

Displacement Determination of a Uniformly Loaded Concrete Beam using Finite Element Analysis

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Abstract— We analyzed the displacement state of a concrete beam subjected to a uniform load. The Mass (m), according to quantum theory is energy (E). This energy is in a constant state of flux. The continuous changes brought about by the displacement of particles in a solid, seems to deter normal explanation until a visible deformation occurs. Deformation usually occurs if displacement at the atomic and sub-atomic level are inhomogeneous. That is, there is variance in the path-lines of displacement. This differences in the path-line travelled by the particles between two points within a solid creates an extrema condition. Hence, the particles displaced can either take a straight path denoted $u(x)$, or a family of trajectories $\bar{u}(x)$. The variation between these two paths of displacements $u(x)$ and $\bar{u}(x)$, within a solid, is what determines the equilibrium configuration of a solid. If the variance is maximum, the solid will deform. If minimum or equal, the solid will then be stationary. This displacement analysis lays the mathematical foundation for the derivation of Euler-Lagrange equation of solid mechanics; with which, we derived the equivalent form of the governing differential equation from any given functional (F). Whose solution (displacement) $u(x)$ is obtained using finite element method.

Keywords— Displacement, Extrema, Stationary, Beam, Governing equation, Finite element.

I. INTRODUCTION

Displacement determination is the most preferred means for the dynamic analysis of solid structures. The determination of the primary field variable such as displacement is of utmost importance. There are different approaches used to determine the primary field variable some of which are as follows: [1] used a generalized method called linear variable differential transformer (LVDT) for displacement analysis and also highlights its limitation with structural systems as a function of height. [2] worked-on the inhomogeneous displacement of particles by numerically correlating a selected subset from the digitized intensified nature of the un-deformed object. Also, [3], and [4] centered their research on the digital image correlation (DIC) and particle image velocimetry (PIV), as some of the image base displacement analysis methods. Furthermore, [5] investigated the image processing based automated grid to determine the displacement and strain accuracy limit while [6] proposed a new strategy of obtaining displacement field from two different images at different times using an iterative optical flow method. Finite element analysis (FEA) had proved to be a powerful numerical tool in engineering analysis. In this paper, we deploy FEA to determine the displacement state of a concrete beam, subjected to a uniform load.

II. PROBLEM FORMULATION

A concrete beam of length x , fixed at both ends, with support at the center is subjected to a uniform load-force q . The total potential of the beam is represented by the functional $F = 1/2 AE \left(\frac{du}{dx}\right)^2 - qu$. where, the first term in the functional expresses the strain energy stored in the beam, and the second term represents the potential of the external force as shown in

figure 1. We are to determine the displacement state $u(x)$ in response to the loading force (F).

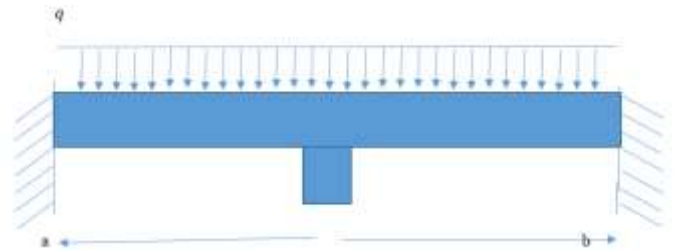


Figure 1: Concrete Beam Subjected to a uniformly Distributed Load

III. MATHEMATICAL FORMULATIONS

The beam, as a static object is mathematically represented by the definite integral [7]

$$I[u(x)] = \int_a^b F\left(u, \frac{du}{dx}, x\right) dx \quad (1)$$

This formulation is backed by the Principle of Stationary Total Potential (PSTP), characterizing the equilibrium configuration of the structure [9]. Which states that the problem of finding $u(x)$ that make "I" stationary with respect to small acceptable changes in $u(x)$ is equivalent to the problem of finding $u(x)$ that satisfies the governing differential equation for the problem [11]. Hence, we wish to find $u(x)$ which makes the functional stationary, subject to the boundary condition $u(a) = u_a$ and $u(b) = u_b$. Then for figure1, the functional $F = 1/2 AE \left(\frac{du}{dx}\right)^2 - qu$, and the prescribed boundary conditions are $u(0) = 0$, and $u(L) = 0$, since both ends of the beam are fixed.

Figure 2 depicts possible trajectories of displacement $\bar{u}(x)$ and also the unknown solution $u(x)$. This will enable to us study what happens to I in equation (1), if $u(x)$ slightly varies to $\bar{u}(x)$. That is,

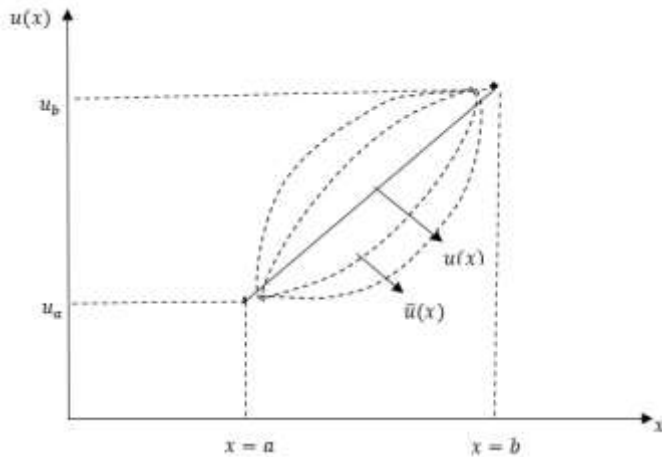


Figure 2: Admissible Solution $u(x)$

$$\bar{u}(x) = u(x) + \varepsilon \quad (2)$$

where ε is a small parameter.

The difference between $\bar{u}(x)$ and $u(x)$ is called the variation in $u(x)$; denoted by,

$$\delta u(x) = \bar{u}(x) - u(x) = \varepsilon \quad (3)$$

The distinction between du and δu at a given position x , is shown in Figure 3. The variation δu refer to the difference between $\bar{u}(x)$ and $u(x)$; while, d refers to the differential change in $u(x)$ as x changes to $x + dx$

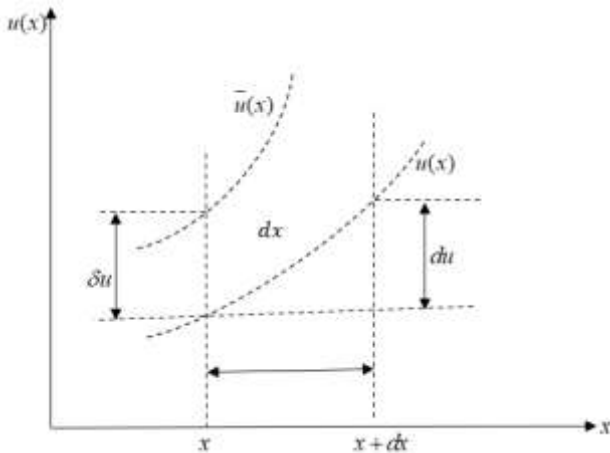


Figure 3: Difference between δu and du .

$\delta(u')$ is the difference in slope of $\bar{u}(x)$ and $u(x)$. That is,

$$\bar{u}'(x) - u'(x) = [\delta u'] \quad (4)$$

where ()' designates differentiation with respect to x [8].

For a given x , as we move from $u(x)$ to $\bar{u}(x)$, using equation (1)

$$\Delta F = F(\bar{u}, \bar{u}', x) - F(u, u', x) = F(u + \delta u, u' + \delta u', x) - F(u, u', x) \quad (5)$$

Expanding the first term in equation (5) by Taylor series [8]; we get,

$$F(u + \delta u, u' + \delta u', x) = F(u, u', x) + \left(\frac{\partial F}{\partial u} \delta u + \frac{\partial F}{\partial u'} \delta u' \right) + \frac{1}{2!} \left(\frac{\partial^2 F}{\partial u^2} \delta u^2 + 2 \frac{\partial^2 F}{\partial u \partial u'} \delta u \delta u' + \frac{\partial^2 F}{\partial u'^2} \delta u'^2 \right) \quad (6)$$

Hence

$$\Delta F = \left(\frac{\partial F}{\partial u} \delta u + \frac{\partial F}{\partial u'} \delta u' \right) + \frac{1}{2!} \left(\frac{\partial^2 F}{\partial u^2} \delta u^2 + 2 \frac{\partial^2 F}{\partial u \partial u'} \delta u \delta u' + \frac{\partial^2 F}{\partial u'^2} \delta u'^2 \right) \quad (7)$$

The first variation of F is defined as:

$$\delta F = \frac{\partial F}{\partial u} \delta u + \frac{\partial F}{\partial u'} \delta u' \quad (8)$$

And the second variation of F is:

$$\delta^2 F = \delta(\delta F) = \frac{\partial^2 F}{\partial u^2} \delta u^2 + 2 \frac{\partial^2 F}{\partial u \partial u'} \delta u \delta u' + \frac{\partial^2 F}{\partial u'^2} \delta u'^2 \quad (9)$$

So that,

$$\Delta F = \delta F + \frac{1}{2!} \delta^2 F$$

Studying what happens to ' I ', in the neighborhood of $u(x)$.

That is,

$$\begin{aligned} \Delta I &= I(\bar{u}, \bar{u}', x) - I(u, u', x) \\ &= \int_a^b F(\bar{u}, \bar{u}', x) dx - \int_a^b F(u, u', x) dx \\ &= \int_a^b \Delta F dx = \int_a^b \left(\delta F + \frac{1}{2!} \delta^2 F + \dots \right) dx \end{aligned} \quad (11)$$

The first variation of I is defined as:

$$\delta I = \int_a^b \delta F dx \quad (12)$$

And the second variation of I is given as

$$\delta^2 I = \int_a^b \delta^2 F dx \quad (13)$$

So that

$$\Delta I = \delta I + \frac{1}{2!} \delta^2 I \quad (14)$$

If the variation of the stationary I expresses the total potential of a structure, and we are looking for a stable equilibrium configuration; then, we wish to find $u(x)$ that minimizes I . Since $u(x)$ minimizes I , $\Delta I \geq 0$ as ε is reduced. $\Delta I \rightarrow 0$ when $\bar{u}(x) = u(x)$. Then I attains a minimum and $\Delta I \equiv \delta I = 0$ [7]

From equation (8) and (12); we get,

$$\delta I = \int_a^b \left(\frac{\partial F}{\partial u} \delta u + \frac{\partial F}{\partial u'} \delta u' \right) dx \quad (15)$$

Also, from equation (4); we have,

$$\delta(u') = (\delta u') = \frac{d}{dx} \delta u \quad (16)$$

Hence, substituting equation (16) into the second term in equation (15); that is,

$$\int_a^b \frac{\partial F}{\partial u'} \delta u' dx = \int_a^b \frac{\partial F}{\partial u'} \frac{d}{dx} \delta u dx = \int_a^b \frac{\partial F}{\partial u'} d(\delta u) \quad (17)$$

Performing integration by parts on the right hand term of equation (17); we get,

$$\int_a^b \frac{\partial F}{\partial u'} d(\delta u) = \left[\frac{\partial F}{\partial u'} \delta u \right]_a^b - \int_a^b (\delta u) \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) dx \quad (18)$$

Substituting equation (18) into equation (15); we obtain,

$$\delta I = \int_a^b \left[\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) \right] \delta u dx + \left[\frac{\partial F}{\partial u'} \delta u \right]_a^b \quad (19)$$

Since the beam is fixed at both ends, the changes in displacement $\delta u(a) = \delta u(b) = 0$.

Making the second term in equation (19) to vanish; leaving,

$$\delta I = \int_a^b \left[\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) \right] \delta u dx \quad (20)$$

$u(x)$ minimizes I when $\bar{u}(x) = u(x)$; implying, the variation in I ; i. e. $\delta I = 0$.

Leaving equation (20)

$$\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) = 0$$

Equation (21) is the famous Euler-Lagrange equation of solid mechanics [7] [10], the necessary condition for $I[u(x)] = \int_a^b F \left(u, \frac{\partial u}{\partial x}, x \right) dx$ to be an extremum (minimum or maximum). Where

$$F = \frac{1}{2} AE \left(\frac{du}{dx} \right)^2 - qu \quad (22)$$

Differentiating equation (22) with respect to the terms in equation (21).

That is,

$$\frac{\partial F}{\partial u} = -q \quad (23)$$

And

$$\frac{\partial F}{\partial u'} = AE \frac{du}{dx} \quad (24)$$

Substituting equation (23) and (24) into equation (21); we have,

$$-q - \frac{d}{dx} \left(AE \frac{du}{dx} \right) = 0$$

$$-\left(q + AE \frac{d^2u}{dx^2} \right) = 0$$

Rearranging the above; we get,

$$AE \frac{d^2u}{dx^2} + q = 0 \quad (25)$$

Equation (25) is the governing differential equation for the beam subjected to the uniformly distributed load q .

IV. SOLUTION TO THE DIFFERENTIAL EQUATION BY FINITE ELEMENT METHOD

The beam in Figure 1, is of length $L = 2l$ is discretized into two finite line elements as:

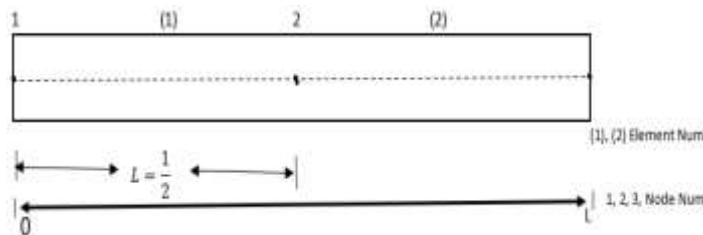


Figure 4: Describing the Shape Function for the Beam

Then, we introduce the general representation of interpolation or shape function for the field variable within an element valid for any number of nodes as [11].

$$u(x) = \left(1 - \frac{x}{l} \right) u_k + \left(\frac{x}{l} \right) u_{k+1} \quad (26)$$

We can, then, deduce the weighting function from equation (26) as:

Note: In figure 4, the beam has three load concentration points called nodes. Hence, we describe the displacement state at these three different points with the weight function $W(x)$ as [11]:

$$W_1(x) = 1 - \frac{x}{l} \quad 0 < x < l \quad \text{in the first element}$$

$$= 0 \quad 0 < x < l$$

l , in the second element

$$W_2(x) = \frac{x}{l} \quad 0 < x < l, \text{ in the first element}$$

$$= 1 - \frac{x}{l} \quad 0 < x < l, \text{ in the second element}$$

$$W_3(x) = 0 \quad 0 < x$$

$< l$, in the first element

$$= \frac{x}{l} \quad 0 < x < l, \text{ in the second element}$$

Also, we describe the primary field variable (displacement) in three states u_1, u_2 and u_3 corresponding to the weighting functions W_1, W_2 , and W_3 .

Substituting the above mentioned two components to the weak form representation of the governing differential equation.

That is,

The governing differential equation is,

$$AE \frac{d^2u}{dx^2} + q = 0$$

The weighted residual statement is given as,

$$\int_0^L W(x) \left(AE \frac{d^2u}{dx^2} + q \right) dx = 0 \quad (27)$$

Then, the weak form can be derived from the weighted residual statement [11] as follows:

Expanding equation (27); we get,

$$\int_0^L W(x) AE \frac{d^2u}{dx^2} dx + \int_0^L W(x) q dx = 0 \quad (28)$$

Applying integration by part, on the first integral of equation (28); that is,

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx} \quad (29)$$

$$u = W(x); \quad \frac{du}{dx} = \frac{dW(x)}{dx}$$

$$\frac{dv}{dx} = AE \frac{d^2u}{dx^2}; \quad v = AE \frac{du}{dx}$$

Coupling the above into equation (29); we have,

$$\int_0^L W(x) AE \frac{d^2u}{dx^2} dx = \left[W(x) AE \frac{du}{dx} \right]_0^L - \int_0^L AE \frac{dW}{dx} \frac{du}{dx} dx \quad (30)$$

Substituting the right hand part of equation (30) into equation (28); we get,

$$\left[W(x) AE \frac{du}{dx} \right]_0^L - \int_0^L \frac{dW}{dx} \frac{du}{dx} dx + \int_0^L W(x) q dx = 0$$

Re-arranging the above, we have the weak form of our governing as:

$$\int_0^L AE \frac{dW}{dx} \frac{du}{dx} dx = \int_0^L W(x) q dx + \left[W(x) AE \frac{du}{dx} \right]_0^L \quad (31)$$

Recalling from the general shape function in equation (26)

$$u(x) = \left(1 - \frac{x}{l}\right)u_k + \left(\frac{x}{l}\right)u_{k+1}$$

K= 1, 2, 3.

$$\frac{du}{dx} = -\frac{1}{l}u_k + \frac{1}{l}u_{k+1} = \begin{bmatrix} -\frac{1}{l} & \frac{1}{l} \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \end{bmatrix}$$

$$= \frac{(-u_k + u_{k+1})}{l}$$

$$= \frac{(u_{k+1} - u_k)}{l} \quad (32)$$

And

$$W_1(x) = 1 - x/l; \quad \frac{dW_1}{dx} = -\frac{1}{l}$$

$$W_2(x) = x/l; \quad \frac{dW_2}{dx} = \frac{1}{l}$$

$$W_3(x) = 0 \quad \frac{dW_3}{dx} = 0$$

Substituting the derivatives $\frac{du}{dx}$ and $\frac{dW}{dx}$ into:

L. H. S. of equation (31)

$$\int_0^L AE \frac{dW_1}{dx} \frac{du}{dx} dx = \sum_1^2 \int_0^l AE \frac{du}{dx} \frac{dW_1}{dx} dx$$

$$= \int_0^l AE \left(\frac{u_2 - u_1}{l}\right) \left(-\frac{1}{l}\right) dx + 0$$

$$= \frac{AE}{l} [u_1 - u_2] \quad (31(i))$$

$$\int_0^L AE \frac{dW_2}{dx} \frac{du}{dx} dx = \sum_1^2 \int_0^l AE \frac{du}{dx} \frac{dW_2}{dx} dx$$

$$= \int_0^l AE \left(\frac{u_2 - u_1}{l}\right) \left(\frac{1}{l}\right) dx$$

$$+ \int_0^l AE \left(\frac{u_3 - u_2}{l}\right) \left(-\frac{1}{l}\right) dx$$

$$= \frac{AE}{l} [(-u_1 + u_2) + (u_2 - u_3)] \quad (31(ii))$$

$$\int_0^L AE \frac{dW_3}{dx} \frac{du}{dx} dx = \sum_1^2 \int_0^l AE \frac{du}{dx} \frac{dW_3}{dx} dx$$

$$= 0 + \int_0^l AE \left(\frac{u_3 - u_2}{l}\right) \left(\frac{1}{l}\right) dx$$

$$= \frac{AE}{l} [u_3 - u_2] \quad (31(iii))$$

Right hand first term of equation (31):

$$\int_0^L W_1(x) q dx = \sum_1^2 \int_0^l W_1(x) q dx = \int_0^l \left(1 - \frac{x}{l}\right) q dx + 0$$

$$= \frac{ql}{2} \quad (31(iv))$$

$$\int_0^L W_2(x) q dx = \sum_1^2 \int_0^l W_2(x) q dx$$

$$= \int_0^l \left(\frac{x}{l}\right) q dx + \int_0^l \left(1 - \frac{x}{l}\right) q dx = \frac{ql}{2}$$

$$+ \frac{ql}{2} \quad (31(v))$$

$$\int_0^L W_3(x) q dx = \sum_1^2 \int_0^l W_3(x) q dx = 0 + \int_0^l \left(\frac{x}{l}\right) q dx$$

$$= \frac{ql}{2} \quad (31(vi))$$

Right hand second term of equation (31):

$$\left[W_1(x) AE \frac{du}{dx}\right]_0^L = \sum_1^2 \left[W_1(x) AE \frac{du}{dx}\right]_0^l = \left[\left(1 - \frac{x}{l}\right) P\right] + 0$$

$$= -P_0 \quad (31(vii))$$

$$\left[W_2(x) AE \frac{du}{dx}\right]_0^L = \sum_1^2 \left[W_2(x) AE \frac{du}{dx}\right]_0^l$$

$$= \left[\frac{x}{l} P\right] + \left[\left(1 - \frac{x}{l}\right) P\right] + 0$$

$$= P_l - P_0 \quad (31(viii))$$

$$\left[W_3(x) AE \frac{du}{dx}\right]_0^L = \sum_1^2 \left[W_3(x) AE \frac{du}{dx}\right]_0^l = 0 + \left[\frac{x}{l} P\right]$$

$$= P_l \quad (31(ix))$$

Where

$$AE \frac{du}{dx} = P, \text{ is the body force.}$$

Hence assembling the system level equation of equation (31) (i.e. from eqn 31(i) to 31(ix))

$$\frac{AE}{l} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{ql}{2} \\ \frac{ql}{2} + \frac{ql}{2} \\ \frac{ql}{2} \end{bmatrix} + \begin{bmatrix} -P_0 \\ P_l - P_0 \\ P_l \end{bmatrix} \quad (33)$$

$$\frac{AE}{l} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} + \begin{bmatrix} f_B \\ f_B \\ f_B \end{bmatrix} \quad (34)$$

K U = F

$$F = KU \quad (35)$$

Where, F is the loading force, K is the global stiffness matrix, and U is the primary field variable (displacement)

Hence, the condense equation is;

$$K \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_2 \\ F_3 \end{bmatrix}$$

On simplifying

$$Ku_2 = F_2 \quad (36)$$

Assuming the stiffness constant $K = 2$; then,

$$F = 2U$$

Equation (37) is used to generate the following table:

TABLE 1: Force and Displacement Values.

F	0	1	2	3	4	5
U	0	2	4	6	8	10

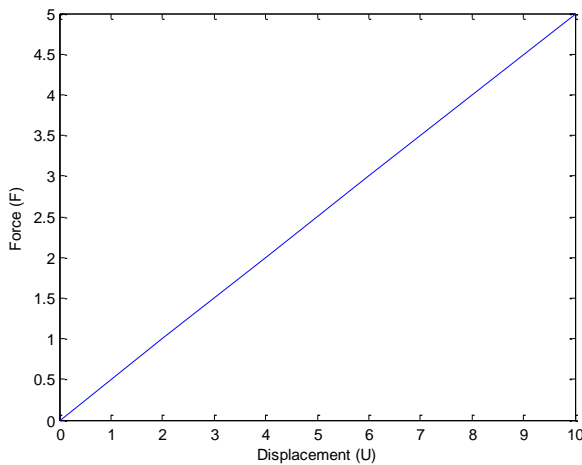


Figure 5: Graph of Force (F) against Displacement (U)

V. RESULT/ DISCUSSION

The graph in figure 5 shows a proportional relationship, between the loading force (F), and the displacement state (U) of the beam [12]. This relationship is purely elastic in nature. The beam which is made of concrete, cannot exceed its elastic limit, into its yielding region, before deformation. Confirming a phenomenon common to all brittle materials.

VI. CONCLUSION

The simulation of a problem, formulated in analytic form as a functional and its equivalent governing differential equation is made possible, by deploying finite element analysis. Hence FEA enables the simplification of a complex differential equation into a linear system of equations, whose primary field variables we have determine in this research. In the future, we would compute also the secondary field variables (strain and stress).

Declaration of Competing Interest The authors declare that they have no known competing of interest

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