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On the Strong Convergence and Stability Results of Some Modified Iterative Scheme for a General Class of Operators

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Abstract— *In this article, we introduce and establish the strong convergence and stability results of some modified iterative schemes for a general class of operators introduced by Bosede and Rhoades [4] in an arbitrary Banach space. Numerical example is given to prove that our results are significant refinement and improvement to results obtained by several authors in literature.*

Keywords— *Fixed Point, Ishikawa Iterative Scheme, Mann Iterative Scheme, Noor Iterative Scheme, T-Stable, Zamfirescu Operators.*

I. Introduction

Definition 1.1: Let (E, d) be a metric space and T: E → E a self map of E. Suppose F_T = {p ∈E: T_p= p} is the set of fixed points of T in E.Let u_0∈E, then the sequence $\{u_n\}_{n=1}^{\infty}$ defined by

$$u_{n+1} = Tu_n, \quad n \geq 0 \tag{1}$$

is called Picard Iterative Scheme.

Definition 1.2: Let E be a Banach space and T: E → E a self map of E. For u_0∈E, the sequence $\{u_n\}_{n=1}^{\infty}$ defined by

$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n T u_n \tag{2}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence in [0, 1) such that $\sum_{n=0}^{\infty}\alpha_n = \infty$ refers to as Mann Iterative scheme.

Definition 1.3: Let E be a Banach space and T: E → E a self map of E. For u_0∈E, the sequence $\{u_n\}_{n=1}^{\infty}$ defined by

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$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n T v_n$$
$$v_n = (1-\beta_n)u_n + \beta_n T u_n \tag{3}$$

where sequences $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}\subset$ [0, 1) such that $\sum_{n=0}^{\infty}\alpha_n = \infty$ refers to as Ishikawa Iterative scheme.

Definition 1.4: Let E be a Banach space and T: E → E a self map of E. For u_0∈E, the sequence $\{u_n\}_{n=1}^{\infty}$ defined by

$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n T v_n$$
$$v_n = (1-\beta_n)u_n + \beta_n T w_n \tag{4}$$
$$w_n = (1-\gamma_n)u_n + \gamma_n T u_n$$

where sequences$\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}\subset$ [0, 1) such that $\sum_{n=0}^{\infty}\alpha_n = \infty$ refers to as Noor Iterative scheme.

Remark 1.5: Obviously in (4), if $\gamma_n = 0$ it reduces to (3). If $\beta_n = \gamma_n = 0$, it also reduces to (2) and if $\alpha_n = 1$, $\beta_n = \gamma_n = 0$ in (4) it is becomes Picards Iterative Scheme (1).

Over the years, several authors have been modifying Iterative Processes for different classes of operators. In 2006, Rafiq A.[13] analyzed a modified three-step iterative scheme for solving nonlinear operators in Banach spaces. Olaleru J. and Mogbademu A.[10] proved the convergence results for modified Noor iteration when applied to three generalized strongly pseudocontrative maps defined on a Banach space. Results obtained generalized works of several authors. Okeke G. A. and Olaleru J. O.[9] introduced a new three step iterative scheme with errors to approximate the unique common fixed point of a family of three strongly pseudocontractive (accretive) mappings on Banach spaces which generalized and improved the result of Olaleru J. & Mogbademu A.[10] and corrected the results of Rafiq A.[14]. Modified Noor Iteration for non expansive semi groups with generalized contraction in Banach spaces was investigated by Pakkapan P. A and Rabian W. [13]. Akewe H. and Olaleru J. O.[2] established some strong convergence and stability results of multistep iterative scheme for a general class of operators. Their convergence results generalized and extended the results of Berinde [6], Bosede [3], Olaleru [9], Rafiq [14] among others. A modified mixed Ishikawa iteration for common fixed points of two asymptotically Quasi Pseudo contrative type non-self mapping was studied by Wang Y. and Suthep S. [16]. Mohammad A. and Mohammad Z. A.[8] established the strong convergence theorem of Noor iterative scheme for a class of Zamfirescu operators in arbitrary Banach spaces. Their results was an extension and generalization of the recent results of Xu B. L. et al. [18], Berinde V. [6], Zhou H. et al. [20] and many other authors. Withun P. and Suthep S. [17] proposed the following modified iterative scheme in which they proved the strong convergence of the proposed scheme to a fixed point of a weak contraction:

$$x_{n+1} = (1-\alpha_n-\lambda_n)y_n + \alpha_n T y_n + \lambda_n T z_n$$
$$y_n = (1-\beta_n)z_n + \beta_n T z_n \tag{5}$$
$$z_n = (1-\gamma_n)x_n + \gamma_n T x_n$$

where sequences $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\lambda_n\}$ and $\{\alpha_n+\lambda\}$ are sequences in [0, 1]. They showed that their proposed scheme converges faster than Mann, Ishikawa and Noor iterations.

Motivated mostly by the work of Withun P. et al [17], we proposed the following iterative schemes:

Algorithm 1.6: Let (E, d) be a complete metric space and T: E → E a self map. Let F_T be the set of fixed point of T, that is,F_T = {u ∈E: T_u= u}. The sequence$\{u_n\}_{n=0}^{\infty}$defined by

$$u_{n+1} = [1-(\alpha_n+\delta_n)]u_n + \delta_n T u_n + \alpha_n T u_n \tag{6}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\delta_n\}_{n=0}^{\infty}$ and $\{\alpha_n+\delta_n\}_{n=0}^{\infty}\subset$[0, 1) is called the modified Mann Iterative Scheme.

If for $u_n \in E$, the sequence$\{u_n\}_{n=0}^{\infty}$defined by

$$u_{n+1} = [1-(\alpha_n+\delta_n)]u_n + \delta_n T u_n + \alpha_n T v_n$$

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$$v_n = [1-(\beta_n+\delta_n)]u_n + \delta_n T u_n + \beta_n T u_n \quad (7)$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\delta_n\}_{n=0}^{\infty}$, $\{\alpha_n+\delta_n\}_{n=0}^{\infty}$ and $\{\beta_n+\delta_n\}_{n=0}^{\infty} \subset [0,1)$ is called the modified Ishiawa Iterative Scheme.

For $u_n \in E$, the sequence $\{u_n\}_{n=0}^{\infty}$ defined by

$$u_{n+1} = [1-(\alpha_n+\delta_n)]u_n + \delta_n T u_n + \alpha_n T v_n$$
$$v_n = [1-(\beta_n+\delta_n)]u_n + \delta_n T u_n + \beta_n T w_n \quad (8)$$
$$w_n = [1-(\gamma_n+\delta_n)]u_n + \delta_n T u_n + \gamma_n T u_n$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$, $\{\delta_n\}_{n=0}^{\infty}$, $\{\alpha_n+\delta_n\}_{n=0}^{\infty}$, $\{\beta_n+\delta_n\}_{n=0}^{\infty}$ and $\{\gamma_n+\delta_n\}_{n=0}^{\infty} \subset [0,1)$ is called the modified Noor Iterative Scheme.

So many authors have studied the convergence and stability of iterative processes in an arbitrary Banach space. Berinde [6] studied the convergence theorem of Mann and Ishikawa iterative schemes for Zamfirescu operators. Bosede [3] investigated the convergence of Noor iterative process. Of recent, Mohammed A. and Mohammed Z. A. [8] extended and generalized the convergence theorem of 3-step Noor iterative scheme for more generalized Zamfirescu operators in arbitrary Banach space.

In 1972, Zamfirescu T. [18] obtained an interesting fixed point theorem stated as follows:

Theorem 1.7: Let E be a Banach space and T: E → E be a self map for which there exist the real number a, b and c satisfying $0 \le a < 1$, $0 \le b < \frac{1}{2}$ and $0 \le c < \frac{1}{2}$ respectively such that for each x, y ϵ E at least one of the following is true:

$$d(T_x, T_y) \le ad(x, y);$$
$$d(T_x, T_y) \le b[d(x, T_x) + d(y, T_y)]; \quad (9)$$
$$d(T_x, T_y) \le c[d(x, T_y) + d(y, T_x)].$$

Then T has a unique fixed point p and the Picard iterative scheme $\{x_n\}_{n=0}^{\infty}$, defined by $x_{n+1} = Tx_n$, n = 0, 1, 2 ... converges to p for any $x_0 \epsilon E$. (9) was later generalized as follows:

Theorem 1.8: Let E be a complete metric space and T: E → E a Zamfirescu operator satisfying:

$$d(T_x, T_y) \le h\max\{d(x.y), \tfrac{1}{2}[(d(x, T_x) + (y, T_y)], \tfrac{1}{2}[d(x, T_y) + d(y, T_x)]\} \quad (10)$$

where $0 \le h < 1$.

Several generalizations (see Bosede and Rhoades [4], Berinde [6], Osilike [12], Bosede [3] and many others) have been proposed. Nevertheless, Akewe H. and Olaleru J. O. [2] pointed that Bosede and Rhoades [4] is more general than all these generalizations subject to a condition. Bosede and Rhoades [4] generalized as follows

Theorem 1.9: Let E be a Banach space and T: E → E a self map, for each x, y ϵ E there exist $a \epsilon [0.1)$ such that

$$d(p, T_y) \le ad(p, y) \quad (11)$$

Bosede [4] and Akewe H. [2] established some stability results using (11). All these generalizations were been used to test for the convergence and stability of conventional iterative schemes, however, in this paper we use (11) to investigate the strong convergence and stability results of the proposed modified iterative scheme while the convergence and stability results of modified Ishikawa and Mann schemes follow as corollaries. The following Lemma will be needed to establish our results.

Lemma 1.10: Let ∂ be a real number satisfying the condition $0 \le \partial < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$, so for any sequence of positive numbers $\{x_n\}_{n=0}^{\infty}$ satisfying $x_{n+1} \le \partial x_n + \epsilon_n$, where n = 0, 1,…,we have $\lim_{n\to\infty} x_n = 0$.

II. MAIN RESULTS

In this section, we will establish some strong convergence and stability results of the modified iterative scheme for a general class of operators introduced by Bosede and Rhoades [5]. While the convergence and stability results of the Modified Mann and Ishikawa Iterative Schemes will follow as corollaries.

Theorem 2.1: Let T: E → E be a self map of E and (E, ‖.‖) be a Banach space with fixed point p satisfying:

$$\| p - T_z \| \le a \| p - z \| \quad (12)$$

where $z \in E$ and $0 \le a < 1$. Let $\{u_n\}_{n=0}^{\infty}$ be the Modified iterative scheme defined by *(8)* which converges to p; $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$, $\{\delta_n\}_{n=0}^{\infty}$, $\{\alpha_n+\delta_n\}_{n=0}^{\infty}$, $\{\beta_n+\delta_n\}_{n=0}^{\infty}$ *and* $\{\gamma_n+\delta_n\}_{n=0}^{\infty}$ are sequences in *[0, 1)* .Then we say that the Modified iterative scheme converges strongly to p.

Proof: From (11) and (8), we have

$$\begin{aligned} ||u_{n+1}+p|| &\le ||[1-(\alpha_n+\delta_n)]u_n + \delta_n T u_n + \alpha_n T v_n - (1-\alpha_n-\delta_n+\alpha_n+\delta_n)p|| \\ &\le [1-(\alpha_n+\delta_n)]||u_n-p|| + \delta_n||Tu_n-p|| + \alpha_n||Tv_n-p|| \end{aligned} \quad (13)$$

Making use of (12) where $z = v_n = u_n$ and substituting it in (13), we have

$$||u_{n+1}+p|| \le [1-\alpha_n+\delta_n(a-1)]||u_n-p|| + a\alpha_n||v_n-p|| \quad (14)$$

Further, we have

$$\begin{aligned} ||v_n-p|| &\le ||[1-(\beta_n+\delta_n)]u_n + \delta_n T u_n + \beta_n T w_n - (1-\beta_n-\delta_n+\beta_n+\delta_n)p|| \\ &\le [1-(\beta_n+\delta_n)]||u_n-p|| + \delta_n||Tu_n-p|| + \beta_n||Tw_n-p|| \end{aligned} \quad (15)$$

Again by (12) and $z = w_n$, we find that

$$||Tw_n-p|| \le a||w_n-p|| \quad (16)$$

Making use of (16) in (15) we have

$$||v_n-p|| \le [1-\beta_n+\delta_n(a-1)]||u_n-p|| + a\beta_n||w_n-p|| \quad (17)$$

Substituting (17) in (14), we obtain

$$||u_{n+1}+p|| \le [1-\alpha_n+\delta_n(a-1)]||u_n-p|| + a\alpha_n[[1-\beta_n+\delta_n(a-1)]||u_n-p|| + a\beta_n||w_n-p||] \quad (18)$$

Further again we have

$$\begin{aligned} ||w_n-p|| &\le ||[1-(\gamma_n+\delta_n)]u_n + \delta_n T u_n + \gamma_n T u_n - (1-\gamma_n-\delta_n+\gamma_n+\delta_n)p|| \\ &\le [1-(\gamma_n+\delta_n)]||u_n-p|| + \delta_n||Tu_n-p|| + \gamma_n||Tu_n-p|| \end{aligned} \quad (19)$$

Again by (12), this time with $z = u_n$, we find that

$$||Tu_n-p|| \le a||u_n-p|| \quad (20)$$

Using (20) in (19), we have

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$$||w_n - p|| \leq \left[1-\gamma_n+\delta_n(a-1)\right]||u_n-p|| + a\gamma_n||u_n-p|| \quad (21)$$

Substituting (21) in (18), we obtain

$$||u_{n+1}+p|| \leq [1-\alpha_n+\delta_n(a-1)]||u_n-p|| + a\alpha_n[1-\beta_n+\delta_n(a-1)]||u_n-p|| + a^2\alpha_n\beta_n[1-\gamma_n+\delta_n(a-1) + a\gamma_n]||u_n-p|| \quad (22)$$

$$||u_{n+1}+p|| \leq [1-\alpha_n+a\alpha_n(1-\beta_n)+a^2\alpha_n\beta_n(1-\gamma_n) + a^3c_n]||u_n-p|| \leq [1-(1-a)\alpha_n]||u_n-p|| \quad (23)$$

Inductively, we obtain

$$||u_{n+1}+p|| \leq \prod_{k=0}^{n}[1-(1-a)\alpha_k]||u_0-p||. \quad (24)$$

Making use of the fact that $0 \leq a < 1$; $\alpha_n, \beta_n, \gamma_n, \delta_n \subset (0,1]$and $\sum_{n=0}^{\infty}\alpha_n = \infty$, it follows that

$$\lim_{n\to\infty}\prod_{k=0}^{n}[1-(1-a)\alpha_k] = 0 \quad (25)$$

Therefore, $\lim_{n\to\infty}||u_{n+1}+p|| = 0$.

In other word,$\{u_n\}_{n=0}^{\infty}$ converges strongly to the fixed point p. This completes the proof.

Corollary 2.2: Let T: E →E be a self map of E and (E, ||.||) a Banach space with fixed point p satisfying:

$$||p - T_z|| \leq a||p - z|| \quad (26)$$

where z ϵ E and 0 ≤ a < 1. Let $\{u_n\}_{n=0}^{\infty}$ be the Modified Ishikawa iterative scheme defined by (7) which converges to p and $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\delta_n\}_{n=0}^{\infty}$are sequences in [0, 1), then we say that the Modified Ishikawa iterative scheme converges strongly to p.

Corollary 2.3: Let T: E →E be a self map of E and (E, ||.||) a Banach space with fixed point p satisfying:

$$||p - T_z|| \leq a||p - z|| \quad (27)$$

where z ϵ E and 0 ≤ a < 1. Let$\{u_n\}_{n=0}^{\infty}$be the Modified Mann iterative scheme defined by (6) which converges to p and$\{\alpha_n\}_{n=0}^{\infty}and\{\delta_n\}_{n=0}^{\infty}$are sequences in [0, 1), then we say that the Modified Mann iterative scheme converges strongly to p.

Theorem *2.4*: Let T: E → E be a self map of E and (E, ||.||) be a Banach space with fixed point p satisfying:

$$||p - T_z|| \leq a|| p - z|| \quad (28)$$

for each z ϵ E and 0 ≤ a < 1. Let $\{u_n\}_{n=0}^{\infty}$ be the Modified iterative scheme which converges to p with $0 < \alpha < \alpha_n$,$0 < \beta < \beta_n$,$0 < \gamma < \gamma_n$ and $0 < \delta < \delta_n$ for all n. Then the Modified iterative scheme is T-stable.

Proof:

Suppose$\{x_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$are real sequences in E and $\epsilon_n = ||x_{n+1} - [1-(\alpha_n+\delta_n)]x_n - \delta_nTx_n - \alpha_nTy_n||$

where

$y_n = [1-(\beta_n+\delta_n)]x_n + \delta_nTx_n + \beta_nTz_n$,

$z_n = [1-(\gamma_n+\delta_n)]x_n + \delta_nTx_n + \gamma_nTx_n$

and let $\lim_{n\to\infty}\epsilon_n = 0$. We will now prove that $\lim_{n\to\infty} z_n = p$ using (11).

$$||x_{n+1}-p|| \leq ||x_{n+1}-[1-(\alpha_n+\delta_n)]x_n - \delta_nTx_n - \alpha_nTy_n|| + ||[1-(\alpha_n+\delta_n)]x_n + \delta_nTx_n + \alpha_nTy_n - (1-\alpha_n-\delta_n+\alpha_n+\delta_n)p|| \leq \epsilon_n + [1-(\alpha_n+\delta_n)]||x_n-p|| + \delta_n||Tx_n-p|| + \alpha_n||Ty_n-p|| \quad (29)$$

Making use of (28) with $z = x_n = y_n$, we have

$$||x_{n+1}-p|| \leq \epsilon_n + [1-(\alpha_n+\delta_n-a\delta_n)]||x_n-p|| + a\alpha_n||y_n-p||$$

$$||x_{n+1}-p| \leq \epsilon_n + [1-(1-a)\delta_n]||x_n-p|| + a\alpha_n||y_n-p||. \quad (30)$$

$$||y_n-p|| = ||[1-(\beta_n+\delta_n)]x_n + \delta_nTx_n + \beta_nTz_n - (1-\beta_n-\delta_n+\beta_n+\delta_n)p|| \leq [1-(\beta_n+\delta_n)]||x_n-p|| + \delta_n||Tx_n-p|| + \beta_n||Tz_n-p|| \quad (31)$$

Using (28) with $z = x_n = z_n$ in (31), we have

$$||y_n-p|| \leq [1-(\beta_n+\delta_n-a\delta_n)]||x_n-p|| + a\beta_n||z_n-p|| \leq [1-(\beta_n+\delta_n-a\delta_n)]||x_n-p|| \leq [1-(1-a)\delta_n]||x_n-p|| \quad (32)$$

Substituting (32) in (30), we have

$$||x_{n+1}-p| \leq \epsilon_n + [1-(1-a)\delta_n]||x_n-p|| + a\alpha_n[1-(1-a)\delta_n]||x_n-p|| = [1-\delta_n+a\delta_n+a\alpha_n-a\alpha_n\delta_n+a^2\alpha_n\delta_n]||x_n-p|| + \epsilon_n \quad (33)$$

Given that $0 < \delta < \delta_n$ and $0 < \alpha < \alpha_n$, then (33) becomes

$$||x_{n+1}-p|| \leq \epsilon_n + [1-(1-a)\delta_n]||x_n-p|| \quad (34)$$

Making use Lemma (1.10) in (34), we have $\lim_{n\to\infty} x_n = p$.

Conversely, suppose $\lim_{n\to\infty} x_n = p$, we will now show that $\lim_{n\to\infty}\epsilon_n = 0$. Given that

$\epsilon_n = ||x_{n+1} - [1-(\alpha_n+\delta_n)]x_n - \delta_nTx_n - \alpha_nTy_n||$,

then

$$\epsilon_n \leq ||x_{n+1}-p|| + ||(1-\alpha_n-\delta_n+\alpha_n+\delta_n)p - [1-(\alpha_n+\delta_n)] - \delta_nTx_n - \alpha_nTy_n|| \leq ||x_{n+1}-p|| + [1-(\alpha_n+\delta_n)]||x_n-p|| + \delta_n||Tx_n-p|| + \alpha_n||Ty_n-p|| \leq ||x_{n+1}-p|| + [1-(\alpha_n+\delta_n)]||x_n-p|| + a\delta_n||x_n-p|| + a\alpha_n||y_n-p|| \quad (35)$$

Substituting (32) in (35), we have

$$\epsilon_n \leq ||x_{n+1}-p|| + [1-(\alpha_n+\delta_n)]||x_n-p|| + a\delta_n||x_n-p|| + a\alpha_n[1-(1-a)\delta_n]||x_n-p|| \leq ||x_{n+1}-p|| + [1-(1-a)\delta]||x_n-p|| \quad (36)$$

Given that$\lim_{n\to\infty}||x_n-p|| = 0$, then $\lim_{n\to\infty}\epsilon_n = 0$.

Hence, the modified iterative scheme is T-stable.

Corollary 2.5: Let T: E → Ebe a self map of E and (E, ||.||) be a Banach space with fixed point p satisfying:

$|| p - T_z|| \leq a|| p - z||$

for each z ϵ Eand *0 ≤ a< 1*.Let$\{u_n\}_{n=0}^{\infty}$ be the Modified Ishikawa iterative scheme defined by (7) which converges to p

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with $0 < \alpha < \alpha_n$, $0 < \beta < \beta_n$ and $0 < \delta < \delta_n$ for all n. Then the Modified Ishikawa iterative scheme is T-stable.

Corollary 2.6: Let T: E → E be a self map of E and (E, ||.||) be a Banach space with fixed point p satisfying:

$|| p - T_z || \leq a || p - z ||$

for each z ϵ E and 0 ≤ a < 1. Let $\{u_n\}_{n=0}^{\infty}$ be the Modified Mann iterative scheme defined by (7) which converges to p with $0 < \alpha < \alpha_n$ and $0 < \delta < \delta_n$ for all n. Then the Modified Mann iterative scheme is T-stable.

III. NUMERICAL EXPERIMENT

Example 3.1(See [8]): Let E = [0,1] and T: E →E be defined by $T_x = cos(x)$ with fixed point $p = 0.739$ and a ϵ [0,1]. Choosing $\alpha_n = \frac{1}{n+1}$, $\beta_n = \frac{1}{n+2}$, $\gamma_n = \frac{1}{n+3}$ & $\delta_n = \frac{1}{n-1}$ and $x_0 = 0.2$. The comparison results to the fixed point $p = 0.739$ are shown in the table below with the help of Matlab.

Table 1: Comparison Results of Noor Iterative Scheme and Modified Iterative scheme.

Iterative number(n)	Noor Iterative Scheme	Modified Iterative Scheme
1	0.552163335	0.610671022
2	0.639644297	0.688110423
3	0.675862087	0.702654195
4	0.694724978	0.706206552
5	0.705939597	0.710029877
6	0.713211744	0.716180014
7	0.718226827	0.720095445
⋮	⋮	⋮
10	0.726646147	0.729822172
⋮	⋮	⋮
50	0.730846343	0.734622151
⋮	⋮	⋮
100	0.738767719	0.739155383
⋮	⋮	⋮
150	0.738922907	0.739000872
⋮	⋮	⋮
198	0.739005907	0.739
⋮	⋮	⋮
224	0.739	0.739

Remark 3.2:

From the table above, we can deduce that our Modified Iterative Scheme converges faster to the fixed point p = 0.739 than the Noor iterative scheme, which proved that this article is an improvement on the results obtained in [8].

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