Volume 5, Issue 11, pp. 79-81, 2021.
International Journal of Scientific Engineering and Science
ISSN (Online): 2456-7361

Direct Signal-Based DoA Estimation Using URA

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Abstract— *This paper proposes a computationally efficient 2-D direction-of-arrival (DoA) estimation method with a uniform rectangular array (URA). This method is called the direct signal-based method in the sense that it is based directly on the relationships among the signals arriving at each antenna. The simulation results show that the direct signal-based method yields a bit better performance compared with the MUSIC method in terms of the root-mean-square error (RMSE) and the maximum absolute error.*

Keywords— *Direct signal-based mehtod: spectral estimation: drectional of arrival; uniform rectangular array.*

I. Introduction

There are many existent DoA estimation methods [1-5] to estimate the azimuth and zenith angles of signals arriving on various structures of antenna array [6-7], such as the delay-and-sum method, Capon's minimum variance method, the MUSIC (MUltiple SIgnal Classification) algorithm, the ESPRIT (Estimation of Signal Parameters via Rational Invariance Techniques) algorithm, etc. Most of the existing methods are involved in eigen-decompositions [8-10] and/or matrix operations, which require lots of computations. This paper proposes a new DoA estimation method to estimate the azimuth and zenith angles of a signal arriving on a URA, which does not require such a heavy computation as eigen-decompositions or any matrix operations. We will call the method the direct signal-based method in the sense that it is based directly on the relationships among the signals arriving at each antenna rather than collectively on the whole set of the signals.

Figure 1 shows an electromagnetic plane wave signal $s(t)$ and an $M \times N$ URA composed of MN antenna sensor elements with equidistances d_1/d_2 each along the x_1/x_2-axes, respectively. The signal $s_{n,m}(t)$ reaching the (n,m)th antenna element at time t can be expressed as [1]

$$s_{n,m}(t) = s(t)e^{j\beta\sin\theta(md_1\cos\phi + nd_2\sin\phi)} \tag{1}$$

$$= s(t)a_{n,m}(\phi,\theta) \tag{2}$$

with the steering phase factor of the (n,m)th antenna

$$a_{n,m}(\phi,\theta) = e^{j\beta\sin\theta(md_1\cos\phi + nd_2\sin\phi)} \tag{3}$$

where ϕ, θ, and β are the azimuth angle, the zenith angle, and the wave factor, respectively.

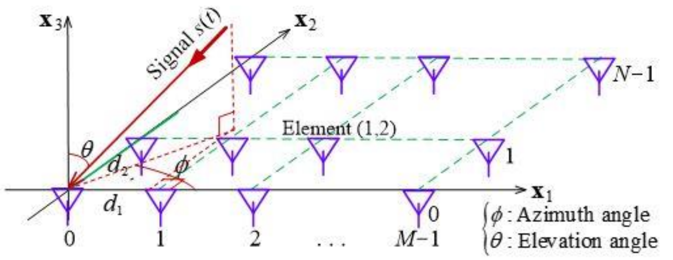

Fig. 1. Uniform rectangular array (URA)

The signals received by the $N \times M$ antenna elements can be described by

$$\begin{bmatrix} u_{0,0}(\phi,\theta,t) & \cdots & u_{0,M-1}(\phi,\theta,t) \\ u_{1,0}(\phi,\theta,t) & \cdots & u_{1,M-1}(\phi,\theta,t) \\ \vdots & \ddots & \vdots \\ u_{N-1,0}(\phi,\theta,t) & \cdots & u_{N-1,M-1}(\phi,\theta,t) \end{bmatrix}$$
$$= \begin{bmatrix} a_{0,0}(\phi,\theta) & \cdots & a_{0,M-1}(\phi,\theta) \\ a_{1,0}(\phi,\theta) & \cdots & a_{1,M-1}(\phi,\theta) \\ \vdots & \ddots & \vdots \\ a_{N-1,0}(\phi,\theta) & \cdots & a_{N-1,M-1}(\phi,\theta) \end{bmatrix} s(t)$$
$$+ \begin{bmatrix} v_{0,0}(t) & \cdots & v_{0,M-1}(t) \\ v_{1,0}(t) & \cdots & v_{1,M-1}(t) \\ \vdots & \ddots & \vdots \\ v_{N-1,0}(t) & \cdots & v_{N-1,M-1}(t) \end{bmatrix} \tag{4}$$

where $v_{n,m}(t)$ is an additive white Gaussian noise (AWGN) for the signal $s_{n,m}(t)$ incident on the (n,m)th antenna element at time t.

The steps of *MUSIC algorithm* [9-10], which is one of the most popular DoA estimation methods, are summarized as follows [1]:

Step 0: Collect the samples of the $MN\times 1$ input vector $\{\ \mathbf{u}(nT),\ n=0,\cdots,K-1\ \}$ (K: the number of snapshots, T: the time sampling period)

$$\mathbf{u}(t) = \begin{bmatrix} u_{0,0}(\phi,\theta,t) \\ \vdots \\ u_{0,M-1}(\phi,\theta,t) \\ \vdots \\ u_{N-1,0}(\phi,\theta,t) \\ \vdots \\ u_{N-1,M-1}(\phi,\theta,t) \end{bmatrix} \tag{5}$$

and estimate their $MN\times MN$ *spatial correlation matrix*:

$$\hat{R}_{uu} = \frac{1}{K}\sum_{n=0}^{K-1}\mathbf{u}(nT)\mathbf{u}^H(nT) \quad (^H\text{: the conjugate transpose}) \tag{6}$$

Step 1: Perform eigendecomposition on the estimated input correlation matrix $\hat{R}_{uu}$:

$\hat{R}_{uu}V = V\Lambda$ with $V = [\boldsymbol{v}_0 \quad \boldsymbol{v}_1 \quad \cdots \quad \boldsymbol{v}_{MN-1}]$

International Journal of Scientific Engineering and Science
Volume 5, Issue 11, pp. 79-81, 2021. ISSN (Online): 2456-7361

and $$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_0 & 0 & \cdots & 0 \\ 0 & \lambda_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{MN-1} \end{bmatrix} \quad (7)$$

Step 2: Estimate the number $\hat{L} = MN - N_n$ of signals where N_n is the number of the smallest eigenvalues forming a cluster and form the $MN \times N_n$ matrix V_n made of the (presumed) noise eigenvectors:

$$\boldsymbol{V_n} = [\boldsymbol{v_0} \quad \boldsymbol{v_1} \quad \cdots \quad \boldsymbol{v_{N_n-1}}] \quad (8)$$

(corresponding to the $\boldsymbol{N_n}$ smallest eigen values)

Step 3: Compute the MUSIC spatial spectrum for possible ranges of the azimuth angle ϕ and the zenith angle θ:

$$P_{MUSIC}(\phi,\theta) = \frac{1}{\mathbf{a}^H(\phi,\theta) V_n V_n^H \mathbf{a}(\phi,\theta)} \quad (9)$$

with the steering vector $$\mathbf{a}(\phi,\theta) = \begin{bmatrix} a_{0,0}(\phi,\theta) \\ \vdots \\ a_{0,M-1}(\phi,\theta) \\ \vdots \\ a_{N-1,0}(\phi,\theta) \\ \vdots \\ a_{N-1,M-1}(\phi,\theta) \end{bmatrix} \quad (10)$$

Step 4: Pick the maximum (peak) of $P_{MUSIC}(\phi,\theta)$ to obtain the azimuth angle estimate $\hat{\boldsymbol{\phi}}$ and the zenith angle estimate $\hat{\theta}$:

$$(\hat{\phi}, \hat{\theta}) = \underset{\phi,\theta}{\operatorname{Arg\,Max}} P_{MUSIC}(\phi,\theta) \quad (11)$$

This MUSIC algorithm requires lots of multiplications and additions, especially for the eigen-decomposition (performed in Step 1) having a computational complexity of $4(MN)^3/3$ [11]. Note that this computational complexity amounts to more than $4(MN)^3/3 = 43{,}691$ complex multiplications.

II. Direct Signal-Based DoA Estimation with URA

Based on (3) and (4), the phase angles of the signals (without the noise) received by each antenna element of the URA can be written as the following $N \times M$ matrix:

$$B = [b_{n,m}(\phi,\theta,t)] = \begin{bmatrix} \beta\sin\theta(0+0) + \varphi(t) & \beta\sin\theta(d_1\cos\phi + 0) + \varphi(t) & \cdots & \beta\sin\theta((M-1)d_1 \cdot \cos\phi + 0) + \varphi(t) \\ \beta\sin\theta(0+d_2 \cdot \sin\phi) + \varphi(t) & \beta\sin\theta(d_1\cos\phi + d_2\sin\phi) + \varphi(t) & \cdots & \beta\sin\theta((M-1)d_1 \cdot \cos\phi + d_2\sin\phi) + \varphi(t) \\ \vdots & \vdots & \ddots & \vdots \\ \beta\sin\theta(0+(N-1)d_2\sin\phi) + \varphi(t) & \beta\sin\theta(d_1\cos\phi + (N-1)\sin\phi) + \varphi(t) & \cdots & \beta\sin\theta((M-1)d_1 \cdot \cos\phi + (N-1)d_2 \cdot \sin\phi) + \varphi(t) \end{bmatrix} \quad (12)$$

where $\varphi(t)$ is the phase angle of the signal $u_{0,0}(t)$ received by the antenna element at the origin (0, 0) of the URA.

Noting that the (phase) differences between neighboring entries of the above matrix B along its row and column are

$$\boldsymbol{x_m} = \boldsymbol{b_{n,m+1}} - \boldsymbol{b_{n,m}} \approx \boldsymbol{\beta d_1 \sin\theta \cos\phi} \text{ for } \boldsymbol{m} = \mathbf{1}, \cdots, \boldsymbol{M} - \mathbf{1} \quad (13a)$$

$$\boldsymbol{y_n} = \boldsymbol{b_{n+1,m}} - \boldsymbol{b_{n,m}} \approx \boldsymbol{\beta d_2 \sin\theta \sin\phi} \text{ for } \boldsymbol{n} = \mathbf{1}, \cdots, \boldsymbol{N} - \mathbf{1} \quad (13b)$$

we can take their mean values

$$\boldsymbol{m_x} = \frac{1}{M-1}\sum_{m=1}^{M-1} \boldsymbol{x_m} \quad (14a)$$

$$\boldsymbol{m_y} = \frac{1}{N-1}\sum_{n=1}^{N-1} \boldsymbol{y_n} \quad (14b)$$

and use them to get the azimuth angle estimate $\hat{\boldsymbol{\phi}}$ and the zenith angle estimate $\hat{\boldsymbol{\theta}}$ as

$$\text{Azimuth angle estimate: } \hat{\boldsymbol{\phi}} = \tan^{-1}\left(\frac{\mathbf{m_y}/\mathbf{d_2}}{\mathbf{m_x}/\mathbf{d_1}}\right) \quad (15)$$

$$\text{Zenith angle estimate: } \hat{\boldsymbol{\theta}} = \sin^{-1}\left(\frac{\mathbf{m_x}/\boldsymbol{\beta}\mathbf{d_1}}{\cos\hat{\boldsymbol{\phi}}}\right) \quad (16a)$$

$$\text{or } \hat{\boldsymbol{\theta}} = \sin^{-1}\left(\frac{\mathbf{m_y}/\boldsymbol{\beta}\mathbf{d_2}}{\sin\hat{\boldsymbol{\phi}}}\right) \quad (16b)$$

where (16a) or (16b) is used to estimate the zenith angle θ depending on whether the azimuth angle estimate $\hat{\phi}$ is close to 0^o or 180^o or it is close to $\pm 90^o$. Here, since the azimuth angle estimate computed by (15) is constrained into the interval $(-180^o, 180^o]$ (larger than -180^o and not larger than 180^o), the outliers may have to be excluded to remove the influence of the angle ambiguity when the mean values are computed by (14a) and (14b).

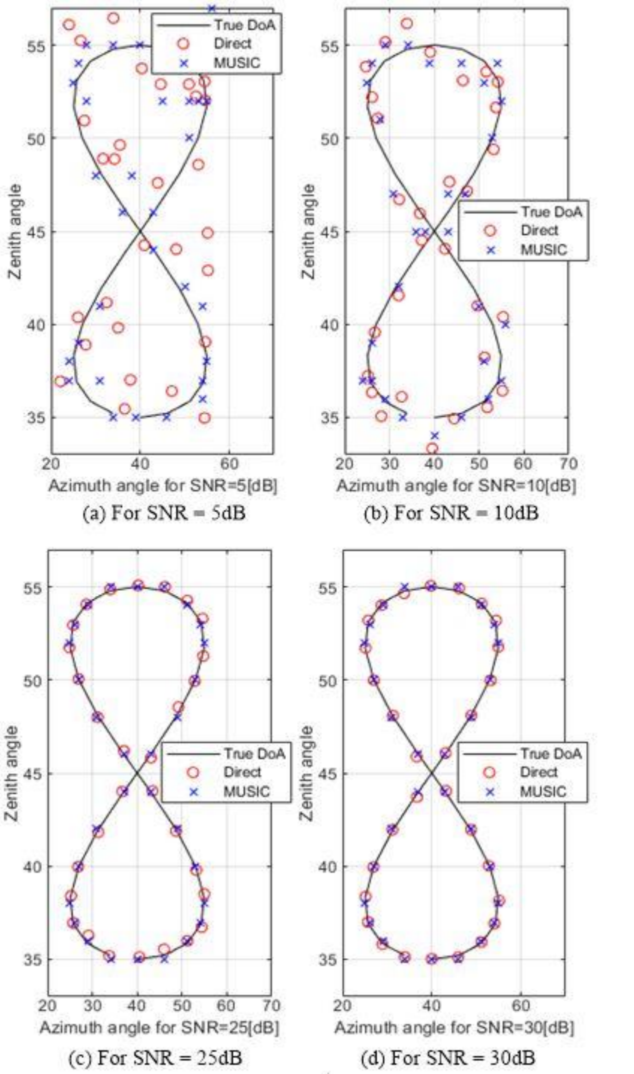

Fig. 2. Simulation results of tracking the azimuth/zenith angles for several values of SNR

International Journal of Scientific Engineering and Science
Volume 5, Issue 11, pp. 79-81, 2021. ISSN (Online): 2456-7361

III. Simulation Results

Figure 2 depicts the simulation results of using the MUSIC method and the direct signal-based method (proposed by this work) to get the estimates of the azimuth and zenith angles (of a signal arriving on an 8×4 URA) each varying within $[25^o, 55^o]$ and $[35^o, 55^o]$, respectively, for several SNR values from 5dB to 30dB. Figures 3 and 4 depict the RMSE and maximum absolute errors (vs. the several SNR values) of the azimuth/zenith angle estimates obtained using the MUSIC method and the direct signal-based method. Overall, they show that the direct signal-based method yields a bit better performance compared with the MUSIC method in terms of the root-mean-square error (RMSE) and the maximum absolute error.

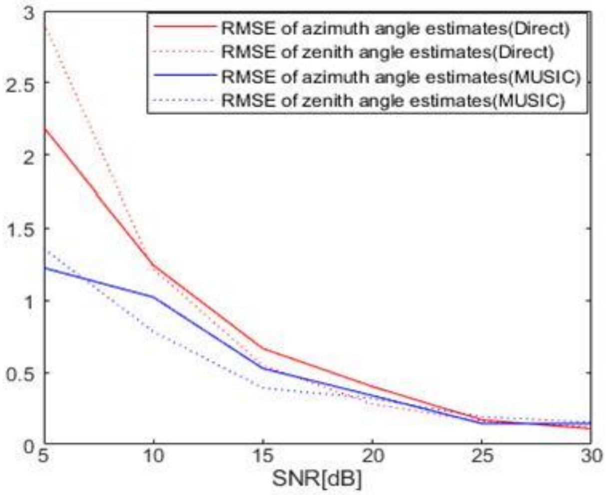

Fig. 3. Simulation results of the RMSEs for the azimuth/zenith angle estimates vs. SNR

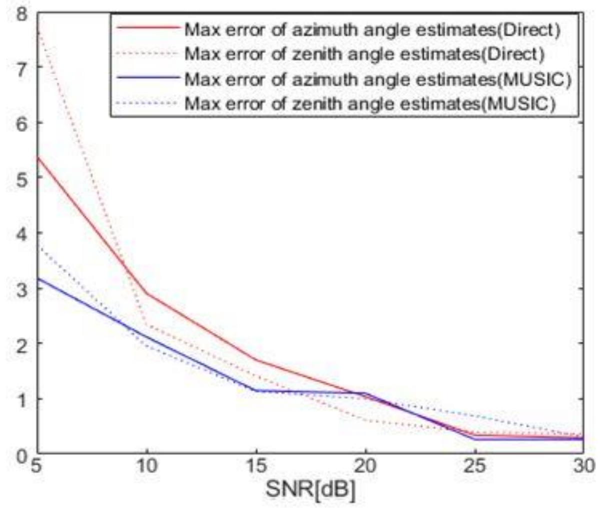

Fig. 4. Simulation results of the maximum absolute errors for the azimuth/zenith angle estimates vs. SNR

IV. Concluding Remarks

As a means to estimate the azimuth and zenith angles of an electromagnetic planewave signal arriving on a URA, the direct signal-based method has been shown to yield the RMSE and maximum absolute errors less than those of the MUSIC method and that with much less computations. One drawback of this method is that it can find the DoA of just one signal while most DoA estimation methods can make the estimates of multiple signals infringing on the array of antennas although the estimation accuracy may be degraded to some degree. For this reason, it is desirable to use the direct signal-based method at every iteration and a DoA estimation method like the MUSIC method (for cross-check) intermittently, hopefully when the mobile device equipped with the URA for DoA estimation is moving around the boundary among eighboring cells where there are multiple incident signals.

References

[1] Won Y. Yang, Yong S. Cho, and Chang Y. Choo, "Spectral Estimation", in *MATLAB/Simulink for Digital Signal Processing*, Hongreung Science, Seoul, 2012.
[2] Barabell, Arthur J., "Improving the resolution performance of eigenstructure-based direction-finding algorithms," *Proc. of the IEEE Int'l Conf. on* Acoustics, Speech, and Signal Processing (ICSAAP)-83, pp. 336-339, 1983.
[3] Roy, R. T. Kailath, "ESPRIT - estimation of signal parameters via rotational invariance techniques," *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. 37, pp. 984-995, 1989.
[4] Schmidt, R. O., "Multiple emitter location and signal parameter estimation," *IEEE Trans. on Antennas and Propagation*, vol. AP-34, no. 3, pp. 276-280, 1986.
[5] Evans, J. E., Johnson, J. R., Sun, D.F., "High resolution angular spectrum estimation techniques for terrain scattering analysis and angle of arrival estimation in ATC navigation and surveillance system," *Proceedings of the 1st ASSP Workshop Spectral Estimation*, pp. 134–139, Lexington, KY, USA, 1981.
[6] Van Trees, Harry L., *Optimum Array Processing*, John Wiley & Sons, Inc., New York, 2002.
[7] Sullivan, Mark C., *Practical Array Processing*, McGraw-Hill Book Company, New York, 2009.
[8] Chen, F. J., Kwong, S., Kok, C. W., "ESPRIT-like two-dimensional DOA estimation for coherent signals," *IEEE Transactions on Aerospace and Electronic Systems*, 46(3), pp. 1477-1484, 2010.
[9] Liberty, Joseph C., Jr. and Theodore S. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications*, Prentice Hall PTR, New Jersey, 1999.
[10] Devendra, M., Manjunathachari, K., "DOA estimation of a system using MUSIC method," *International Conference on Signal Processing and Communication Engineering Systems*, pp. 309-313, doi: 10.1109/SPACES.2015.7058272, 2015.
[11] Haidar, Azzam, Piotr Luszczek, Jack Dongarra, "New algorithm for computing eigenvectors of the symmetric eigenvalue problem," *IEEE 28th International Parallel & Distributed Processing Symposium Workshop*, pp. 1150-1159, 2014.