i^n=U(x_i,t_n),\ f_i^n=f(x_i,t_n,U(t_n),V(t_n)),\ \delta_x^2U_i^n=\frac{U_{i+1}^n-2U_i^n+U_{i-1}^n}{h^2},$$

and there exists a normal number of c_1, c_2, which makes

$$\begin{aligned}
&|(r_1)_i^n|\leq c_1(\tau^{2-\alpha}+h^2),\quad 1\leq i\leq M-1,\ 1\leq n\leq N\\
&|(r_2)_i^n|\leq c_2(\tau^{2-\beta}+h^2),\quad 1\leq i\leq M-1,\ 1\leq n\leq N
\end{aligned}.\quad (8)$$

Note the initial boundary condition (2), there are

$$\begin{cases}
U_i^0=\varphi_1(x_i),\ U_0^n=\mu_1(t_n),\ U_M^n=\psi_1(t_n),\ 0\leq n\leq N\\
V_i^0=\varphi_2(x_i),\ V_0^n=\mu_2(t_n),\ V_M^n=\psi_2(t_n),\ 0\leq n\leq N
\end{cases}.$$

Omit the small term $(r_1)_i^n$, $(r_2)_i^n$ in equation (7), by using numerical solution u_i^n instead of exact solution U_i^n, the following difference scheme can be obtained for solving problems (1)-(2):

International Journal of Scientific Engineering and Science
Volume 9, Issue 3, pp. 15-19, 2025. ISSN (Online): 2456-7361

$$\begin{cases} \dfrac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\left[a_0u_i^n - \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})u_i^k - a_{n-1}u_i^0\right] \\ = \delta_x^2 u_i^n + f_i^n,\ 1\le i\le M-1,\ 1\le n\le N \\ \dfrac{\tau^{-\beta}}{\Gamma(2-\beta)}\left[a_0v_i^n - \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})v_i^k - a_{n-1}v_i^0\right] \\ = \delta_x^2 v_i^n + g_i^n,\ 1\le i\le M-1,\ 1\le n\le N \end{cases}, \quad (9)$$

where the initial boundary conditions are as follows

$$\begin{cases} u_i^0=\varphi_1(x_i),\ u_0^n=\mu_1(t_n),\ u_M^n=\psi_1(t_n),\ 0\le n\le N \\ v_i^0=\varphi_2(x_i),\ v_0^n=\mu_2(t_n),\ v_M^n=\psi_2(t_n),\ 0\le n\le N \end{cases}, \quad (10)$$

set

$$s_1=\tau^{\alpha}\Gamma(2-\alpha),\ s_2=\tau^{\beta}\Gamma(2-\beta),\ \lambda_1=\frac{s_1}{h^2},\ \lambda_2=\frac{s_2}{h^2}.$$

Note that the right-hand term f_i^n, g_i^n in equation (9) contains u_i^n, v_i^n, which makes it an implicit format. When calculating, construct a display algorithm, namely the estimation correction method. Firstly, when calculating the estimated value, use the previous u_i^{n-1}, v_i^{n-1} instead of u_i^n, v_i^n in f_i^n, g_i^n to calculate an estimated value u_i^p, v_i^p. Then, substitute the estimated value u_i^p, v_i^p into the right end f_i^n, g_i^n of equation (9) to calculate the correction value.

IV. STABILITY OF DIFFERENTIAL FORMATS

4.1 Auxiliary results

In order to better discuss the stability of differential formats (9)-(10), we consider the right-hand form equivalent to (1), namely

$$\begin{aligned} f(t,u(t),v(t)) &:= \eta u(x,t) \\ g(t,u(t),v(t)) &:= \mu v(x,t) \end{aligned}. \quad (11)$$

Then the difference format (9) can be rewritten as follows

$$\begin{cases} \dfrac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\left[a_0u_i^n - \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})u_i^k - a_{n-1}u_i^0\right] \\ = \delta_x^2 u_i^n + \eta u_i^n,\ 1\le i\le M-1,\ 1\le n\le N \\ \dfrac{\tau^{-\beta}}{\Gamma(2-\beta)}\left[a_0v_i^n - \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})v_i^k - a_{n-1}v_i^0\right] \\ = \delta_x^2 v_i^n + \mu v_i^n,\ 1\le i\le M-1,\ 1\le n\le N \end{cases}, \quad (12)$$

where initial boundary value conditions in equation (10).

4.2 stability analysis

Theorem 1 Let $\{u_i^n, v_i^n \mid 0\le i\le M,\ 0\le n\le N\}$ is the solution of difference schemes (12) and (10), then

$$\|u^n\|_\infty \le C\|u^0\|_\infty$$
$$\|v^n\|_\infty \le C\|v^0\|_\infty$$

Proof: First, it is proved that the numerical solution u_i^n satisfies the conclusion
equation (12) can be rewritten as follows

$$\begin{aligned} a_0u_i^n &= \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})u_i^k + a_{n-1}u_i^0 \\ &+ \lambda(u_{i-1}^n - 2u_i^n + u_{i+1}^n) + s\mu u_i^n,\ 1\le i\le M-1,\ 1\le n\le N \end{aligned},$$

namely

$$\begin{aligned} (1+2\lambda-s\mu)u_i^n &= \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})u_i^k + a_{n-1}u_i^0 \\ &+ \lambda(u_{i-1}^n + u_{i+1}^n),\ 1\le i\le M-1,\ 1\le n\le N \end{aligned}. \quad (13)$$

Let $\|u^n\|_\infty = |u_{i_n}^n|$, where $i_n\in\{1,2,\dots,M-1\}$, In equation (13), let $i=i_n$, take absolute values on both sides, and use trigonometric inequality to obtain

$$\begin{aligned} (1+2\lambda-s\mu)\|u^n\|_\infty &\le \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})\|u^k\|_\infty + a_{n-1}\|u^0\|_\infty \\ &+ 2\lambda\|u^n\|_\infty,\ 1\le i\le M-1,\ 1\le n\le N \end{aligned},$$

therefore

$$\begin{aligned} \|u^n\|_\infty &\le \frac{1}{s\mu}\sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})\|u^k\|_\infty \\ &+ \frac{a_{n-1}}{s\mu}\|u^0\|_\infty,\ 1\le i\le M-1,\ 1\le n\le N \end{aligned}. \quad (14)$$

Starting from inequality (14), applying the discrete form of Gronwall inequality, we can obtain

$$\|u^n\|_\infty \le C\|u^0\|_\infty$$

Similarly, it can be concluded that

$$\|v^n\|_\infty \le C\|v^0\|_\infty$$

V. CONVERGENCE OF DIFFERENTIAL FORMAT

Theorem 2 Assume $\{u_i^n, v_i^n \mid 0\le i\le M,\ 0\le n\le N\}$ and $\{U_i^n, V_i^n \mid 0\le i\le M,\ 0\le n\le N\}$ are the solutions to problem (1) and differential format (12), respectively. Let

$$e_i^n = U_i^n - u_i^n,\ 0\le i\le M,\ 0\le n\le N,$$
$$E_i^n = V_i^n - v_i^n,\ 0\le i\le M,\ 0\le n\le N,$$

there have

$$\|e^n\|_\infty \le C(\tau^{2-\alpha}+h^2),\ 1\le n\le N,$$
$$\|E^n\|_\infty \le C(\tau^{2-\alpha}+h^2),\ 1\le n\le N.$$

Proof: Firstly, prove that the numerical solution u_i^n satisfies the conclusion.
Subtracting equation (7) from equation (9) yields the error equation

International Journal of Scientific Engineering and Science
Volume 9, Issue 3, pp. 15-19, 2025. ISSN (Online): 2456-7361

$$\begin{cases} \frac{1}{s}\left[a_0 e_i^n - \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})e_i^k - a_{n-1}e_i^0\right] \\ = \delta_x^2 e_i^n + (r_1)_i^n,\ 1 \le i \le M-1,\ 1 \le n \le N \\ \frac{1}{s}\left[a_0 E_i^n - \sum_{k=1}^{n-1}(a_{n-k-1}-a_{n-k})E_i^k - a_{n-1}E_i^0\right] \\ = \delta_x^2 E_i^n + (r_2)_i^n,\ 1 \le i \le M-1,\ 1 \le n \le N \end{cases}.$$

Referring to the proof process of reference [15, Theorem 2.3.3] and equation (8), we can obtain

$$\begin{aligned} \| e^n \|_\infty &\le \| e^0 \|_\infty + t_n^\alpha \Gamma(1-\alpha) \max_{1\le m\le n} \| (r_3)^m \|_\infty \\ &\le t_n^\alpha \Gamma(1-\alpha) c_1 (\tau^{2-\alpha} + h^2) \\ &\le c_1 T^\alpha \Gamma(1-\alpha)(\tau^{2-\alpha} + h^2) \\ &= C(\tau^{2-\alpha} + h^2),\ 1 \le n \le N \end{aligned}.$$

Similarly, it can be concluded that

$\| E^n \|_\infty \le C(\tau^{2-\alpha} + h^2),\ 1 \le n \le N$.

6. Numerical examples

Example 1 Consider (1) the right-hand function as the following linear term,

$$\begin{cases} f(x,t,u,v) = \frac{2}{\Gamma(3-\alpha)} t^{2-\alpha}\sin(x) + t^2 \sin(x) - u + v \\ g(x,t,u,v) = \frac{2}{\Gamma(3-\beta)} t^{2-\beta}\sin(x) + t^2 \sin(x) + u - v \end{cases},$$

its exact solution is

$$\begin{cases} u(t) = t^2 \sin(x) \\ v(t) = t^2 \sin(x) \end{cases}.$$

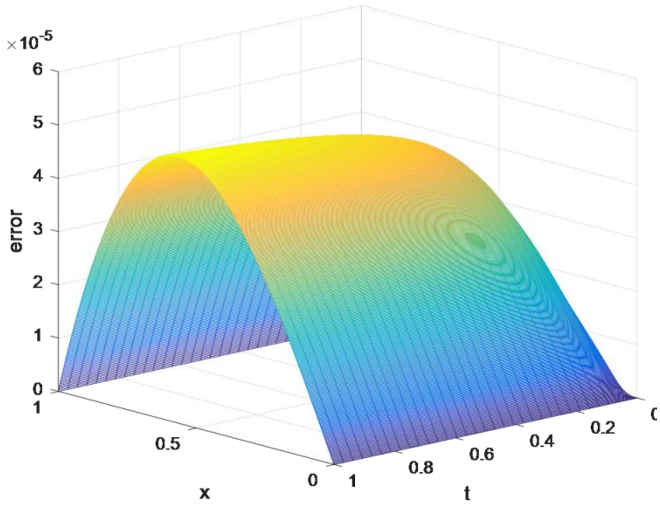

Figure 1 The U error surface of $\alpha = 0.3, \beta = 0.5, h = 1/800, \tau = 1/32$.

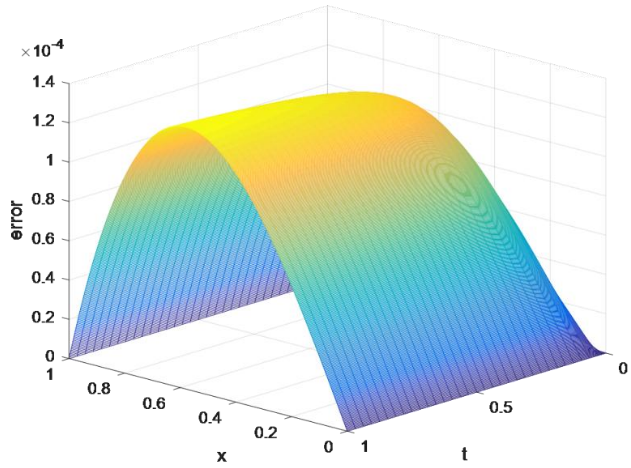

Figure 2 The V error surface of $\alpha = 0.3, \beta = 0.5, h = 1/800, \tau = 1/32$.

Table 1. The maximum error and convergence order of U under different step length h, τ

h	τ	$\alpha = 0.2$, $\beta = 0.4$	$Rate$	$\alpha = 0.6$, $\beta = 0.8$	$Rate$
1/4	1/16	3.5001e-04		9.6400e-04	
1/8	1/64	7.6546e-05	2.1930	1.7456e-04	2.4654
1/16	1/256	1.8132e-05	2.0778	3.3702e-05	2.3728
1/32	1/1024	4.4187e-06	2.0368	6.8333e-06	2.3022

TABLE 2. The maximum error and convergence order of V under different step length h, τ

h	τ	$\alpha = 0.2$, $\beta = 0.4$	$Rate$	$\alpha = 0.6$, $\beta = 0.8$	$Rate$
1/800	1/8	0.0019		0.0079	
1/800	1/16	6.4888e-04	1.5340	0.0035	1.1924
1/800	1/32	2.2114e-04	1.5530	0.0015	1.1992
1/800	1/64	7.4684e-05	1.5661	6.5689e-04	1.2020

Example 2 Consider (1) the right-hand function as the following nonlinear term

$$\begin{cases} f(x,t,u,v) = \frac{\Gamma(4+\alpha)}{6} t^3 \sin(x) + t^{3+\alpha}\sin(x) - u^2 + v + t^{6+2\alpha}\sin^2(x) - t^{3+\beta}\sin(x) \\ g(x,t,u,v) = \frac{\Gamma(4+\alpha)}{6} t^3 \sin(x) + t^{3+\beta}\sin(x) - v^2 + u + t^{6+2\beta}\sin^2(x) - t^{3+\alpha}\sin(x) \end{cases},$$

its exact solution is

$$\begin{cases} u(t) = t^{3+\alpha} \sin(x) \\ v(t) = t^{3+\beta} \sin(x) \end{cases}.$$

TABLE 3. The maximum error and convergence order of U under different step length h, τ

h	τ	$\alpha = 0.2$, $\beta = 0.4$	$Rate$	$\alpha = 0.6$, $\beta = 0.8$	$Rate$
1/4	1/16	5.7587e-04		0.0105	
1/8	1/64	1.0601e-05	2.4416	0.0024	2.1129
1/16	1/256	2.1858e-05	2.2780	5.3908e-04	2.1692
1/32	1/1024	4.7752e-06	2.1945	1.1820e-04	2.1893

IJSES

International Journal of Scientific Engineering and Science

Volume 9, Issue 3, pp. 15-19, 2025.

ISSN (Online): 2456-7361

TABLE 4. The maximum error and convergence order of V under different step length h, τ

h	τ	$\alpha = 0.3$, $\beta = 0.5$	$Rate$	$\alpha = 0.6$, $\beta = 0.8$	$Rate$
1/800	1/8	0.0035		0.0145	
1/800	1/16	0.0013	1.4354	0.0066	1.1307
1/800	1/32	4.6359e-04	1.4763	0.0030	1.1626
1/800	1/64	1.6357e -04	1.5029	0.0013	1.1797

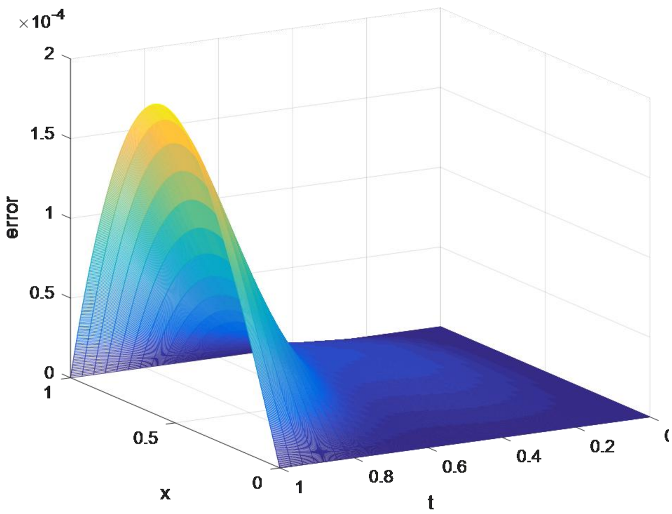

Figure 3 The U error surface of $\alpha = 0.3, \beta = 0.5, h = 1/800, \tau = 1/32$.

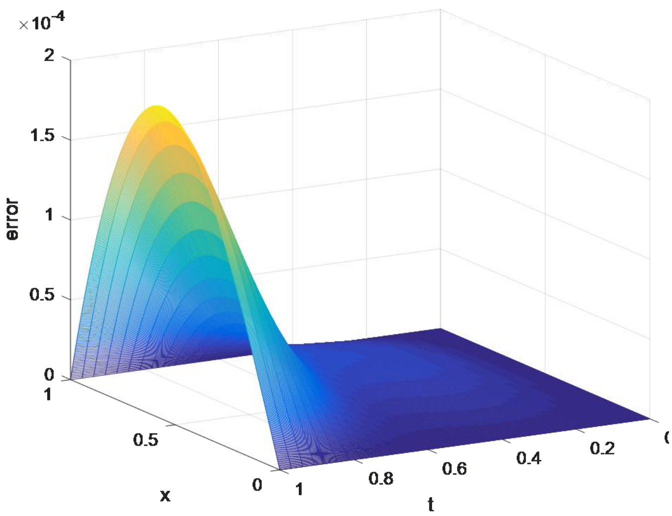

Figure 4 The V error surface of $\alpha = 0.3, \beta = 0.5, h = 1/800, \tau = 1/32$.

VI. CONCLUDING REMARKS

For the nonlinear coupled time fractional slow diffusion equations, the predictor corrector method is used to linearize the right-hand function. At the same time, the time fractional derivative is approximated by L1 scheme, and the spatial second derivative is discretized by difference method. Then a stable convergence numerical scheme is established. The results of theoretical analysis are verified by the above numerical examples, and the error accuracy of the corresponding method is obtained.

ACKNOWLEDGMENTS

The authors would like to thank the referees and editor for their valuable comments and suggestions which helped to improve the results of this paper.

REFERENCES

[1] Jean-Philippe Bouchaud,Antoine Georges. Anomalous diffusion in disordered media: Statistical mechanisms, models and physical applications[J]. Physics Reports, 1990, 195(4-5).
[2] Hilfer, R. (ed.): Applications of Fractional Calculus in Physics. World Scientific, Singapore (2000).
[3] Hilfer R. Applications Of Fractional Calculus In Physics[M]. WORLD SCIENTIFIC: 2000-03.
[4] Oldham, K.B., Spanier, J. The Fractional Calculus Theory and Applications of Differentiation and Integration to Arbitrary Order. Academic Press, New York (1974).
[5] Podlubny, I. Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of their Solution and Some of their Applications. Academic Press, San Diego (1999).
[6] Povstenko, Y. Signaling problem for time-fractional diffusion-wave equation in a half-space in the case of angular symmetry. Nonlinear Dyn. 59, 593–605 (2010).
[7] R. C. KOELLER, Applcation of fractional calculus to the theory of viscoelasticity, J. Appl. Mech., 51 (1984), pp. 229–307.
[8] D. A. Benson, S. W. Wheatcraft And M. M. Meerschaert, Application of a fractional advection-dispersion equation, Nonlinear Anal., 36 (2006), pp. 1403–1412.
[9]A spectral collocation method for the coupled system of nonlinear fraction-al differential equations, Xiaojun Zhou and Yue Dai, AIMS Mathematics, 2022,7(4): 5670–5689.
[10] Huseynov I T, Ahmadova A, Fernandez A , et al. Explicit analytical solutions of incommensurate fractional differential equation systems[J]. Applied Mathematics and Computation, 2020.
[11] Zhou Xiaojun, Xu Chuanju. Numerical Solution of the Coupled System of Nonlinear Fractional Ordinary Differential Equations[J]. Advances in Applied Mathematics and Mechanics, 2017, 9(3).
[12] Nabil T. Ulam stabilities of nonlinear coupled system of fractional differential equations including generalized Caputo fractional derivative[J]. AIMS Mathematics, 2021, 6(5): 5088-5105.
[13] M. A. Zaky and I. G. Ameen. A priori error estimates of a jacobi spectral method for nonlinear systems of fractional boundary value problems and related volterra-fredholm integral equations with smooth solutions. Numerical Algorithms, (2): 1–27, 2019.
[14] Shiri B, Wu G C, Baleanu D. Terminal value problems for the nonlinear systems of fractional differential equations[J]. Applied Numerical Mathematics, 2021.
[15] Dai Yue, Zhou xiaojun Euler method for Nonlinear Fractional Ordinary Differential Equations [j]. Journal of Guizhou Normal University (Natural Science Edition), 2020,38(06):68-74.