

# Finite Time Control of Parameterized Systems with Dead Zone and State Constraints

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**Abstract**—This paper investigates the problem of adaptive finite-time tracking control for nonlinear parameterized systems subject to full state constraints and dead-zone. By utilizing the finite-time stability theory, one to one nonlinear mapping and dynamic surface control (DSC), a novel adaptive tracking control is proposed. By using the defined compact set in stability analysis, all the signals in the close-loop system are proved to be semi-globally practical finite-time bounded (SGPFB), and state constraint is not violated. Numerical simulation has verified the effectiveness of the control strategy.

**Keywords**— Adaptive finite-time control, Dead-zone, Full state constraints.

## I. INTRODUCTION

In recent years, dynamic surface control technology has been widely applied in nonlinear system control and has achieved good results in [1-4]. Based on the DSC method, reference [3] solved the controller design problem of a class of strict feedback systems and provides Lyapunov stability analysis. With the help of the implicit function theorem, an adaptive control strategy is proposed for a class of pure feedback nonlinear systems via DSC in [4].

In addition, in real-world systems, due to factors such as safety and location, the system's state must be limited within a certain range. For example, the literature [5] defined a set of Barrier Lyapunov functions that constrain the state at each step of the design process. However, in practical applications, this method cannot accurately constrain the state. On this basis, people have proposed the method of introducing nonlinear mapping to transform constrained problems into unconstrained problems, thereby enabling controller design in [6-8]. The nonlinear mappings in references [6] and [7] are based on logarithmic functions, while reference [8] adopts a new fractional form.

Specifically, control systems in reality often exhibit nonlinear phenomena in the controller, with the most common form being dead zones. Reference [9] linearized the dead zone and successfully separated the control law  $u$  for controller design. At the same time, specific systems also have time requirements, leading to the emergence of finite time control in [10-12]. Inspired by the above achievements, this paper studies the finite time control problem of a class of constrained parameterized systems with dead zones. The main work is as follows: (1) Introducing non logarithmic mapping to transform the constrained system into an unconstrained system, and using dynamic surface control method for controller design to avoid the problem of parameter explosion. (2) After transformation, the parameterized system becomes a strict feedback system, and neural networks are used to approximate unknown functions to handle the unknown nonlinear terms of the system.

## II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Consider the following nonlinear systems:

$$\begin{cases} \dot{x}_i = v_i^T f_i(\bar{x}_i) + \phi_i x_{i+1}, i = 1, L, n - 1 \\ \dot{x}_n = v_n^T f_n(\bar{x}_n) + \phi_n Q(u(t)) \\ y = x_1 \end{cases} \quad (1)$$

where  $\bar{x}_i = [x_1 \cdots x_i]^T \in R^i$ ,  $y$  denote the system states and output.  $v_i \in R^m$  is the unknown constant vector,  $\phi_i(\bar{x}_i)$  is known nonlinear function vector. All the states are constrained in predefined compact set, i.e.,  $|x_i| < k_{ci} (i = 1, \dots, n)$  are predefined positive constants. The dead-zone input  $u(t)$  can be described as

$$Q(u(t)) = \begin{cases} m_r(u(t) - b_r), & u(t) \geq b_r \\ 0, & -b_l < u(t) < b_r \\ m_l(u(t) + b_l), & u(t) \leq -b_l \end{cases} \quad (2)$$

**Control objective:** Design a controller  $u$  that enables the output of system (1) to track the desired trajectory and satisfy the constraint conditions.

**Assumption 1** [5] The desired trajectory vector  $x_d = [y_d, \dot{y}_d, \ddot{y}_d]^T \in \Omega_d$  is constants and available with known compact set  $\Omega_d = \{x_d : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\} \in R^3$ , and  $|y_d| < B_1 \leq \min\{k_{b_1}, k_{b_2}\}$ , where  $B_0, B_1$  are two known positive constants.

**Lemma 1** [110] considering the system  $\dot{x} = f(x, u)$ , for positive function  $V(x)$ , if there exist scalars  $\alpha > 0$ ,  $\frac{1}{2} < \beta < 1$  and  $\mu > 0$ , such that  $\dot{V}(x) \leq -\alpha V^\beta(x) + \mu$

then the nonlinear system  $\dot{x} = f(x, u)$  is SGPFB.

**Lemma 2** [10] For any real variables  $x, y$ , there any given positive constants  $a, b$  and  $c$ , it holds the inequality such that

$$|x|^a |y|^b \leq \frac{a}{a+b} c |x|^{a+b} + \frac{b}{a+b} c^{-\frac{a}{b}} |y|^{a+b} \quad (3)$$

**Lemma 3** [10] For  $x_j \in \mathbb{R}, j=1 \cdots n, 0 < p \leq 1$ , it holds the inequality such that

$$\left( \sum_{j=1}^n |x_j| \right)^p \leq \sum_{j=1}^n |x_j|^p \leq n^{1-p} \left( \sum_{j=1}^n |x_j| \right)^p \quad (4)$$

### III. CONTROLLER DESIGN AND STABILITY ANALYSIS

In order to carry out full state constraints, we introduce the following one to one mapping:

$$s_i = \ln \frac{k_{ci} + x_i}{k_{ci} - x_i}, i = 1, \dots, n \quad (5)$$

Then we get

$$\dot{s}_i = \frac{e^{s_i} + e^{-s_i} + 2}{2k_{ci}} \dot{x}_i, i = 1, \dots, n \quad (6)$$

The system (1) can be rewritten as follows:

$$\begin{cases} \dot{s}_i = f_i(\bar{s}_{i+1}) + s_{i+1}, i = 1, \dots, n-1 \\ \dot{s}_n = f_n(\bar{s}_n) + k_n(s_n) \varphi_n Q(u(t)) \end{cases} \quad (7)$$

where  $\bar{s}_i = [s_1, \dots, s_i]^T$ ,  $k_i(s_i) = (e^{s_i} + e^{-s_i} + 2) / 2k_{ci}$ ,

$$f_i(s_{i+1}) = k_i(s_i) [v_i^T \phi_i(\bar{x}_i) + \varphi_i x_{i+1}] - s_{i+1}, i = 1, \dots, n-1,$$

$$f_n(\bar{s}_n) = k_n(s_n) v_n^T \phi_n(\bar{x}_n).$$

Let  $\hat{y}_d = \ln \frac{k_{c1} + y_d}{k_{c1} - y_d}$ . Suppose  $\Omega_{z_i} = \mathbb{R}^{i+3}$  be a given

compact set, and  $W_{hi}^{*T} S_i(Z_i)$  be the approximation of BRN NNs over the compact set  $\Omega_{z_i}$  to  $h_i(Z_i)$ , where  $h_i(Z_i)$  will be given later. Then,  $h_i(Z_i) = W_{hi}^{*T} S_i(Z_i) + \varepsilon_{hi}(Z_i)$ , where  $Z_i = [Z_{i1}, \dots, Z_{iq}]^T \in \mathbb{R}^{iq}$ , and the basis function vector  $S_i(Z_i) = [s_{i1}(Z_i), \dots, s_{il_i}(Z_i)]^T \in \mathbb{R}^{l_i}$  with  $s_{ij}(Z_i)$  being choose as follows:

$$s_{ij}(Z_i) = \exp \left[ -\frac{(Z_i - \mu_{ij})^T (Z_i - \mu_{ij})}{\phi_{ij}^2} \right], j = 1, \dots, l_i, i = 1, \dots, n,$$

$\mu_{ij} = [\mu_{ij1}, \mu_{ij2}, \dots, \mu_{ijq}]^T$  is the center of the receptive field

with  $q_{ij} = n + 3$ , and  $\phi_{ij}$  is width of the Gaussian function.

For clarity, some notations are defined as follows:

$$\bar{z}_i = [z_1, \dots, z_i]^T, \bar{y}_j = [y_1, \dots, y_j]^T, y_j = \omega_j - \alpha_{j-1}, j = 2, \dots, n.$$

$$\theta_i = \|W_{hi}^*\|^2, \hat{\theta}_i = [\hat{\theta}_1, \dots, \hat{\theta}_i]^T, V_{z_i} = \frac{1}{2} z_i^2, z_i = s_i - \omega_i$$

$i = 1, \dots, n.$

**Step 1:** Let  $\omega_1 = \hat{y}_d$ , we obtain the time derivative of  $z_1$

$$\dot{z}_1 = s_2 + f_1(\bar{s}_2) - \dot{\omega}_1$$

Then the derivative of  $V_{z_1}$  with respect to  $t$  is

$$\begin{aligned} \dot{V}_{z_1} &= z_1 [z_2 + y_2 + \alpha_1 + h_1(Z_1)] - 3z_1^2 \\ &= z_1 [z_2 + y_2 + \alpha_1 + W_{h1}^{*T} S_1(Z_1) + \varepsilon_{h1}(Z_1)] - 3z_1^2 \\ &\leq z_1 [z_2 + y_2 + \alpha_1] + \frac{1}{2a_1^2} z_1^2 \theta_1 \|S_1(Z_1)\|^2 \end{aligned} \quad (8)$$

$$+ \frac{a_1^2}{2} + z_1 \varepsilon_{h1}(Z_1) - 3z_1^2$$

where  $h_1(Z_1) = f_1(\bar{s}_2) - \dot{\omega}_1 + 3z_1$  and  $a_1 > 0$  is a design constant.

Design the virtual control  $\alpha_1$  as follows:

$$\alpha_1 = -k_1 z_1^{2\beta-1} - \frac{z_1}{2a_1^2} \hat{\theta}_1 \|S_1(Z_1)\|^2 \quad (9)$$

where  $k_1$  is a design constant that we will choose later,  $\hat{\theta}_1$  is the estimate of  $\theta_1$  at time  $t$ .

The following adaptive law is used to update the unknown parameter  $\theta_1$  for  $i = 1$ . (2)

$$\dot{\hat{\theta}}_1 = \gamma_1 \left[ \frac{1}{2a_1^2} z_1^2 \|S_1(Z_1)\|^2 - \sigma_1 \hat{\theta}_1 \right] \quad (10)$$

where  $\gamma_1, a_i$  and  $\sigma_i$  are strictly positive constants and  $\hat{\theta}_1$  is the estimate of  $\theta_1$  at time  $t$ .

Define  $\omega_2$  in such a way that

$$\tau_2 \dot{\omega}_2 + \omega_2 = \alpha_1, \omega_2(0) = \alpha_1(0) \quad (11)$$

where  $\tau_2$  is a design constant that we will choose later.

Since  $s_2 = z_2 + y_2 - k_1 z_1^{2\beta-1} - z_1 \hat{\theta}_1 \|S_1(Z_1)\|^2 / 2a_1^2$ , using young's inequality, we obtain

$$\begin{aligned} \dot{V}_{z_1} &\leq -k_1 z_1^{2\beta} - \frac{z_1^2 \tilde{\theta}_1 \|S_1(Z_1)\|^2}{2a_1^2} \\ &\quad + \frac{z_2^2}{4} + \frac{y_2^2}{4} + \frac{a_1^2}{2} + z_1 \eta_1 - z_1^2 \end{aligned} \quad (12)$$

where continuous function  $\eta_1$  satisfies  $|\varepsilon_{h1}(Z_1)| \leq \eta_1$ .

From young's inequality, we have  $|z_1| \eta_1 \leq z_1^2 + \frac{1}{4} \eta_1^2$ , then we get (4)

$$\dot{V}_{z_1} \leq -k_1 z_1^{2\beta} - \frac{z_1^2 \tilde{\theta}_1 \|S_1(Z_1)\|^2}{2a_1^2} + \frac{z_2^2}{4} + \frac{y_2^2}{4} + \frac{a_1^2}{2} + \frac{1}{4} \eta_1^2 \quad (13)$$

Noting Assumption 1, we have  $\left| \dot{y}_2 + \frac{y_2}{\tau_2} \right| \leq \xi_2$ , and  $\xi_2$

is a continuous function.

Further, we obtain

$$y_2 \dot{y}_2 \leq -\frac{y_2^2}{\tau_2} + |y_2| \xi_2 \leq -\frac{y_2^2}{\tau_2} + y_2^2 + \frac{1}{4} \xi_2^2 \quad (14)$$

Step i ( $2 \leq i \leq n-1$ )

The time derivative of  $z_i$  is

$$\dot{z}_i = f_i(\bar{s}_{i+1}) + s_{i+1} - \dot{\omega}_i \quad (15)$$

Therefore, the derivative of  $V_{z_i}$  with respect to  $t$  is

$$\begin{aligned} \dot{V}_{z_i} &= z_i [z_{i+1} + y_{i+1} + \alpha_i + h_i(Z_i)] - 3 \frac{1}{4} z_i^2 \\ &= z_i [z_{i+1} + y_{i+1} + \alpha_i + W_{h_i}^{*T} S_i(Z_i) + \varepsilon_{h_i}(Z_i)] - 3 \frac{1}{4} z_i^2 \\ &\leq z_i [z_{i+1} + y_{i+1} + \alpha_i] + \frac{1}{2a_i^2} z_i^2 \theta_i \|S_i(Z_i)\|^2 \\ &\quad + \frac{a_i^2}{2} + z_i \varepsilon_{h_i}(Z_i) - 3 \frac{1}{4} z_i^2 \end{aligned} \quad (16)$$

where  $h_i(Z_i) = f_i(\bar{s}_{i+1}) - \dot{\omega}_i + 3 \frac{1}{4} z_i$ , and  $a_i > 0$  is a design constant.

Construct the virtual control  $\alpha_i$  as follows:

$$\alpha_i = -k_i z_i^{2\beta-1} - \frac{z_i}{2a_i^2} \hat{\theta}_i \|S_i(Z_i)\|^2 \quad (17)$$

where  $k_i$  is a positive design constant that we will choose later.

The adaptive law of the parameter  $\hat{\theta}_i$  is determined by (10) for  $i$ .

Define  $\omega_{i+1}$  in such a way that

$$\tau_{i+1} \dot{\omega}_{i+1} + \omega_{i+1} = \alpha_i, \omega_{i+1}(0) = \alpha_i(0),$$

where  $\tau_{i+1}$  is a positive design constant.

$$s_{i+1} = z_{i+1} + y_{i+1} - k_i z_i^{2\beta-1} - \frac{z_i}{2a_i^2} \hat{\theta}_i \|S_i(Z_i)\|^2 \quad (18)$$

Using young's inequality, we obtain

$$\begin{aligned} \dot{V}_{z_i} &\leq -k_i z_i^{2\beta} - \frac{z_i^2 \tilde{\theta}_i \|S_i(Z_i)\|^2}{2a_i^2} + \frac{z_{i+1}^2}{4} + \frac{y_{i+1}^2}{4} \\ &\quad + \frac{a_i^2}{2} + z_i \eta_i - 1 \frac{1}{4} z_i^2 \end{aligned} \quad (19)$$

where continuous function  $\eta_i$  satisfies  $|\varepsilon_i(Z_i)| \leq \eta_i$ .

From young's inequality, we have

$$|z_i| \eta_i \leq z_i^2 + \frac{1}{4} \eta_i^2 \quad (20)$$

Therefore, we obtain

$$\begin{aligned} \dot{V}_{z_i} &\leq -k_i z_i^{2\beta} - \frac{z_i^2 \tilde{\theta}_i \|S_i(Z_i)\|^2}{2a_i^2} + \frac{z_{i+1}^2}{4} \\ &\quad + \frac{y_{i+1}^2}{4} + \frac{a_i^2}{2} + \frac{1}{4} \eta_i^2 - \frac{1}{4} z_i^2 \end{aligned} \quad (21)$$

Then, one has

$$y_{i+1} \dot{y}_{i+1} \leq -\frac{y_{i+1}^2}{\tau_{i+1}} + y_{i+1}^2 + \frac{1}{4} \xi_{i+1}^2 \quad (22)$$

**Step n:** The control law  $u$  will be design in this step.

The time derivative of  $z_n$  is

$$\dot{z}_n = f_n(\bar{s}_n) + k_n(s_n) \varphi_n Q(u) - \dot{\omega}_n \quad (23)$$

Therefore, the derivative of  $V_{z_n}$  with respect to  $t$  is

$$\dot{V}_{z_n} = z_n [f_n(\bar{s}_n) + k_n(s_n) \varphi_n (mu(t) + d(t)) - \dot{\omega}_n] \quad , \quad \text{since } d(t) \leq \rho, \text{ then we obtain}$$

$$\begin{aligned} \dot{V}_{z_n} &\leq z_n k_n(s_n) \varphi_n mu + \frac{g_0 z_n^2 \theta_n \|S_n(Z_n)\|^2}{2a_n^2} \\ &\quad + \frac{a_n^2}{2g_0} + \varepsilon_{h_n}(Z_n) - 1 \frac{1}{4} z_n^2 \end{aligned} \quad (24)$$

where  $a_n > 0$  is a design constant and  $g_0 > 0$  we will give later, and

$$\begin{aligned} h_n(Z_n) &= f_n(\bar{s}_n) - \dot{\omega}_n + k_n(s_n) \varphi_n \rho \\ &\quad + 1 \frac{1}{4} z_n, Z_n = [\bar{s}_n^T, z_n, \dot{\omega}_n]^T \in R^{n+2}. \end{aligned}$$

Design the control law  $u$  as follows:

$$u = \frac{1}{m \varphi_n} \left[ -k_n z_n^{2\beta-1} - \frac{z_n}{2a_n^2} \hat{\theta}_n \|S_n(Z_n)\|^2 \right] \quad (25)$$

$\hat{\theta}_n$  is the estimate of  $\theta_n$ , the function as

$$\dot{\hat{\theta}}_n = \gamma_n \left[ \frac{1}{2a_n^2} z_n^2 \|S_n(Z_n)\|^2 - \sigma_n \hat{\theta}_n \right] \quad (26)$$

Let  $\frac{2}{k_{cn}} = g_0$ , then we have

$$\begin{aligned} \dot{V}_{z_n} \leq & -g_0 k_n z_n^{2\beta} - \frac{g_0 z_n^2 \hat{\theta}_n \|S_n(Z_i)\|^2}{2a_n^2} \\ & + \frac{g_0 z_n^2 \theta_n \|S_n(Z_i)\|^2}{2a_n^2} + \frac{a_n^2}{2g_0} + z_n \eta_n - 1 \frac{1}{4} z_n^2 \end{aligned} \quad (27)$$

where continuous function  $\eta_n$  satisfies  $|\varepsilon_{hm}(Z_n)| \leq \eta_n$ .

Using young's inequality, we have

$$|z_n| \eta_n \leq z_n^2 + \frac{1}{4} \eta_n^2 \quad (28)$$

From the above, it can be concluded that

$$\begin{aligned} \dot{V}_{z_n} \leq & -g_0 k_n z_n^{2\beta} - \frac{g_0 z_n^2 \tilde{\theta}_n \|S_n(Z_i)\|^2}{2a_n^2} \\ & + \frac{a_n^2}{2g_0} - \frac{1}{4} z_n + \frac{1}{4} \eta_n^2 \end{aligned} \quad (29)$$

**Theorem 1** Consider the closed-loop system consisting of system (1) under Assumption 1, the controller (25), and adaption law (10). For bounded initial conditions, satisfying  $V_n(0) \leq p$ ,  $k_{c1} \leq B_1$ , and  $x_i(0) \in \Omega_{x_i}$ , there exist constants  $k_i > 0, \tau_i > 0, \gamma_i > 0, \sigma_i > 0$ , such that the overall closed-loop neural control system is SGPFSS in the sense that all of the signals in the closed-loop system are finite-time bounded, and  $x_i \in \Omega_{x_i}, \forall t \geq 0$ , i.e., the full state constraints are never violated, in addition,  $k_i$  and  $\tau_i$  satisfy

$$\begin{cases} k_i \geq \frac{\alpha_0}{2} 2^\beta, k_n \geq \frac{\alpha_0}{2g_0} 2^\beta, i = 1, \dots, n-1 \\ \frac{1}{\tau_i} \geq 1 \frac{1}{4} + \frac{\alpha_0}{2} \\ \alpha_0 = \min\{\gamma_1 \sigma_1, \dots, \gamma_n \sigma_n\} \end{cases} \quad (30)$$

**Proof.** Consider the overall Lyapunov function candidate as follows:

$$V = V_n = \sum_{i=1}^n [V_{z_i} + \frac{1}{2\gamma_i} g_{i0} \tilde{\theta}_i^2] + \frac{1}{2} \sum_{i=1}^{n-1} y_{i+1}^2 \quad (31)$$

Differentiating  $V(t)$  with respect to time t leads to

$$\dot{V} = \sum_{i=1}^n [\dot{V}_{z_i} + \frac{1}{\gamma_i} \tilde{\theta}_i \dot{\tilde{\theta}}_i] + \sum_{i=1}^{n-1} y_{i+1} \dot{y}_{i+1} \quad (32)$$

Substituting (13), (14), (21), (22) and (29) into (32), and applying (10) and (26), it follows that

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^{n-1} [-k_i z_i^{2\beta} + g_0 k_n + \sum_{i=1}^{n-1} [-\frac{y_{i+1}^2}{\tau_{i+1}} + 1 \frac{1}{4} y_{i+1}^2 + \frac{1}{4} \xi_{i+1}^2] \\ & + \sum_{i=1}^{n-1} \frac{a_i^2}{2g_{i0}} + \frac{a_n^2}{2g_0} + \sum_{i=1}^n \frac{1}{4} \eta_i^2 + \sum_{i=1}^n [-\sigma_i \tilde{\theta}_i \hat{\theta}_i] \end{aligned} \quad (33)$$

If  $V = V_n = p$ , then  $\eta_i^2 \leq H_i^2$  and  $\xi_{i+1}^2 \leq M_{i+1}^2$ , By completion of squares, we have

$$-\sigma_i \tilde{\theta}_i \hat{\theta}_i = -\sigma_i \tilde{\theta}_i (\tilde{\theta}_i + \theta_i) \leq \sigma_i (-\frac{\tilde{\theta}_i^2}{2} + \frac{\theta_i^2}{2}) \quad (34)$$

Now we apply lemma 2 and Lemma 3, one has

$$\dot{V} \leq -\alpha V^\beta + \mu \quad (35)$$

where  $\alpha = \min\{\alpha_0, \alpha_0^\beta\}$ ,

$$\begin{aligned} \mu = & \sum_{i=1}^n \frac{1}{4} \eta_i^2 + \sum_{i=1}^{n-1} \frac{a_i^2}{2} + \frac{a_n^2}{2g_0} + \sum_{i=1}^n -\gamma_i \sigma_i \frac{1}{2\gamma_i} \tilde{\theta}_i^2 \\ & + \sum_{i=1}^n \frac{1}{2} \sigma_i \theta_i^2 + 2(1-\beta)l \end{aligned}$$

It means that all the signals in the closed-loop system are SGPFSS. Therefore,  $z_i$ ,  $y_i$  and  $\hat{\theta}_i$  are finite time bounded.  $s_i$ ,  $\alpha_i$ ,  $\omega_{i+1}$  are also finite time bounded.

#### IV. SIMULATION RESULTS

Consider a nonlinear system, the dynamic equation is given as follows:

$$\begin{cases} \dot{x}_1 = 0.1x_1^2 + x_2 \\ \dot{x}_2 = 0.2 \sin(x_1) + x_1 + 2u \\ y = x_1 \end{cases} \quad (37)$$

For conducting the simulation, the initial conditions are  $x_1(0) = 0.13, x_2(0) = 1.43, \tau_2 = 0.01, \theta_1(0) = 0.1, \theta_2(0) = 0.3, \omega_2(0) = 0.1$ .

The reference signal is  $y_d = 0.3 \cos(0.57t + 0.6) - 0.24$ , and the states are constrained in the regions  $|x_1| < 0.16$  and  $|x_2| < 1.5$ . The parameters are selected as  $k_1 = 4, k_2 = 15, a_1 = 1, a_2 = 1, \sigma_1 = \sigma_2 = 0.01, \gamma_1 = 50, \gamma_2 = 30, b_r = 0.3, b_l = 0.5, m_r = m_l = 0.8, l = 0.5, \tau_2 = 0.01, \beta = 99/101$ .

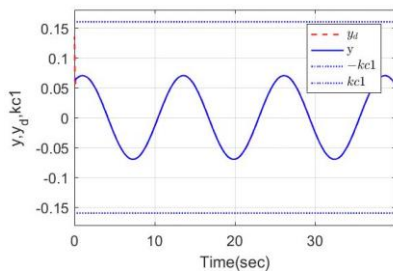


Fig. 1. Output  $y$ , desired trajectory  $y_d$  and constraints  $kc1$

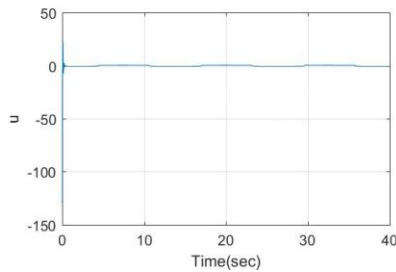


Fig. 2. Control law  $u$

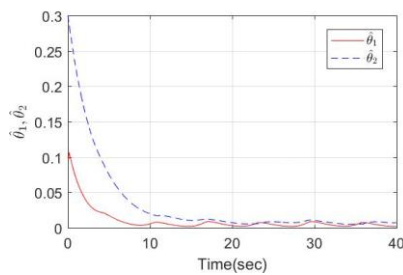


Fig. 3. The curve of  $\hat{\theta}_1$  and  $\hat{\theta}_2$

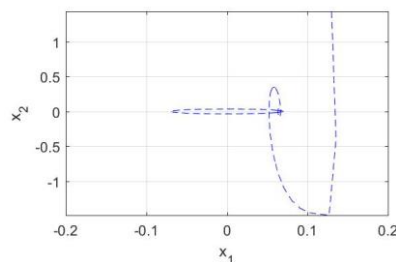


Fig. 4. Phase portrait of states  $x_1, x_2$

The simulation results are shown in the Fig. 1–4. From above Figures, we can see that the all states abide by extended constraint conditions, and the control objective can be well implemented. Moreover, all signals in systems are bounded.

## V. CONCLUSION

Adaptive finite-time tracking control for nonlinear systems subject to full state constraints and dead-zone is studied in this paper. Base on DSC, control, a novel adaptive finite time tracking control is proposed by utilizing the finite-time

stability theory and NM. All the signals in the close-loop system are proved to be SGPF and state constraints have not been triggered. The simulation further verified the effectiveness of the proposed finite time control strategy.

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