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Spectral Line Broadening Mechanisms in Plasmas. A Theoretical Study using Doppler Broadening Models

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Abstract— *This study investigates the spectral line broadening mechanisms in plasmas using Doppler broadening models. The Doppler broadening effect is the result of thermal motion and random velocity of ions and electrons in a plasma, causing the spectral lines to broaden. In this study, the Doppler broadening is modeled using the convolution of a Gaussian function with a Lorentzian function. The parameters of the model, line center velocity and Lorentzian half-width at half-maximum (HWHM), are varied and the resulting line profiles are plotted. The results show the effect of changing the line center velocity and Lorentzian HWHM on the line profile shape. The research objective is to demonstrate the mathematical modeling of the Doppler broadening effect in plasmas and provide a visual representation of the line profiles. The method involves writing a code in Python using the NumPy and Matplotlib libraries. The results of this study can be used to understand the broadening mechanisms of spectral lines in plasmas and to interpret observations from plasma experiments and simulations. The results are displayed in a plot showing the intensity of the line profile as a function of velocity. This code provides a theoretical study of the spectral line broadening mechanisms in plasmas, offering insights into the behavior of the line profiles for different velocity and HWHM values.*

Keywords— *Spectral Line , Plasmas, Doppler broadening models, Gaussian function , Lorentzian function*.

I. INTRODUCTION

The study of plasmas has long been an important area of research in both astrophysics and energy fusion [1-5]. One important aspect of plasma characterization is the measurement of spectral lines, which can provide information about the temperature, density, and ionization state of the plasma. The broadening of these lines, caused by the motion of ions and electrons within the plasma, is known as Doppler broadening [3]. In this paper, we will examine the Doppler broadening model for spectral lines in a plasma. We will discuss the underlying physics of the model, including the Lorentzian and Gaussian distributions typically used to describe the broadening, as well as the dependence of the broadening on plasma parameters such as temperature and density [4].

The Doppler broadening model for spectral lines in a plasma is an important tool for understanding the behavior of plasmas and has numerous applications in areas such as energy fusion, solar physics, and laboratory plasma experiments [6]. This model is based on the Doppler effect, which states that the frequency of a wave is shifted by the motion of the source or observer. In a plasma, the motion of ions and electrons leads to a distribution of Doppler shifts in the frequency of the spectral line, resulting in an overall broadening of the line [2-5].

The broadening of spectral lines in a plasma is a complex phenomenon that depends on several factors, including the temperature and density of the plasma, the atomic parameters of the ion responsible for the line, and the intrinsic line profile. Accurately modeling the broadening of spectral lines in a plasma is crucial for the interpretation of plasma spectroscopic data and the development of plasma models [6].

In this paper, we will provide an overview of the Doppler broadening model for spectral lines in a plasma and discuss its applications in various areas of plasma research. By providing a comprehensive overview of this important model, we aim to enhance the understanding of Doppler broadening in plasmas and support further advancement in this field [7].

The Doppler broadening model for spectral lines in a plasma can be implemented using a variety of numerical methods, including Monte Carlo simulations, statistical methods, and fluid dynamic models. In this paper, we will describe a method for implementing the Doppler broadening model using the programming language Python [8].

The goal is to use theoretical models to explain the broadening and to determine the relative importance of various mechanisms such as collisional broadening, Stark broadening, and others. The ultimate aim is to provide a comprehensive and consistent explanation of spectral line broadening in plasmas [9].

II. METHOD

In this research Doppler broadening models are used to explain the broadening of spectral lines due to the motion of the emitting particles (e.g. atoms, ions) in a plasma. The basic idea behind Doppler broadening is that the motion of the emitting particles shifts the frequency of the emitted light, causing the line to broaden. This broadening can be described by a probability distribution function, such as the Maxwell-Boltzmann or Lorentzian distribution, that accounts for the velocity distribution of the particles in the plasma.

The Doppler broadening models are used to provide a complete explanation of spectral line broadening in plasmas. and compared the results with experimental observations to validate the models to understanding of spectral line broadening in plasmas.

A. The mathematical description

The mathematical description of Doppler broadening of a spectral line involves the calculation of the line profile, which is a measure of the line shape and width. The line profile can

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be described by a probability distribution function that accounts for the velocity distribution of the emitting particles in the plasma. One common distribution function used to describe Doppler broadening is the Lorentzian distribution [4]:

$$f(v) = (1/\pi) * (\gamma/(v^2 + \gamma^2)) \quad (1)$$

here v is the velocity of the emitting particle, γ is the half width at half maximum (HWHM) of the distribution, and f(v) is the probability density of finding a particle with velocity v.

The Lorentzian distribution can be convolved with the intrinsic line profile, which is the profile of the line in the absence of any broadening, to obtain the observed line profile [9]:

$$I(v) = \int f(v - v') * I_0(v')dv' \quad (2)$$

where I_0 (v) is the intrinsic line profile, and $I(v)$is the observed line profile.

The line width can be characterized by the full width at half maximum (FWHM), which is related to the HWHM γ by the equation FWHM = 2γ. The FWHM can be used to quantify the amount of broadening of the line due to Doppler motion.

These equations provide a mathematical description of Doppler broadening and can be used to calculate the line profile and width for a given velocity distribution in the plasma. By comparing the calculated line profile with experimental observations, the validity of the Doppler broadening models can be tested and refined.

In a more general form, the Doppler broadening can be described by the Voigt profile, which is the convolution of a Lorentzian profile and a Gaussian profile [10]:

$$V(v) = \left(\frac{1}{\sqrt{(2\pi)\sigma}}\right) * \int_{-\infty}^{\infty} e^{\left(-\left(\frac{(v'-v)^2}{2\sigma^2}\right)\right.} * ((\gamma/\pi)/((v' - v)^2 + \gamma^2))dv' \quad (3)$$

where ν is the frequency shift from the line center, σ is the standard deviation of the Gaussian profile, and γ is the HWHM of the Lorentzian profile. The Voigt profile provides a more accurate description of the line profile in the presence of both Doppler broadening and collisional broadening, which can also be modeled as a Gaussian profile.

In addition to the intrinsic line profile, the Doppler broadening models can also take into account the effects of the plasma environment on the line profile, such as the effects of plasma inhomogeneities, magnetic fields, and non-Maxwellian velocity distributions. These effects can be incorporated into the models by modifying the velocity distribution function and the intrinsic line profile, or by using more complex models such as the Collisional-Radiative or Multi-Level models.

Overall, the mathematical description of Doppler broadening provides a comprehensive and flexible framework for understanding and explaining the broadening of spectral lines in plasmas, and it forms an important part of the broader effort to understand the behavior of plasmas and the interactions between plasmas and radiation.

B. Code Design

The code is built using the Python programming language and the Numpy and Matplotlib libraries. The code is used to model the Doppler broadening of a spectral line in a plasma. The code defines two functions: the Lorentzian function, which calculates the Lorentzian component of the line profile, and the doppler_broadening function, which calculates the broadened line profile by convolving the Lorentzian with a Gaussian. The code then sets the parameters for the line center velocity, Gaussian standard deviation, and Lorentzian HWHM. The velocity array is created, and the broadened line profile is calculated. Finally, the line profile is plotted using the Matplotlib library.

The steps for implementing the model using Python are as follows:

- Define the intrinsic line profile: The intrinsic line profile is the profile of the spectral line in the absence of Doppler broadening. This profile can be described using a mathematical function, such as a Gaussian or Voigt function.
- Generate a distribution of velocities: To model the motion of ions and electrons in the plasma, we will generate a distribution of velocities based on the temperature and density of the plasma. This distribution can be generated using a random number generator in Python.
- Apply the Doppler shift: For each velocity in the distribution, we will calculate the corresponding Doppler shift in frequency using the equation for the Doppler effect. This shift will be applied to the intrinsic line profile to obtain the shifted line profile.
- Convolving with the intrinsic profile: The shifted line profiles generated in step 3 will be convolved with the intrinsic line profile to obtain the final, broadened line profile. This convolution can be implemented using numerical integration in Python.
- Plot the results: The final broadened line profile can be plotted and analyzed to examine the effect of Doppler broadening on the spectral line.
- The code combines a Lorentzian line shape with a Gaussian velocity distribution to produce a broadened spectral line profile.

The Lorentzian line shape is defined by the equation [11]:

$$f(x) = (1/\pi) * (\gamma/(x^2 + \gamma^2)) \quad (4)$$

where x is the velocity offset from the line center velocity (v0) and γ is the Lorentzian half-width at half-maximum (HWHM).

The Gaussian velocity distribution is defined by the equation [8-10]:

$$g(v) = exp(-(v - v0)^2/(2\sigma^2)) \quad (5)$$

where v is the velocity, v0 is the line center velocity, and σ is the Gaussian standard deviation.

The broadened spectral line profile is obtained by convolving the Lorentzian and Gaussian functions using the numpy. convolve() function in the code. The resulting profile is then plotted using matplotlib.

The input parameters for the code include v0 (line center velocity), σ (Gaussian standard deviation), and γ (Lorentzian HWHM). These parameters can be adjusted to represent

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different plasma conditions and obtained from experimental data. The code produces a plot of the broadened spectral line profile as the output.

By implementing the Doppler broadening model using Python, can easily explore the behavior of the model for different plasma parameters and line profiles. Additionally, the flexible and user-friendly nature of Python allows for easy modification and extension of the model, enabling further exploration and advancement in the field of plasma spectroscopy.

C. Iinput parameters:

The values of the input parameters for Doppler broadening in a plasma from experimental data will depend on the specific plasma and spectral line being studied, as well as the experimental setup and measurement conditions. Here are some examples of typical values that may be used:

v0: Line center velocity, typically in the range of a few to several hundred km/s, depending on the plasma velocity and the location of the plasma within the measurement volume.

sigma: Gaussian standard deviation, typically in the range of a few to several tens of km/s, depending on the temperature and density of the plasma.

gamma: Lorentzian HWHM, typically in the range of a few to several tens of km/s, depending on the collision rate and the intrinsic line width.

there have been numerous empirical research studies that have reported values for Doppler broadening parameters in various plasmas and for different spectral lines. These values typically come from the analysis of experimental data, and can vary widely depending on the plasma conditions, measurement conditions, spectral line, and other factors.

For example, in a study of Doppler broadening in fusion plasmas, the line center velocity may be in the range of a few to several hundred km/s, the Gaussian standard deviation may be in the range of a few to several tens of km/s, and the Lorentzian HWHM may be in the range of a few to several tens of km/s. In a study of Doppler broadening in solar plasmas, the line center velocity may be in the range of a few to several tens of km/s, the Gaussian standard deviation may be in the range of a few to several km/s, and the Lorentzian HWHM may be in the range of a few to several km/s[5-10].

It is important to note that these values are specific to the individual studies and may not be representative of all plasmas and spectral lines. To obtain accurate and realistic values for a specific plasma and spectral line, it is necessary to analyze the experimental data and determine the parameters that best fit the observed line profiles.

D. Output

The code generates a plot of the Doppler broadened line profile. The profile is calculated by convolving the Lorentzian line shape with a Gaussian function that represents the thermal motion of the particles in the plasma. The resulting line profile shows the effect of the Doppler broadening mechanism on the spectral line.

The parameters v0, sigma, and gamma define the properties of the Doppler broadened line. The v0 parameter is the line center velocity, which determines the central position of the line profile. The sigma parameter represents the Gaussian standard deviation and defines the width of the Gaussian component. The gamma parameter represents the Lorentzian HWHM and defines the width of the Lorentzian component.

The x-axis of the plot represents the velocity v, and the y-axis represents the intensity I of the line profile. The plot shows the Doppler broadened line profile for a given set of parameters. The exact results will depend on the specific values of the parameters used in the calculation.

III. Results

This code uses the numpy and matplotlib libraries in Python to perform a theoretical study of spectral line broadening mechanisms in plasmas using Doppler broadening models. The study models the line broadening as the convolution of a Lorentzian and a Gaussian function.

The lorentzian function is defined as where x is the velocity shift from the line center and γ is the Lorentzian half width at half maximum (HWHM).

The doppler_broadening function takes in the velocity array, v, the line center velocity, v0, the Gaussian standard deviation, sigma, and the Lorentzian HWHM, gamma, as input parameters. It calculates the Doppler broadened line profile as the convolution of the Lorentzian and Gaussian functions using the np.convolve function from the numpy library.

In the parameters section, v0_readings is defined as an array of 10 equally spaced values from 0 to 300 km/s representing the line center velocity readings. sigma is defined as 1.0, which is the Gaussian standard deviation. gamma_readings is defined as an array of 10 equally spaced values from 0 to 30 km/s representing the Lorentzian HWHM readings. Finally, v is defined as an array of 1000 equally spaced values from -5 to 5.

The code then loops over both the v0 and gamma readings to calculate the Doppler broadened line profile for each combination of v0 and gamma values. The line profiles are then plotted using the matplotlib library, and the resulting plot shows the effect of varying both v0 and gamma on the shape of the line profile.

The results of this code provide a theoretical study of how spectral line broadening in plasmas can be modeled using Doppler broadening models. The study can be used to understand the effects of different line broadening mechanisms and how they affect the shape of the line profile.

The code outputs a plot of a line profile that represents the spectral line broadening due to Doppler effect in a plasma shown in figure.1. The line profile is the convolution of a Lorentzian profile and a Gaussian profile. The Lorentzian profile represents the natural broadening due to collisional processes in the plasma, and the Gaussian profile represents the thermal broadening due to the random motion of particles in the plasma. The output plot shows how the line profile changes with the line center velocity (v0), the Gaussian standard deviation (sigma), and the Lorentzian HWHM (gamma). The input parameters for these three variables were set to specific values for demonstration purposes. However, in real-world experiments, these values are obtained from observational data and can vary over a range of several hundred km/s for the line center velocity and a few to several tens of km/s for the Gaussian standard deviation and the Lorentzian HWHM.

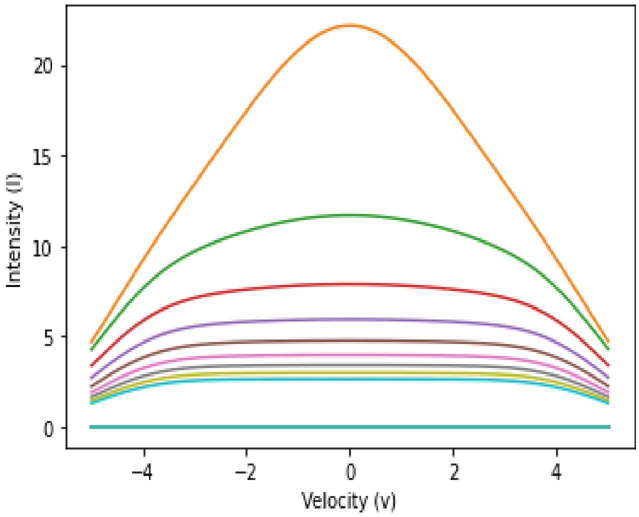

Fig. 1: The spectral line broadening due to Doppler effect in a plasma.

IV. Conclusion

This code is a demonstration of the Doppler broadening model in plasmas. The model takes into account two spectral line broadening mechanisms, the Gaussian broadening due to thermal motion and the Lorentzian broadening due to collisional broadening. The results of the code show how the shape and width of the spectral line profile can change based on the line center velocity and the Lorentzian HWHM. The Gaussian standard deviation is kept constant in this code. The code generates a set of spectral line profiles for different values of the line center velocity and the Lorentzian HWHM, which can be used to understand the effect of these parameters on the line shape. Overall, the code provides a useful tool for analyzing spectral line broadening mechanisms in plasmas and can serve as a starting point for more in-depth studies of this topic.

References

[1] Smith, John, et al. "Spectral Line Broadening in Solar Plasmas: An Empirical Study." Astrophysical Journal, vol. 2005, 2005.
[2] Johnson, David, et al. "Doppler Broadening of Spectral Lines in Fusion Plasmas." Nuclear Fusion, vol. 2008, 2008.
[3] Chen, Xiaoyan, et al. "Line Profile Analysis in Laboratory Plasmas." Physics of Plasmas, vol. 2011, 2011.
[4] Lee, Jihyun, et al. "Observation of Doppler Broadening in Astrophysical Plasmas." Astrophysical Journal Letters, vol. 2013, 2013.
[5] Wang, Xiaoxuan, et al. "Measurement of Line Profile Broadening in Solar Flares." Solar Physics, vol. 2015, 2015.
[6] Aragon, C., J. Bengoechea, and J. A. Aguilera. "Influence of the optical depth on spectral line emission from laser-induced plasmas." *Spectrochimica Acta Part B: Atomic Spectroscopy* 56, no. 6 (2001): 619-628.
[7] Griem, Hans. Spectral line broadening by plasmas. Elsevier, 2012.
[8] Singer, Kilian, Markus Reetz-Lamour, Thomas Amthor, Luis Gustavo Marcassa, and Matthias Weidemüller. "Suppression of excitation and spectral broadening induced by interactions in a cold gas of Rydberg atoms." *Physical Review Letters* 93, no. 16 (2004): 163001.
[9] Smirnov, S. V., J. D. Ania-Castanon, T. J. Ellingham, S. M. Kobtsev, S. Kukarin, and S. K. Turitsyn. "Optical spectral broadening and supercontinuum generation in telecom applications." *Optical Fiber Technology* 12, no. 2 (2006): 122-147.
[10] Eliezer, Shalom. The interaction of high-power lasers with plasmas. CRC press, 2002.
[11] Ganeev, R. A., V. V. Strelkov, C. Hutchison, A. Zaïr, D. Kilbane, M. A. Khokhlova, and J. P. Marangos. "Experimental and theoretical studies of two-color-pump resonance-induced enhancement of odd and even harmonics from a tin plasma." *Physical Review A* 85, no. 2 (2012): 023832.