Volume 8, Issue 6, pp. 15-19, 2024.
International Journal of Scientific Engineering and Science
ISSN (Online): 2456-7361

Multiplicative Topological Indices of Aztec Diamond Cross Networks

Irfan Haider[1], Riaz Hussain Khan[2], Jamila Majeed[3], Maria Mehtab[4]
[1]The School of Software, Dalian University of Technology, Dalian 116620, Liaoning, China
[2]Institute of Mathematics, Khwaja Fareed University of Engineering and Information Technology
[3]Government College University Faisalabad, Layyah Campus
[4]Government College University Faisalabad
*Correspondence: Email: irfanhaider7372@gmail.com

Abstract—*In this paper we consider the Aztec diamond cross and then compute the degree based topological indices of Aztec diamond cross networks. These indices include multiplicative first Zagreb index, second Zagreb index, general first Zagreb index, general second Zagreb index, Hyper Zagreb index, multiplicative product and sum connectivity index. we compute the degree based multiplicative topological of Aztec diamond cross networks.*

Keywords— *Zagreb Index, General Zagreb Index, Hyper Zagreb Index, Multiplicative Zagreb Index, Sum Connectivity Index, Molecular descriptor*

I. Introduction

A graph is a representation of a network consisting of vertices (points)(Kulli, 2016) and edges (lines) that connect these vertices. Vertices serve as the points within the graph, while edges depict the relationships or connections between these points. The connection between two vertices, u and v, is denoted by the edge uv, and these vertices are known as adjacent vertices. Through the calculation of topological indices, it is possible to analytically determine various properties of the network, such as connectivity, distance metrics, and other structural characteristics. (Baranyi et al., 2011)Essentially, graphs provide a visual and analytical framework for understanding the relationships and interactions within a network, with topological indices offering a quantitative means to assess and analyze the network's properties and structural features. The Seven Bridges of Konigsberg, a renowned article from 1736 by the distinguished Swiss mathematician Leonhard Euler, is often considered the pioneering work in graph theory.

In 2007, Assad provided a brief analysis(Hayat & Imran, 2014)utilizing graph theory to explore the connections between nodes and edges. Chemical graph theory, a field within mathematical chemistry focusing on topology, was advanced by Triassic, who addressed unresolved and solved issues in this area. The application of chemical graph theory enables the mathematical representation of chemical processes, leading to the development of topological indices, useful tools in understanding molecular structures. (García-Domenech et al., 2008)Balaban introduced topological indices for molecular graphs, which are numerical values linking chemical compositions to physical properties, chemical reactivity, and biological effects. Topological indices play a crucial role in bridging physical qualities, chemical structures, and graph representations, offering valuable insights across disciplines such as computer science and chemistry. In the realm of chemistry, topological descriptors based on polynomials, distances, and degrees are extensively employed to characterize chemical graphs that depict molecular structures and substances.

These descriptors provide a quantitative framework for analyzing the structural features of chemical compounds.

Moreover, the field of graph theory is renowned for its adeptness in modeling and visualizing diverse structures and networks. Let is combination of vertices and edges and is degree of vertex and is the number of edges incident with. For molecular structure we usually take simple undirected graph.(Khan et al., 2020)By utilizing graph theory principles and topological indices, researchers can effectively analyze complex systems, enabling a comprehensive understanding of connectivity, relationships, and structural characteristics within networks and systems in various scientific domains. To predict the bioactivity of chemical compounds, the topological descriptors such as Wiener index, Szeged index, Geometric-arithmetic (GA) index and atom bond connectivity (ABC) index and the physic chemical properties are studied in QSAR/QSPR (Iranmanesh et al., 2012)Mostly these structures and networks are linked with chemistry, biology, electrical circuits etc. The Indices like Multiplicative Zagreb Multiplicative Hyper Zagreb, multiplicative computation connectivity, Multiplicative invention connectivity, General Multiplicative Zagreb, Multiplicative *ABC* and Multiplicative *GA* indices are computed. The applications of dissimilar heliotropes of carbon especially graphite, diamond and bucky balls are discussed in detail (Siddiqui et al., 2016)

II. *Basic Definitions*

The first and second

Multiplicative of graph G Zagreb indices introduce by Todechine.

$$II_1(G) = \prod_{h \in V(G)}$$

II_2 (G)=∏_{hi∈E(G)}{(d_G (h)×d_G (i))}

International Journal of Scientific Engineering and Science
Volume 8, Issue 6, pp. 15-19, 2024. ISSN (Online): 2456-7361

The new version of multiplicative first Zagreb Indices which are discussed in ELIASI proposed as $II_1{}^*(G)$ index as: of graph G is

II_1^{*} (G)=∏_{hi∈E(G)}{(d_G (h)+d_G (i))}

Now we discussed the first and second Multiplicative Hyper Zagreb indices of a Graph G which introduce by (Kulli, 2016)

HII_{1}(G) =∏_{hi∈E(G)}(d_G (h)+ d_G (i))^{2}

We discuss the general first and second Zagrab which are introduce also by Kulli in [14].

MZ_1^{α}{G} = ∏_{hi ∈ E(G)}{(d_G (h) + d_G (i))}^{α}

MZ_2^{α}{G} = ∏_{hi ∈ E(G)}{(d_G (h) × d_G (i))}^{α}

The sum multiplicative connectivity index of a graph G could be defined as which introduce by Kulli (Kulli, 2019)

$$XII(G) = \prod_{\{pq\in E(G)\}\backslash frac\{1\}\left\{\sqrt{\{d_{G(p)}+d_{G(q)}\}}\right\}}$$

The Atom Bond which introduces by (Gao et al., 2012)

$$\mathrm{ABCII}(G) = \prod_{\{ih\in E(G)\}\sqrt{\{\backslash frac\{d_{G(h)}+d_{G(i)}-2\}\{d_{G(h)}\times d_{G(i)}\}\}}}$$

The Geometric Arithmetic Index of G introduces (Rosen & Krithivasan, 1999)

$$\mathrm{GAII}(G) = \prod_{\{hi\in E(G)\}\backslash frac\left\{2\sqrt{\{d_{G(h)}\times d_{G(i)}\}}\right\}\{d_{G(h)}+d_{G(i)}\}}$$

III. MTIs Calculated Result for Diamond Cross Networks

An Aztec diamond cross with order n containing all squares lattice. Here n is fixed, and square lattice contains unit squares with the origin as vertex of degree 8, so that both p and q are half integer.

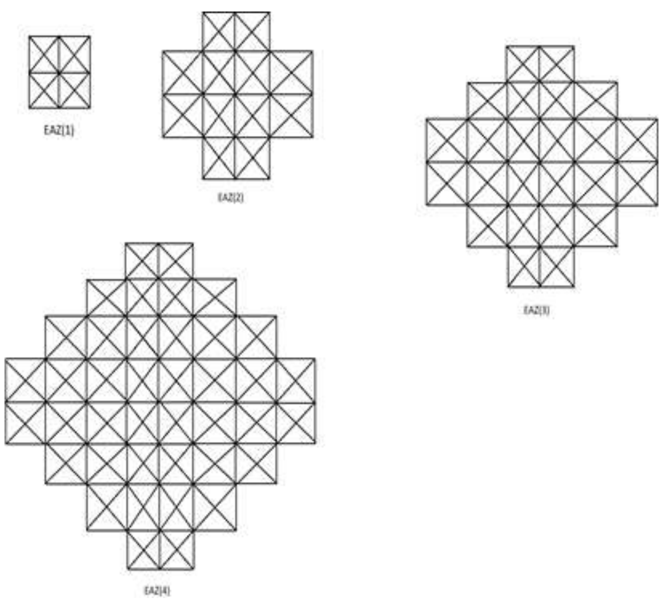


Figure1: Aztec diamond Cross

In previous work, explored the application of Multiplicative Topological Indices (MTIs) to analyze Aztec Diamond Networks (Imran et al., 2017) Building upon this foundation, this study delves deeper into the empire of Diamond Cross Networks.

MTIs proved valuable in characterizing Aztec Diamond Networks. Here, we extend this approach by investiga3wting MTIs for Aztec Diamond Cross Networks.

Our aim to identify similarities and potential differences in the network behavior as captured by these indices. This study ventures into uncharted territory by introducing Diamond Cross Networks.

In this paper we calculate the multiplicative topological indices of Aztec Diamond Cross Networks using mathematical formulas. The results are then visualized as a 3D graph using Maple software, providing a clear representation of the network's structural properties

Explore the applicability of MTIs to this novel network structure and develop new indices specifically personalized to capture their unique characteristics.

TABLE 1: Frame Separation of Aztec diamond cross

(d_u,d_v)	Frequency
(3,4)	$4n$
(3,5)	08
(3,7)	8n-8
(4,5)	08
(4,7)	12n-12
(4,8)	$8n^2-8n+4$
(5,8)	04
(7,8)	*8n-8*
(8,8)	$4n^2-8n+4$

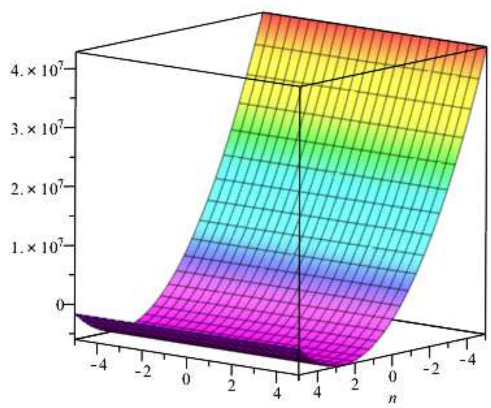
Theorem 1. *The second multiplicative Zagreb index is by Todechine*

$$II_2(G) = 824320\,n^2 - 4444160\,n - 512$$

Proof: *The 2nd multiplicative Zagreb index*

$$\mathrm{II}_2\,(G) = \prod_{\{hi\in E(G)\}\{(d_G(h)\times d_G(i))\}}$$

$=4n(3\times4)\times08(3\times5)\times(8n-8)(3\times7)\times08(4\times5)\times12n-12(4\times7\times8n\ -8n+4(4\times8)\times04(5\times8)$
$\times8n-8(7\times8)\times4n\ -8n+4(8\times8)$
$=824320n^2-4444160n-512$



Theorem 2. *The new version of multiplicative first Zagreb*

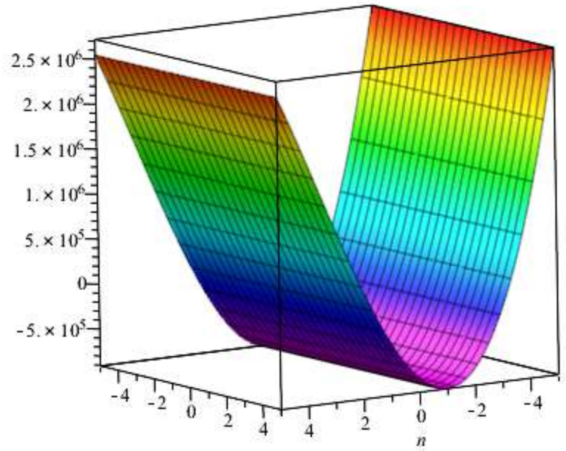
$$II_1^{*}(G) = 135680n^2 - 760256 - 12672n^3 + 299296n$$

Proof*: The new version of multiplicative first Zagreb*

$$II_1^{\{*\}}(G) = \prod_{\{hi\in E(G)\}\{(d_G(h)+d_G(i))\}}$$

$=4n(3+4)\times 08(3+5)\times(8n-8)(3+7)\times 08(4+5)$
$\times 12n-12(4+7)\times 8n \;\; -8n+4(4+8)\times 04(5+8)$
$\times 8n-8(7+8)\times 4n \;\; -8n+4(8+8)$

$$= 135680n^2 - 760256 - 12672n^3 + 299296n$$



Theorem 3. *The 1st and 2nd Multiplicative Hyper Zagreb index of a graph G is compute*

$HII_1(G) = 60381696\, n^2 - 51843200\, n + 1024$

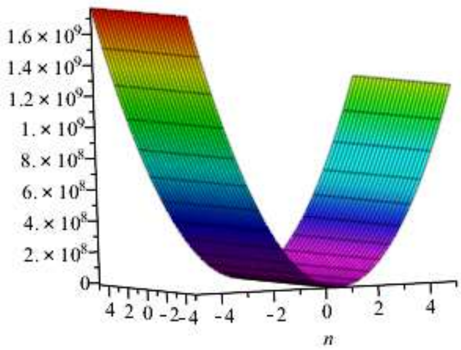
$HII_2(G) = 3203436544\, n^2 + 597922295808\, n + 15872$

Proof:

The first and second Multiplicative Hyper Zagreb Index

HII_{1}(G) = ∏_{hi ∈ E(G)}(d_G (h) + d_G (i))^{2}

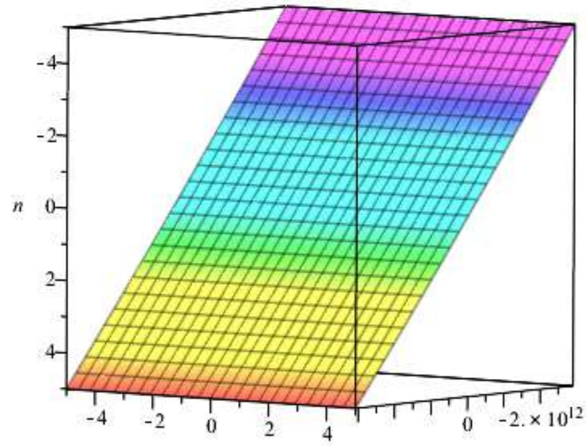
$= 4n(3+4)^2 \times 08(3+5)^2 \times (8n-8)(3+7)^2 \times 08(4+5)^2$
$\times 12n-12(4+7)^2 \times 8n\ 2 - 8+4(4+8)^2 \times 04(5+8)^2$
$\times 8n-8(7+8)^2 \quad \times 4n\ 2 - 8n+4(8+8)^2$
$= 60381696\, n^2 - 51843200\, n + 1024$



The second Multiplicative Hyper Zagreb Index

HII_{2}(G) = ∏_{hi ∈ E(G)}(d_G (h) × d_G (i))^{2}

$= 4n\ (3\times 4)^2 \times 08(3\times 5)^2 \times (8n-8)(3\times 7)^2$
$\times 08(4\times 5)^2 \times 12\text{-}12(4\times 7)^2 \times 8\ n^2 - 8+4(8\times 8)^2$
$= 3203436544\ n^2 + 597922295808n + 15872$



Theorem 4. *The multiplicative universal 1st Zagreb index is computed as*

$MZ_1^{\alpha}(G) = 2240\ n^{2\alpha}\ 64\alpha - 132\ n^{\alpha}5760^{\alpha} - n^2 12672^{\alpha}$
$\quad -96^{\alpha} \quad n + 120n^{\alpha}2496^{\alpha} - n^2 7680^{\alpha} - 128\ n^{\alpha} + 64^{\alpha}$

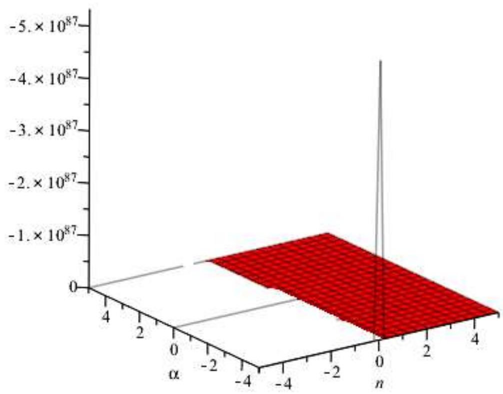
$MZ_2^{\alpha}(G) = 8064120^{\alpha}\ n^{2\alpha} - 336\ n^{\alpha}26880^{\alpha} - n^2 86016^{\alpha}$
$\quad -256^{\alpha}n + n9175040^{\alpha} - n^{\alpha}114688^{\alpha} - 512\ n^{\alpha} + 256^{\alpha}$

Proof:

The multiplicative general first Zagreb index

MZ_1^{α}{G} = ∏_{hi ∈ E(G)}{(d_G (h) + d_G (i))}^{α}

$= 4n(3+4)^{\alpha} \times 08(3+5)^{\alpha} \times (8n-8)(3+7)^{\alpha} \times 08(4+5)^{\alpha}$
$\times 12n-12(4+7)^{\alpha} \quad \times 8n \quad -8+4(4+8)^{\alpha} \times 04(5+8)^{\alpha}$
$\times 8n-8(7+8)^{\alpha} \quad \times 4n^2 \text{-}8n+4\ (8+8)^{\alpha}$
$= 2240\ n^{2\alpha}\ 64\alpha - 132\ n^{\alpha}5760^{\alpha} - n^2 12672^{\alpha}$
$-96^{\alpha} \quad n + 120n^{\alpha}2496^{\alpha} - n^2 7680^{\alpha} - 128\ n^{\alpha} + 64^{\alpha}$



The multiplicative general second Zagreb index

International Journal of Scientific Engineering and Science
Volume 8, Issue 6, pp. 15-19, 2024. ISSN (Online): 2456-7361

MZ_2^{α}{G} = ∏_{hi ∈ E(G)}{(d_G (h) × d_G (i))}^{α}

$= 4n(3\times4)^{\alpha} \times 08(3\times5)^{\alpha} \times (8n-8)(3\times7)^{\alpha} \times 08(4\times5)^{\alpha} \times 12n-12(4\times7)^{\alpha} \times 8n\ -8+4(4\times8)^{\alpha} \times 04(5\times8)^{\alpha} \times 8n-8(7\times8)^{\alpha} \times 4n^2 -8n+4\ (8\times8)^{\alpha}$

$= 8064120^{\alpha}\, n^{2\alpha} - 336n^{\alpha}26880^{\alpha} - n^2 86016^{\alpha}$
$-256^{\alpha}n + n9175040^{\alpha} - n^{\alpha}114688^{\alpha} - 512\, n^{\alpha} + 256^{\alpha}$

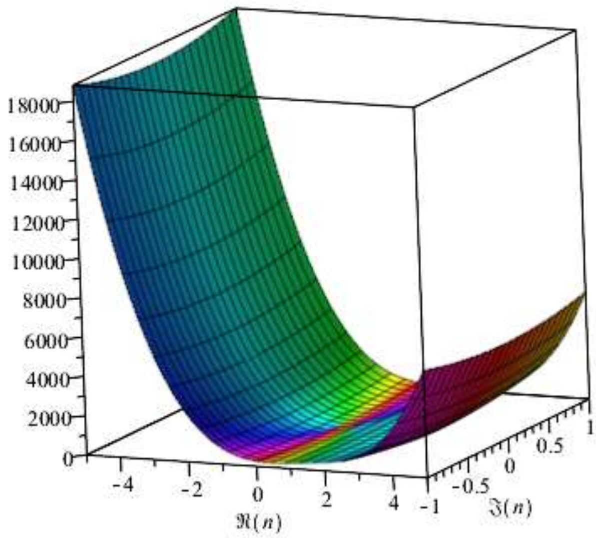
Theorem 5. *The sum connectivity index of a graph G is computed as*

$$XII(G) = \prod_{\{pq \in E(G)\}\backslash frac\{1\}\{\sqrt{\{d_{G(p)}+d_{G(q)}\}}\}}$$

Proof: *The Multiplicative summation connectivity*

$$=4n\frac{1}{\sqrt{3+4}} \times 08\frac{1}{\sqrt{3+5}} \times 8n - 8\frac{1}{\sqrt{3+7}} \times 08\frac{1}{\sqrt{4+5}} \times 12n - 12\frac{1}{\sqrt{4+7}} \times 8n^2 - 8n + 4\frac{1}{\sqrt{4+8}} \times 04\frac{1}{\sqrt{5+8}} \times 8n - 8\frac{1}{\sqrt{7+8}} \times 4n^2 - 8n + 4\frac{1}{\sqrt{8+8}}$$

$$=\frac{131072}{15015}\, n\sqrt{101}(n(2n^2 - 2n+1)-1^3(n^2-2n+1)$$



Theorem 6. *The graph G then the multiplicative connectivity indices*

$$ABCII(G) = \frac{128}{3}n^2\sqrt{6} - \frac{256}{5}\sqrt{30n} - \frac{144}{7}\sqrt{7}\,n^2 - 16n + 8\sqrt{22n}\,\frac{8}{7}\sqrt{182}\,n^2 + \frac{1}{2}\sqrt{14}$$

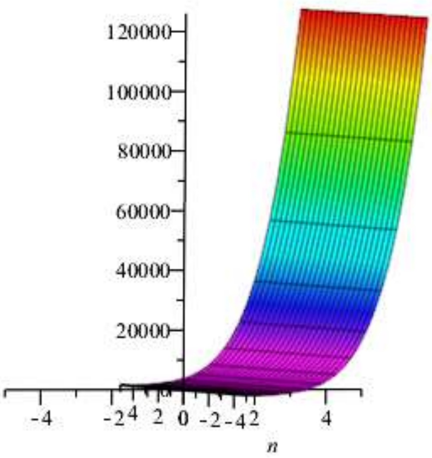
Proof:

The multiplicative connectivity Zagreb index

$$ABCII(G) = \prod_{\{ih \in E(G)\}\sqrt{\{\backslash frac\{d_{G(h)}+d_{G(i)}-2\}\{d_{G(h)}\times d_{G(i)}\}\}}}$$

$$= 4n\sqrt{\frac{3+4-2}{3\times4}} \times 08\sqrt{\frac{3+5-2}{3\times5}} \times 08 - 8\sqrt{\frac{3+7-2}{3\times7}} \times 08\sqrt{\frac{4+5-2}{4\times5}} \times 12n - 12\sqrt{\frac{4+7-2}{4\times7}} \times 8n^2 - 8n + 4\sqrt{\frac{4+8-2}{4\times8}} \times 4\sqrt{\frac{5+8-2}{5\times8}} \times 8n - 8\sqrt{\frac{7+8-2}{7\times8}} \times 4n^2 - 8n + 4\sqrt{\frac{8+8-2}{8\times8}}$$

$$\frac{128}{3}n^2\sqrt{6} - \frac{256}{5}\sqrt{30n} - \frac{144}{7}\sqrt{7}\,n^2 - 16n + 8\sqrt{22n}\,\frac{8}{7}\sqrt{182}\,n^2 + \frac{1}{2}\sqrt{14}$$



theorem 7. *The multiplicative geometric Index*

$$GAII(G) = \sqrt{3}\,n^2 - 16n\frac{2048}{39}\sqrt{5}\,n - \frac{128}{15}\sqrt{14}\,n^2 + 4$$

Proof:

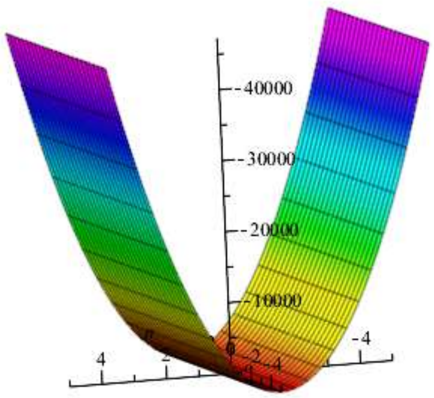
The algebraic arithmetic Index of graph

$$GAII(G) = \prod_{\{hi \in E(G)\}\backslash frac\{2\sqrt{\{d_{G(h)}\times d_{G(i)}\}}\}\{d_{G(h)}+d_{G(i)}\}}$$

$$= 4n\frac{\sqrt[2]{3\times4}}{3+4} \times 08\frac{\sqrt[2]{3\times5}}{3+5} \times 8n - 8\frac{\sqrt[2]{3\times7}}{3+7} \times 08\frac{\sqrt[2]{4\times5}}{4+5} \times 12n - 12\frac{\sqrt[2]{4\times7}}{4+7} \times 8n^2 - 8n + 4\frac{\sqrt[2]{4\times8}}{4+8} \times 04\frac{\sqrt[2]{5\times8}}{5+8} \times 8n - 8\frac{\sqrt[2]{7\times8}}{7+8} \times 4n^2 - 8n + 4\frac{\sqrt[2]{8\times8}}{8+8}$$

$$= \sqrt{3}\,n^2 - 16\,n\frac{2028}{39}\sqrt{5}\,n - \frac{128}{15}\sqrt{14}\,n^2 + 4$$

International Journal of Scientific Engineering and Science
Volume 8, Issue 6, pp. 15-19, 2024. ISSN (Online): 2456-7361



ABBREVIATION

Abbreviations	Words
$II_1(G)$	*Ist multiplicative Zagreb index*
$II_2(G)$	*2nd multiplicative Zagreb index*
$II_1{}^*(G)$	*A new multiplicative Ist Zagreb*
$HII_1(G)$	*Ist multiplicative Hyper Zagreb*
$HII_2(G)$	*2nd multiplicative Hyper Zagreb*
$MZ_1{}^\alpha(G)$	*General Ist multiplicative Zagreb*
$MZ_2{}^\alpha(G)$	*General 2nd Zagreb index*
$XII_2(G)$	*sum connectivity index*
$ABCII(G)$	*Atom-Bond connectivity index*
$GAII(G)$	*Geometric-Arithmetic index*
$GA^\alpha II(G)$	*General Geometric-Arithmetic*
$V(G)$	*vertex set of a graph*
$E(G)$	*Edge set of a graph*
CGT	*Chemical Graph Theory*

IV. CONCLUSION

In this paper we calculated a comprehensive analysis of the degree-based topological indices for Aztec diamond cross networks. By computing various Indices, including the multiplicative first Zagreb Index, second Zagreb Index, general first Zagreb Index, general second Zagreb Index, Hyper Zagreb Index, and sum connectivity indices, we provide a detailed understanding of the structural properties of these networks. These calculations provide insights into the structural properties of these networks, enhancing our understanding of their mathematical characteristics.

ACKNOWLEDGMENT

Although I am now pursuing my “Ph.D.” at a different institution, the lessons learned in Master under Dr. A.Q. Baig supervision continue to influence my research approach and academic growth.

REFERENCE

[1]. Baranyi, G., Saura, S., Podani, J., & Jordán, F. (2011). Contribution of habitat patches to network connectivity: Redundancy and uniqueness of topological indices. *Ecological Indicators*, *11*(5), 1301-1310.

[2]. Gao, H., Song, L., Guo, W., Huang, L., Yang, D., Wang, F., Zuo, Y., Fan, X., Liu, Z., & Gao, W. (2012). A simple method to synthesize continuous large area nitrogen-doped graphene. *Carbon*, *50*(12), 4476-4482.

[3]. García-Domenech, R., Gálvez, J., de Julián-Ortiz, J. V., & Pogliani, L. (2008). Some new trends in chemical graph theory. *Chemical Reviews*, *108*(3), 1127-1169.

[4]. Hayat, S., & Imran, M. (2014). Computation of topological indices of certain networks. *Applied Mathematics and Computation*, *240*, 213-228.

[5]. Imran, M., Baig, A. Q., Siddiqui, H. M. A., & Sarwar, R. (2017). On molecular topological properties of diamond-like networks. *Canadian Journal of Chemistry*, *95*(7), 758-770.

[6]. Iranmanesh, A., Hosseinzadeh, M., & Gutman, I. (2012). On multiplicative Zagreb indices of graphs. *Iranian journal of mathematical Chemistry*, *3*(2), 145-154.

[7]. Khan, R. H., Baig, A. Q., Kiran, R., Haider, I., Rizwan, M., & Elahi, A. (2020). M-polynomials and degree-based topological indices of dexamethasone, chloroquine and hydroxychloroquine; using in COVID-19. *International Journal of scientific Engineering and science*, *4*(7), 47-52.

[8]. Kulli, V. (2016). Multiplicative hyper-Zagreb indices and coindices of graphs: Computing these indices of some nanostructures. *International research journal of pure algebra*, *6*(7), 342-347.

[9]. Kulli, V. (2019). Some new multiplicative status indices of graphs. *International Journal of Recent Scientific Research*, *10*(10), 35568-35573.

[10]. Rosen, K. H., & Krithivasan, K. (1999). *Discrete mathematics and its applications* (Vol. 6). McGraw-Hill New York.

[11]. Siddiqui, M. K., Imran, M., & Ahmad, A. (2016). On Zagreb indices, Zagreb polynomials of some nanostar dendrimers. *Applied Mathematics and Computation*, *280*, 132-139.