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Modeling Nonlinear Partial Differential Equations in an Inductive Electrical Line and Construction of Implicit Wave Solutions

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Abstract— *In this work, we model the nonlinear partial differential equations that describe the dynamics of propagation of signals (waves) in the carefully constructed inductive power line. This modeling is based on the appropriate choice of the magnetic flux of connection between the inductors in the electric line. The signals sought are naturally the solutions of the partial differential equations obtained for this purpose. These equations not being easily integrable, it is advisable for us to make the choice of the forms of solutions to be constructed. Faced with the need to make the right choice, we use the iB-function with unknown characteristics as a basic signal to finally determine with exactness the solution functions (signals) via the notion of probability of appearance of the characteristic indices of the iB-function in the general equation of range.*

Keywords— *field of possible solutions; kink; iB- functions; Inductive electrical line; nonlinear partial differential equations; range equations; solitary waves.*

I. INTRODUCTION

The analytical, experimental and numerical study form the three axes which are most often at the base of the study of a physical phenomenon. If we place ourselves in this perspective, we can start a study by its analytical aspect based on a theoretical design so as to obtain the mathematical equations whose search for solutions allows to know more about the applicability of the study through appropriate numerical simulations. A study can also start with the experimental, that is to say, the manufacture of the practical device which produces certain measurable effects and whose in-depth understanding requires a complete modeling through mathematical equations and whose numerical simulations reinforce the understanding of the study carried out. To circumscribe this approach in the field of the propagation of waves and signals in waveguides, we note that the propagation of a wave or a signal constitutes the basis of information technologies. It is for this reason that for a very long time, researchers from various horizons have been involved in everything that participates in the dynamics of wave propagation in waveguides and everything that goes into their production. It is in this logic that we have listed many works on electric lines [1-9], atomic chains [10-12], microstructures [13,14], optical fibers [15-24], etc. when each waveguide or propagation circuit considered is taken individually; each modification in their constitution induces functional modifications. We can broadly say that the mode of operation of each circuit to remain in the case of the power line is closely related to its constituent elements. In this work, we start from the experimental design of a previously constituted electric line to model the nonlinear partial differential equations likely to govern the dynamics of wave propagation in the electric line. The particularity of this study is the taking into account of a nonlinear magnetic flux linking the inductance coils in the circuit [7]. The global partial differential equation being established, corrections are subsequently made to the chosen flux as a function of the solutions (signals) that one would like to propagate in the line. To obtain signal shapes, we have assumed that the solution signal is analytically embodied by the analytical sequence of the iB-function [25-30]. It is this analytical approach which led us to the identification of solitary wave solutions [31-39] or not, likely to propagate in the circuit constituted by means of the adjustments which one imposed on the initial circuit and especially of the form of the magnetic flux.

This study is globally organized as follows: In section 2, the analytical and experimental device is drawn and the nonlinear partial differential equation generally governing the dynamics of propagation is established. in section 3, the range of coefficients equation on which it the different solutions will be discussed is established. Section 4, determines the characteristics (n, m) of the iB-function for which certain terms of the range equation are grouped. In section 5, the implicit and trigonometric solutions are searched. Section 6 is dedicated to the study of the propagation of some solutions. We naturally end the study with a general conclusion which returns to the important points of the work as well as the perspectives.

II. GENERAL MODELING OF A NONLINIEAR INDUCTIVE ELECTRICAL LINE

The electrical line below is made up of several identical networks where G is the conductance of the resistor, R being the resistance of any resistor connected in series with an inductor whose magnetic flux linkage $\phi(i_n)$ varies nonlinearly with respect to the current flowing through the inductor.

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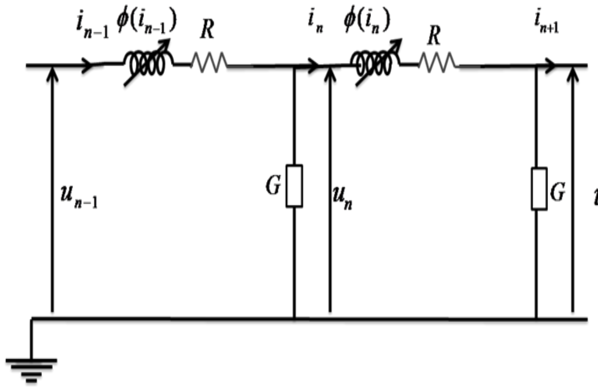

Fig. 1. Presentation of a nonlinear inductive electrical line.

By applying Kirchoff's law on the constituted circuit, we obtain as in [4-9]the following relations on the voltage and the current

$$u_n - u_{n+1} = Ri_n + \frac{\partial \phi_n}{\partial t}, \quad (1)$$

and

$$i_n = i_{n+1} + Gu_{n+1}, \quad (2)$$

where n is a positive integer that numbers each network of the line, i_n the current flowing through the inductor network of order n and i_{n+1} the current flowing through the inductor network of order $n+1$, u_n and u_{n+1} are respectively the voltages across the conductance G of network n and the network of order $n+1$, ϕ_n is the nonlinear magnetic flux linkage of the inductor of network of order n.

Taking into account equation (1) and (2) we obtain

$$i_n = i_{n+1} + Gu_n - G\frac{\partial \phi_n}{\partial t} - RGi_n. \quad (3)$$

Substituting $Gu_n = i_{n-1} - i_n$ of (2) in (3) gives rise to the differential equation

$$i_{n+1} - 2i_n + i_{n-1} = G\frac{\partial \phi_n}{\partial t} + RGi_n. \quad (4)$$

We approximate the left hand side of equation (4) to a spatial partial derivative using Taylor expansion with respect to $x = nh$ which is a distance measure from the beginning of the line. h is the distance that separate two consecutive nodes. The fourth order Taylor expansion of i_{n+1} and i_{n-1} are as follows:

$$i_{n+1} = i_n + \frac{h}{1!}\frac{\partial i_n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 i_n}{\partial x^2} + \frac{h^3}{3!}\frac{\partial^3 i_n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 i_n}{\partial x^4}, \quad (5)$$

and

$$i_{n-1} = i_n - \frac{h}{1!}\frac{\partial i_n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 i_n}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 i_n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 i_n}{\partial x^4}. \quad (6)$$

The left hand side of equation (4) takes the form

$$i_{n+1} - 2i_n + i_{n-1} = h^2\frac{\partial^2 i_n}{\partial x^2} + \frac{h^4}{12}\frac{\partial^4 i_n}{\partial x^4}. \quad (7)$$

Soughing (7) in (4) gives

$$-h^2\frac{\partial^2 i_n}{\partial x^2} - \frac{h^4}{12}\frac{\partial^4 i_n}{\partial x^4} + G\frac{\partial \phi_n}{\partial t} + RGi_n = 0. \quad (8)$$

The continuum model of the nonlinear inductive electrical line of Figure 1 is given by the nonlinear partial differential equation

$$-h^2\frac{\partial^2 i(x,t)}{\partial x^2} - \frac{h^4}{12}\frac{\partial^4 i(x,t)}{\partial x^4} + G\frac{\partial \phi(i(x,t))}{\partial t} + RGi(x,t) = 0. \quad (9)$$

Equation (9) is that which describes the dynamics of the movement of the current in the circuit. Thus, the nonlinear aspect of the circuit can be embodied by the magnetic flux through the circuit. If we choose to introduce a nonlinear flow of the form

$$\phi(i(x,t)) = B_1 i^4(x,t) + B_2 i^2(x,t) + B_3 In(i^2(x,t) - B_0^2), \quad (10)$$

with $|i(x,t)| \succ |B_0|$, $B_1; B_2$ and B_3 are non-nil real numbers. A substitution of (10) in (9) gives

$$\begin{aligned}
&\frac{B_0^2 h^4}{12}\frac{\partial^4 i(x,t)}{\partial x^4} - \frac{h^4}{12} i^2(x,t)\frac{\partial^4 i(x,t)}{\partial x^4} \\
&+B_0^2 h^2\frac{\partial^2 i(x,t)}{\partial x^2} - h^2 i^2(x,t)\frac{\partial^2 i(x,t)}{\partial x^2} \\
&+(2GB_3 - 2B_0^2 GB_2) i(x,t)\frac{\partial i(x,t)}{\partial t} \\
&+(2GB_2 - 4B_0^2 GB_1) i^3(x,t)\frac{\partial i(x,t)}{\partial t} \\
&+4GB_1 i^5(x,t)\frac{\partial i(x,t)}{\partial t} - B_0^2 RGi(x,t) \\
&+RGi^3(x,t) = 0.
\end{aligned} \quad (11)$$

Setting $n_1 = B_0^2 h^4/12$, $n_2 = -h^4/12$, $n_3 = B_0^2 h^2$, $n_4 = -h^2$, $n_5 = 2GB_3 - 2B_0^2 GB_2$, $n_6 = 2GB_2 - 4B_0^2 GB_1$, $n_7 = 4GB_1$, $n_8 = -B_0^2 RG$ and $n_9 = RG$, (11) takes the form

$$\begin{aligned}
&n_1\frac{\partial^4 i(x,t)}{\partial x^4} + n_2 i^2(x,t)\frac{\partial^4 i(x,t)}{\partial x^4} + n_3\frac{\partial^2 i(x,t)}{\partial x^2} + \\
&n_4 i^2(x,t)\frac{\partial^2 i(x,t)}{\partial x^2} + n_5 i(x,t)\frac{\partial i(x,t)}{\partial t} \\
&+n_6 i^3(x,t)\frac{\partial i(x,t)}{\partial t} + n_7 i^5(x,t)\frac{\partial i(x,t)}{\partial t} \\
&+n_8 i(x,t) + n_9 i^3(x,t) = 0.
\end{aligned} \quad (12)$$

Equation (12) can still be written in the following contracted form

$$\begin{aligned} & n_1 i_{xxxx} + n_2 i^2 i_{xxxx} + n_3 i_{xx} + n_4 i^2 i_{xx} + n_5 i i_t \\ & + n_6 i^3 i_t + n_7 i^5 i_t + n_8 i + n_9 i^3 = 0. \end{aligned} \quad (13)$$

Equation (13) is the nonlinear partial differential equation which describes the dynamics of the current in the circuit when the magnetic flux is defined by (10). In the following lines, we will invest ourselves in the search for forms of solutions.

III. THE RANGE EQUATION

Knowing that the iB-function [28-32] presents a certain flexibility and a large maneuver for obtaining solutions, we propose to construct the solution of equation (13) in the form

$$i(x,t) = aJ_{n,m}(\alpha x - \alpha_0 t), \quad (14)$$

where a, α, α_0 are arbitrary non zero constants. Using the change of variable $\xi = \alpha x - \alpha_0 t$, (14) takes the form

$$\begin{aligned} & n_1 \alpha^4 i_{\xi\xi\xi\xi} + n_2 \alpha^4 i^2 i_{\xi\xi\xi\xi} + n_3 \alpha^2 i_{\xi\xi} + n_4 \alpha^2 i^2 i_{\xi\xi} \\ & - n_5 \alpha_0 i i_\xi - n_6 \alpha_0 i^3 i_\xi - n_7 \alpha_0 i^5 i_\xi + n_8 i + n_9 i^3 = 0. \end{aligned} \quad (15)$$

The ansatz (14) takes the form

$$i(\xi) = aJ_{n,m}(\xi). \quad (16)$$

The injection of ansatz (16) into (15) requires the evaluation of the terms of (15). So we have

$$\begin{aligned} i_{\xi\xi\xi} &= am(m-1)(m-2) J_{n-3,m-3} \\ & - a\left[m(m-1)(n-2) + m^2(n-1) + mn(m+1)\right] J_{n-1,m-1} \\ & + a\left[nm(n-1) + n^2(m+1) + n(n+1)(m+1)\right] J_{n+1,m+1} \\ & - an(n+1)(n+2) J_{n+3,m+3}, \end{aligned} \quad (17)$$

$$\begin{aligned} i_{\xi\xi\xi\xi} &= \alpha_1 J_{n-4,m-4} - \alpha_2 J_{n-2,m-2} + \alpha_3 J_{n,m} \\ & - \alpha_4 J_{n+2,m+2} + \alpha_5 J_{n+4,m+4}, \end{aligned} \quad (18)$$

with

$$\alpha_1 = am(m-1)(m-2)(m-3), \quad (19)$$

$$\alpha_2 = a\begin{bmatrix} m(m-1)(m-2)(n-3) + m(m-1)(n-2)(n-1) \\ + m^2(n-1)^2 + mn(m+1)(n-1) \end{bmatrix}, \quad (20)$$

$$\alpha_3 = a\begin{bmatrix} m(m-1)(n-2)(m-1) + m^2(n-1)(m-1) \\ + nm(m+1)(m-1) \\ + nm(n-1)(m+1) + n(m+1)^2 \\ + n(n+1)(m+2)(m+1) \end{bmatrix}, \quad (21)$$

$$\alpha_4 = a\begin{bmatrix} nm(n-1)(n+1) + n^2(m+1)(n+1) \\ + n(n+1)(m+2) + n(n+1)(n+2)(m+3) \end{bmatrix}, \quad (22)$$

$$\alpha_5 = an(n+1)(n+2)(n+3), \quad (23)$$

$$i^5 = a^5 J_{5n,5m}(\xi), \quad (24)$$

and

$$i^5 i_\xi = a^6\left(mJ_{6n-1,6m-1} - nJ_{6n+1,6m+1}\right). \quad (25)$$

Inserting the terms given by (17) to (25) into (15) leads to

$$\begin{aligned} & A_1 J_{n-4,m-4} + A_2 J_{n-2,m-2} + A_3 J_{n,m} + A_4 J_{n+2,m+2} \\ & + A_5 J_{n+4,m+4} - A_6 J_{2n-1,2m-1} + A_7 J_{2n+1,2m+1} \\ & + A_8 J_{3n-4,3m-4} + A_9 J_{3n-2,3m-2} + A_{10} J_{3n,3m} \\ & + A_{11} J_{3n+2,3m+2} + A_{12} J_{3n+4,3m+4} - A_{13} J_{4n-1,4m-1} \\ & + A_{14} J_{4n+1,4m+1} - A_{15} J_{6n-1,6m-1} + A_{16} J_{6n+1,6m+1} = 0, \end{aligned} \quad (26)$$

where the coefficients $A_i\,(i = 1, 2, \ldots, 16)$ are given by:

$$A_1 = n_1 \alpha^4 \alpha_1,$$

$$A_2 = -n_1 \alpha^4 \alpha_2 + n_3 \alpha^2 am(m-1),$$

$$A_3 = n_1 \alpha^4 \alpha_3 - n_3 \alpha^2 a\left[m(n-1) + n(m+1)\right] + n_8 a,$$

$$A_4 = -n_1 \alpha^4 \alpha_4 + n_3 \alpha^2 an(n+1),\ A_5 = n_1 \alpha^4 \alpha_5,$$

$$A_6 = mn_5 \alpha_0 a^2,\ A_7 = n_5 \alpha_0 a^2,\ A_8 = n_2 \alpha^4 a^2 \alpha_1,$$

$$A_9 = n_4 \alpha^2 a^3 m(m-1) - n_2 \alpha^4 a^2 \alpha_2,$$

$$A_{10} = n_9 a^3 - n_4 \alpha^2 a\left[m(n-1) + n(m+1)\right] + n_2 \alpha^4 a^2 \alpha_3,$$

$$A_{11} = n_4 a \alpha^2 n(n+1) - n_2 \alpha^4 a^2 \alpha_4,\ A_{12} = n_2 \alpha^4 a^2 \alpha_5,$$

$$A_{13} = n_6 \alpha_0 a^4 m,\ A_{14} = n_6 \alpha_0 a^4 n,\ A_{15} = n_7 \alpha_0 ma^6$$

and $A_{16} = n_7 \alpha_0 na^6$.

Equation (26) is the range equation which will be at the center of all search for solutions.

IV. PAIRS (n, m) FOR WHICH CERTAIN TERMS OF THE RANGE EQUATION ARE GROUPED TOGETHER AND FIELD OF POSSIBLE SOLUTIONS

Examination of equation (26) allows to identify 86 pairs (n, m) for which there is a grouping of terms. the values of n and m corresponding to these groupings are given by

$$n, m \in \left\{ \begin{matrix} -5, -4, -3, -2, \frac{-5}{3}, -1, \frac{-3}{5}, \frac{-1}{2}, \frac{-1}{3}, 0, \\ \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, 1, \frac{5}{3}, 2, 3, 4, 5 \end{matrix} \right\}. \quad (27)$$

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The combination of the elements of the set (27), globally allows to have 361 pairs (n,m) which form the extended field of search for solutions. Unable to manually pass all these 361 pairs into the range equation before extracting those which lead to non-trivial solutions, we use the probabilities of appearance of each pair to get an idea of its preponderance to lead to an acceptable solution. The probabilities of appearance of the pairs of the principal diagonal of the widened field of the possibilities of the solutions are given by: $P(-3,-3)=8/86$, $P(-2,-2)=2/86$, $P(-1/3,-1/3)=3/86$, $P(-3,-3)=8/86$, $P(4,4)=1/86$, $P(1/2,1/2)=1/86$, $P(-4,-4)=2/86$, $P(-5/3,-5/3)=2/86$, $P(-1/5,-1/5)=1/86$, $P(3/5,3/5)=2/86$, $P(5/3,5/3)=2/86$, $P(0,0)=10/86$, $P(-3/5,-3/5)=2/86$, $P(3,3)=8/86$, $P(1/5,1/5)=1/86$ $P(-5,-5)=2/86$ $P(-1,-1)=15/86$, $P(1,1)=15/86$ $P(2,2)=2/86$, $P(5,5)=3/86$, $P(-1/2,-1/2)=1/86$.

With regard to the different probabilities of appearance, we notice that the greatest probabilities are given by: $P(-1,-1)=15/86$, $P(1,1)=15/86$, $P(0,0)=10/86$, $P(-3,-3)=8/86$ and, $P(3,3)=8/86$.Thus, the dominant pairs are $(-1,-1)$, $(1,1)$, $(0,0)$, $(-3,-3)$ and $(3,3)$. So the values of n and m for which we have a large number of groupings are given by

$$n,m\in\{,-3,-1,0,1,3\}. \tag{28}$$

The combination of the elements of (28) allows to have a restricted number of pairs (n,m) whose review in (26) will allow to detect the possible solutions. The restricted field of search for solutions is given by the following table

TABLE 1: Restricted field of possibilities

(n,m)	-3	-1	0	1	3
-3	(-3,-3)	(-3,-1)	(-3,0)	(-3,1)	(-3,3)
-1	(-1,-3)	(-1,-1)	(-1,0)	(-1,1)	(-1,3)
0	(0,-3)	(0,-1)	(0,0)	(0,1)	(0,3)
1	(1,-3)	(1,-1)	(1,0)	(1,1)	(1,3)
3	(3,-3)	(3,-1)	(3,0)	(3,1)	(3,3)

V. Solving (26) for the Pairs (n,m) of the Restricted Field of Possibilities and Solutions

V.1. Solutions For Pairs of the Main Diagonal of the Restricted Field

- Case of the pair (n,m) = $(-1,-1)$

The constants $\alpha_i\left(i=1,\ldots,4\right)$ and $A_i\left(i=1,2,\ldots,16\right)$ are given by:

$\alpha_1=24a$, $\alpha_2=40a$, $\alpha_3=16a$, $\alpha_4=0$, $\alpha_5=0$

$A_1=24n_1\alpha^4 a$, $A_2=-40n_1\alpha^4 a+2n_3\alpha^2 a$,

$A_3=16n_1\alpha^4 a-2n_3\alpha^2 a+n_8 a$, $A_4=0$, $A_5=0$,

$A_6=-n_5\alpha_0 a^2$, $A_7=n_5\alpha_0 a^2$, $A_8=24n_2\alpha^4 a^3$,

$A_9=2n_4\alpha^2 a^3-40n_2\alpha^4 a^3$,

$A_{10}=n_9 a^3-2\alpha^2 n_4 a+16n_2\alpha^4 a^3$, $A_{11}=0$, $A_{12}=0$,

$A_{13}=-n_6\alpha_0 a^4$, $A_{14}=-n_6\alpha_0 a^4$, $A_{15}=-n_7\alpha_0 a^6$ and

$A_{16}=-n_7\alpha_0 a^6$

The equation (26) with the constants calculated above is written

$$\begin{aligned}&\left(A_1+A_9-A_{13}+A_{16}\right)J_{-5,-5}\\&+\left(A_2-A_6+A_{10}+A_{14}\right)J_{-3,-3}+A_3J_{-3,-3}\\&+\left(A_3+A_7+A_{11}\right)J_{-1,-1}+\left(A_{14}+A_{12}\right)J_{1,1}\\&+A_5J_{3,3}+\left(A_8-A_{15}\right)J_{-7,-7}=0.\end{aligned} \tag{29}$$

Equation (26) holds if we have

$$24n_1\alpha^4 a+2n_4\alpha^2 a^3-40n_2\alpha^4 a^3+n_6\alpha_0 a^4-n_7\alpha_0 a^6=0, \tag{30}$$

$$\begin{aligned}&-40n_1\alpha^4 a+2n_3\alpha^2 a+n_5\alpha_0 a^2+n_9 a^3\\&-2n_4\alpha^2 a+16n_2\alpha^4 a^3-n_6\alpha_0 a^4=0,\end{aligned} \tag{31}$$

$$16n_1\alpha^4 a-2n_3\alpha^2 a+n_8 a+n_5\alpha_0 a^2=0, \tag{32}$$

$$-n_6\alpha_0 a^4=0, \tag{33}$$

and

$$24n_2\alpha^4 a^3+n_7\alpha_0 a^6=0. \tag{34}$$

Solving (30) to (34) requires setting $a\neq 0$, since we are looking for non-trivial solutions. So the only possibility that works in (33) is $n_6=0$, which forces us to fix the constraint $B_2=2B_0^2B_1$, while allowing us to correct the magnetic flux in the form

$$\phi(i(x,t))=B_1 i^4(x,t)+2B_0^2B_1 i^2(x,t)+B_3 In(i^2(x,t)-B_0^2).$$

We get from (34)

$$a=\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}},\ n_7\alpha_0\neq 0. \tag{35}$$

Equation (35) must satisfy equations (30) to (32) for the choice of the other parameters. So the solution of (15) and (13) in this case is given by

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$$i(\xi)=\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} J_{-1,-1}(\xi)$$
$$\Rightarrow i(x,t)=\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} J_{-1,-1}(\alpha x-\alpha_0 t). \quad (36)$$

- Case of the pair $(n,m)=(1,1)$

The constants $\alpha_i\,(i=1,\dots,4)$ and $A_i\,(i=1,2,\dots,16)$ are given by:
$\alpha_1=0,\alpha_2=0,\alpha_3=16a,\alpha_4=34a,\alpha_5=24a$ $A_1=0$,
$A_2=0$, $A_3=16n_1\alpha^4a-2n_3\alpha^2a+n_8a$,
$A_4=-34n_1\alpha^4a+2n_3\alpha^2a$, $A_5=24n_1\alpha^4a$, $A_6=n_5\alpha_0a^2$,
$A_7=n_5\alpha_0a^2$, $A_8=0$, $A_9=0$,
$A_{10}=n_9a^3-2\alpha^2n_4a+16n_2\alpha^4a^3$,
$A_{11}=2n_4a\alpha^2-34n_2\alpha^4a^3$, $A_{12}=24n_2\alpha^4a^3$,
$A_{13}=n_6\alpha_0a^4$, $A_{14}=n_6\alpha_0a^4$, $A_{15}=n_7\alpha_0a^6$ and
$A_{16}=n_7\alpha_0a^6$

Equation (26) with the constants calculated above is written

$$\left(A_3-A_6+A_9\right)J_{1,1}+\left(A_4+A_7+A_{10}-A_{13}\right)J_{3,3}+\left(A_{11}+A_{14}-A_{15}\right)J_{5,5}+\left(A_{12}+A_{16}\right)J_{7,7}=0. \quad (37)$$

Equation (37) holds if we have

$$16n_1\alpha^4a-2n_3\alpha^2a+n_8a-n_5\alpha_0a^2=0, \quad (38)$$
$$-34n_1\alpha^4a+2n_3\alpha^2a+n_5\alpha_0a^2+n_9a^3-2n_4\alpha^2a+16n_2\alpha^4a^3+n_6\alpha_0a^4=0, \quad (39)$$
$$2n_4\alpha^2a-34n_2\alpha^4a^3+n_6\alpha_0a^4-n_7\alpha_0a^6=0, \quad (40)$$

and

$$24n_2\alpha^4a^3+n_7\alpha_0a^6=0. \quad (41)$$

Solving (38) to (41) requires setting $a\neq 0$, since we are looking for non-trivial solutions. So we obtain from (41)

$$a=\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}},\ n_7\alpha_0\neq 0. \quad (42)$$

Equation (42) must satisfy (38) to (40) for the choice of the other parameters. So the solution of (15) and (13) in this case is given by

$$i(\xi)=\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} J_{1,1}(\xi)$$
$$\Rightarrow i(x,t)=\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} J_{1,1}(\alpha x-\alpha_0 t). \quad (43)$$

Apart from the solutions obtained above, we realize as expected by the different probabilities of the pairs of the restricted table of the possibilities of solutions that the other pairs lead in their majority to trivial solutions.

V.2. Trigonometric Solutions

By making the correspondences $\xi\leftarrow j\xi$, $\alpha\leftarrow j\alpha$ and $\alpha_0\leftarrow j\alpha_0$, $j^2=-1$ in (36) and (43) we obtain the following solutions

$$i(\xi)=j\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} \cot an(\xi)$$
$$\Rightarrow i(x,t)=-j\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} \cot an(\alpha x-\alpha_0 t), \quad (44)$$

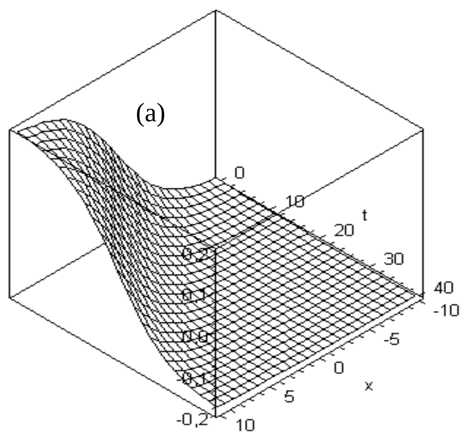
and

$$i(\xi)=j\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} \tan(\xi)$$
$$\Rightarrow i(x,t)=j\left(\frac{-24n_2\alpha^4}{n_7\alpha_0}\right)^{\frac{1}{3}} \tan(\alpha x-\alpha_0 t). \quad (45)$$

Obtaining solutions (44) and (45) is very difficult using direct integration.

VI. Propagation and Profiles of Solutions Likely to Propagate in the Circuit

The curves are drawn for values $a=0.2$, $\alpha=0.2$ and $\alpha_0=0.2$. Thus, we get from the solutions (36) and (43)



http://ijses.com/

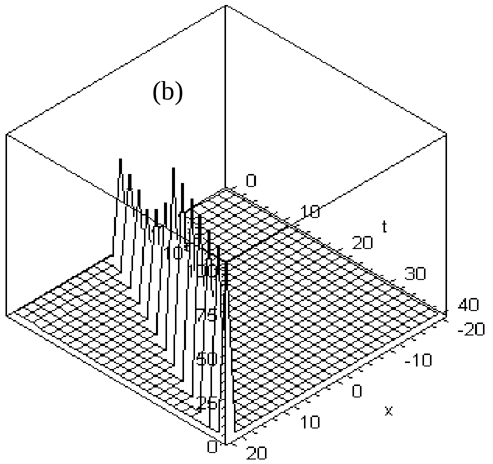


Fig.1. Profiles of the signals likely to propagate in the circuit: (a): curve resulting from the solution (36); (b): curve from solution (43).

The trigonometric solutions (44) and (45) being trigonometric and complex, the curves below are magnitudes of the corresponding current intensities in the circuit

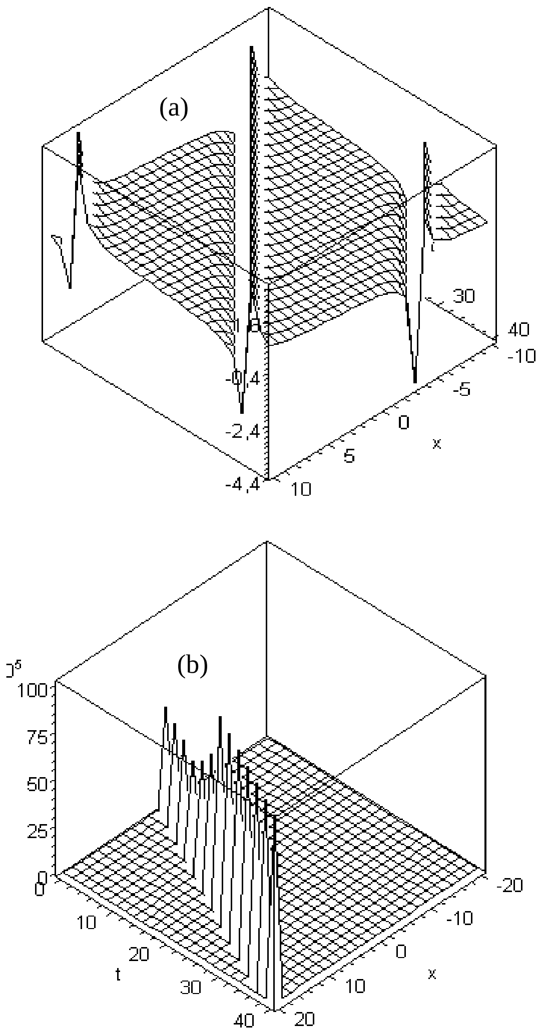


Fig.2. Profiles of the signals likely to propagate in the circuit: (a): curve resulting from the solution (44); (b): curve from solution (45).

These two figures reflect the shapes of the signals during their different propagation in the circuit in the case where the conditions of stability are ensured.

VII. Conclusion

The objective of this work was to model a nonlinear partial differential equation which describes the dynamics of propagation in an inductive electric line and to propose solutions which can be used in the practical case of signals likely to propagate in the proposed circuit. But what is important in the modeling approach used here is that the nonlinear effects generated in the circuit are embodied by the magnetic flux and therefore the analytical form is carefully chosen. But as the most important thing at this level is not only obtaining the partial differential equation, but above all the expression or analytical sequence of the current, then the question arose of the resolution of this nonlinear partial differential equation. Obviously, this equation not being easily integrable, we resorted to the probabilistic technique developed for a very short time in our previous work to construct solutions. Thus, our various investigations and analyzes in the treated subject allowed us to isolate some solutions which are given to (36), (43), (44) and (45). Of all these solutions obtained, we can easily identify that the one given by equation (43) is of the family of solitary waves of the kink type. This solution particularly can be adopted within the framework of the practical study like signal because not presenting a point of discontinuity in its field of study. Knowing that a good solution must also present guarantees of stability; it would be necessary for a stability study to be undertaken to establish all the limits and precautions for its proper use as a propagation signal in the manufactured circuit. Beyond the analytical approach used in this work, we want through this study to verify and compare the results obtained with those already obtained in a previous work [7]. for this purpose, the pleasure for us was to note that in addition to the accuracy of the proven solutions, we had other forms of solutions and which are the only ones which hold with the constituted circuit. We have also noted that the constraint equations linking the coefficients $n_i\left(i=1,\ldots,9\right)$ do not allow us to globally modify the structure of the circuit or the properties in order to claim other forms of solutions. However, the solutions obtained required small corrections to the magnetic flux.

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