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Optimal Control Analysis of a Mathematical Model for Covid-19 Transmission Dynamics in Nigeria

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Abstract— *The outbreak of the novel coronavirus diseases caused by SARS-CoV-2 is causing great challenges to the global health. Non-pharmaceutical interventions are being deployed due to the unavailability of certified effective drugs or vaccine for the virus. In this study, a compartmental model is formulated to explore the transmission dynamics of COVID-19 in Nigeria. The model has six non- linear differential equations which describe the spread of corona virus disease. The disease free equilibrium point and existence of endemic point was determined. The basic reproduction number which is the threshold value for determining the pattern of coronavirus disease was calculated to be R_0=0.9330. Pontryagin Maximum Principle was applied on the model in order to develop strategies to counteract corona virus disease pandemic. Four control methods were used which are combination of social distance, contact tracing, treatment and face masks usage. This study reveals that observation of social distancing and uses of face masks should be strictly comply with by susceptible, exposed and infectious detected individual in order to combat COVID-19. Finally, contact tracing should be force on both exposed and infectious undetected individual with disease. Treatment is only effective at early stage of infectious detected that is between 1-20 days of detection, anything above that treatment is no longer effective.*

Keywords— *Basic Reproduction Number: COVID-19: Disease Free Equilibrium: Mathematical Model and Optimal Control.*

I. INTRODUCTION

Ever since the first reported case of the deadly corona virus (COVID-19) was reported in the china city of Wuhan in late 2019, the whole world has since been experiencing the worst pandemic in the century spreading to over 215 countries. The cause of COVID-19 is a novel Coronavirus named Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2).The World Health organization (WHO), on the 30th of January, 2020 officially announced corona virus outbreak a Global Public Health Emergency of International Concern [1] and later declared the virus a pandemic on the 11thof March, 2020 [2]. As of 3rd September 2020, COVID-19 has accounted for over 26240870 confirmed positive cases with about 868448 deaths globally [3].

Nigeria, Africa's most populous country of over 200 million people [4] had her 1st reported case on February 27, 2020 when an Italian citizen who just returned from Europe to Lagos was tested positive of the virus [5]. Ever since, COVID-19 has spread to all state of the country. As of September 3rd, 2020, data released by the Nigeria Centre for Disease Control (NCDC) shows that out of 411077 total tested samples for the virus, there are 54463 confirmed positive cases of the virus with 1027 deaths in the country. Lagos the epicenter of the virus in the country account for over 33% of the country's reported cases as at the end August 2020 [6].

Scientists are still battling with the search for safe and effective vaccine or antiviral drug for use against the virus. Consequent upon this, non-pharmaceutical intervention are recommended to control the spread of the virus. These include community lockdown, maintenance of social distance, face mask usage in public, contact tracing, confirmed cases isolation and quarantine of suspected cases.

On the 30th of March, in other to control community transmission of the virus, the federal government of Nigeria declared a strict two weeks lockdown of two states (namely, Lagos and Ogun State) together with the Federal Capital Territory Abuja [7]. The lock down was later extended for another two weeks on April 13 and the rest of the country was placed under lock down on April 27 when there are increased numbers of cases being recorded across the country. Despite the continuous rise in the number of reported cases, the government begins easing of imposed restrictions in phases on May 4. The contentious issue is how effective and safe are the non-pharmaceutical interventions in the control of the virus. This study therefore aims on evaluating the effectiveness of some of the control measures using a mathematical model based on the available data on the virus spread.

II. MODEL FORMULATION

The mathematical model formulated is based on the spread and transmission of SARS-CoV-2. The model subdivides the total population size at time t denoted as $N(t)$ into susceptible $S(t)$, exposed $E(t)$, infectious detected $I_D(t)$, infectious undetected $I_U(t)$, infectious isolated $I_S(t)$ and recovered class $R(t)$. The infectious rate λ is defined as:

$$\lambda = \frac{\beta(\eta_D I_D + \eta_U I_U + \eta_S I_S)}{N}$$

The assumption is that the exposed and recovered individual do not transmit the disease i.e. only infected detected, infected undetected and infected isolated are assumed capable of transmitting the COVID-19 to susceptible individual. Putting all this assumptions together, we obtain the following model equations

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$$\begin{aligned}
\frac{dS}{dt} &= \rho - \lambda S - \mu S + \varphi R \\
\frac{dE}{dt} &= \varepsilon\lambda S - (\kappa + \sigma_1 + \mu)E \\
\frac{dI_D}{dt} &= (1-\varepsilon)\lambda S + \omega\kappa E - (\sigma_2 + \delta + \mu)I_D \\
\frac{dI_U}{dt} &= (1-\omega)\kappa E - (\gamma_1 + \mu + \delta)I_U \\
\frac{dI_S}{dt} &= \sigma_1 E + \sigma_2 I_D - (\gamma_2 + \delta + \mu)I_S \\
\frac{dR}{dt} &= \gamma_1 I_U + \gamma_2 I_S - (\varphi + \mu)R
\end{aligned} \tag{2.1}$$

TABLE 1. Description of parameters

Parameter	Description
ρ	Recruitment rate
δ	Disease induced death rate
ω	Endogenous reactivation rate
κ	Progression rate from exposed to detected and undetected individual
ε	Slow progressor
γ_1	Recovery rate of infectious undetected individual
γ_2	Recovery rate of infectious isolated individual
μ	Natural death rate
σ_1	Isolation rate of exposed individual
σ_2	Isolation rate of infectious detected
$\eta_D = \eta_U = \eta_S$	Modification parameters
β	Contact rate
ϕ	Loss of immunity from recovered individual

III. Model Analysis

a. Invariant region

In this aspect, the solutions of the model system are uniformly bounded in the proper subset $\Phi \subset \Re_+^6$.

By the entire population, we have

$$\frac{dN}{dt} = \frac{dE}{dt} + \frac{dI_D}{dt} + \frac{dI_U}{dt} + \frac{dI_S}{dt} + \frac{dR}{dt} \tag{2.2}$$

Therefore

$$\frac{dN}{dt} = \rho - \mu(S + E + I_D + I_U + I_S + R) - \delta(I_D + I_U + I_S) \tag{2.3}$$

In the absence of mortality due to COVID 19, equation (2.3) lead

$$\frac{dN}{dt} \le \rho - \mu N \tag{2.4}$$

$$\therefore \rho - \mu N \ge Be^{-\mu t} \tag{2.5}$$

Where B is a constant independent of t. Further simplification gives

$$N \le \frac{\rho}{\mu} - \left(\frac{\rho - \mu N_0}{\mu}\right)e^{-\mu t} \tag{2.6}$$

As $t \to \infty$ in equation (2.6), the population $N \to \frac{\rho}{\mu}$ that implies $0 \le N \le \frac{\rho}{\mu}$ Thus the feasible solution set of the system equation of the model enter and remain in the region

$$\Phi = \left\{\left(S, E, I_D, I_U, I_S, R\right) \in \Re_+^6 : N \le \frac{\rho}{\mu}\right\}.$$

Therefore, the basic model is well posed epidemiologically and mathematically. The Disease Free Equilibrium E_0 of the model is given by $E_0 = \left(\frac{\rho}{\mu}, 0, 0, 0, 0, 0\right)$.

b. Existence of endemic equilibrium point

Here, we determine the possible existence and stability of endemic (positive) equilibria of the model (2.1). Let $E_1 = \left(S^{**}, E^{**}, I_D^{**}, I_U^{**}, I_S^{**}, R^{**}\right)$ represent any arbitrary endemic equilibrium of the model (2.1), so that $N^{**} = S^{**} + E^{**} + I_D^{**} + I_U^{**} + I_S^{**} + R^{**}$. Force of infection λ^{**} at endemic steady-state is defined as

$$\lambda^{**} = \frac{\beta(\eta_D I_D^{**} + \eta_U I_U^{**} + \eta_S I_S^{**})}{N^{**}}. \tag{2.7}$$

Solving the model (2.1) at steady-state gives

$$\begin{aligned}
S^{**} &= \frac{\rho + \varphi R^{**}}{\lambda^{**} + \mu}, \quad E^{**} = \frac{\varepsilon\lambda^{**}S^{**}}{\kappa + \sigma_1 + \mu} == b_1\lambda^{**}S^{**}, \\
I_D^{**} &= \frac{(1-\varepsilon)\lambda^{**}S^{**} + \omega\kappa E^{**}}{\sigma_2 + \mu + \delta} = \frac{\left[(1-\varepsilon) + \omega\kappa b_1\right]}{\sigma_2 + \mu + \delta}\lambda^{**}S^{**} \\
&= b_2\lambda^{**}S^{**}, \\
I_U^{**} &= \frac{(1-\omega)\kappa E^{**}}{\gamma_1 + \mu + \delta} = \frac{\left[(1-\omega)\kappa b_1\right]\lambda^{**}S^{**}}{\gamma_1 + \mu + \delta} = b_3\lambda^{**}S, \\
I_S^{**} &= \frac{\sigma_1 E^{**} + \sigma_2 I_D^{**}}{\gamma_2 + \mu + \delta} = \frac{\left(\sigma_1 b_1 + \sigma_2 b_2\right)\lambda^{**}S^{**}}{\gamma_2 + \delta + \mu} = b_4\lambda^{**}S, \\
R^{**} &= \frac{\gamma_1 I_U^{**} + \gamma_2 I_S^{**}}{\varphi + \mu} = \frac{\left(\gamma_1 b_3 + \gamma_2 b_4\right)}{\varphi + \mu}\lambda^{**}S^{**} \\
&= b_5\lambda^{**}S
\end{aligned} \tag{2.8}$$

Where

$$b_1 = \frac{\varepsilon}{\kappa + \sigma_1 + \mu}, \quad b_2 = \frac{\left[(1-\varepsilon) + \omega\kappa b_1\right]}{\sigma_2 + \mu + \delta}, \quad b_3 = \frac{\left[(1-\omega)\kappa b_1\right]}{\gamma_1 + \mu + \delta},$$

$$b_4 = \frac{\left(\sigma_1 b_1 + \sigma_2 b_2\right)}{\gamma_2 + \delta + \mu}, \; b_5 = \frac{\left(\gamma_1 b_3 + \gamma_2 b_4\right)}{\varphi + \mu}$$

Substituting the expressions in (2.8) into (2.7) gives,

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$$\lambda^{**}(S^{**}+b_1\lambda^{**}S^{**}+b_2\lambda^{**}S^{**}+b_3\lambda^{**}S^{**}+b_4\lambda^{**}S^{**}+b_5\lambda^{**}S^{**})$$
$$=\beta\lambda^{**}(\eta_D b_2\lambda^{**}S^{**}+\eta_U b_3\lambda^{**}S^{**}+\eta_S b_4\lambda^{**}S^{**})$$
$$\lambda^{**}S^{**}(1+b_1\lambda^{**}+b_2\lambda^{**}+b_3\lambda^{**}+b_4\lambda^{**}+b_5\lambda^{**})$$
$$=\beta\lambda^{**}S^{**}(\eta_D b_2+\eta_U b_3+\eta_S b_4)$$
$$1+(b_1+b_2+b_3+b_4+b_5)\lambda^{**}=\beta(\eta_D b_2+\eta_U b_3+\eta_S b_4)$$
$$1+P\lambda^{**}=R_0$$
$$\Rightarrow \lambda^{**}=\frac{R_0-1}{P}>0 \text{ whenever } R_0>1 \quad (2.9)$$

The components of E_1 can be obtained by substituting the unique value of λ^{**}, obtain in (2.9) into the expression in (2.8), then the result is established.

c. Basic reproduction number

The new infection matrix F, the transition matrix V and the basic reproduction number R_0 defined by the spectral radius of the next generation matrix FV^{-1} [8] are given by:

$$F=\begin{pmatrix}0 & \varepsilon\beta\eta_D & \varepsilon\beta\eta_U & \varepsilon\beta\eta_S\\ 0 & (1-\varepsilon)\beta\eta_D & (1-\varepsilon)\beta\eta_U & (1-\varepsilon)\beta\eta_S\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix},$$

$$V=\begin{pmatrix}a_1 & 0 & 0 & 0\\ -\omega\kappa & a_2 & 0 & 0\\ -(1-\omega)\kappa & 0 & a_3 & 0\\ -\sigma_1 & -\sigma_2 & 0 & a_4\end{pmatrix}$$

$$R_0=\frac{\beta\begin{pmatrix}\omega\varepsilon\kappa a_2a_4\eta_U+\omega\varepsilon\kappa a_3a_4\eta_D+\\ \omega\varepsilon\kappa a_3\eta_s\sigma_2+\varepsilon\kappa a_2a_4\eta_U-\\ \varepsilon a_1a_3a_4\eta_D-\varepsilon a_1a_3\eta_S\sigma_2+\\ \varepsilon a_2a_3\eta_s\sigma_1+a_1a_3a_4\eta_D+a_1a_3\eta_S\sigma_2\end{pmatrix}}{a_1a_2a_3a_4}$$

d. Optimal control analysis

Control variables are strategies to detect and reduce coronavirus disease (COVID-19) transmission especially in Nigeria, this includes social distancing (u_1), contact tracing (u_2), case detection and treatment (u_3) and wearing of face masks (u_4) as recommended by World Health Organization.

Based on the optimal control problem, the formulated mathematical model is re-modified by incorporating the control strategies into the model:

$$\frac{dS}{dt}=\rho-(1-u_1-u_4)\lambda S-\mu S+\varphi R$$
$$\frac{dE}{dt}=(1-u_2-u_4)\varepsilon\lambda S-(\kappa+u_3\sigma_1+\mu)E$$
$$\frac{dI_D}{dt}=(1-u_1-u_4)\lambda S-(1-u_1-u_4)\varepsilon\lambda S+$$
$$\omega\kappa E-(\sigma_2u_3+\delta+\mu)I_D \quad (2.10)$$
$$\frac{dI_U}{dt}=(1-\omega)\kappa E-(\gamma_1+\mu+\delta)I_U$$
$$\frac{dI_S}{dt}=u_3\sigma_1E+u_3\sigma_2I_D-(\gamma_2+\delta+\mu)I_S$$
$$\frac{dR}{dt}=\gamma_1I_U+\gamma_2I_S-(\varphi+\mu)R$$

The goal of the control variables is to find the optimal strategies to minimize the exposed, infected undetected, infected detected and infected isolated individuals. Like the common form of an objective function in an optimal control transversality problem, the objective function of the model to be minimized is a non-linear and quadratic function defined as:

$$J(u_1,u_2,u_3,u_4)=\int_{t0}^{tf}\left[\begin{array}{l}K_1E+K_2I_D+K_3I_U+K_4I_S+\\ \frac{1}{2}\left(K_5u_1^2+K_6u_2^2+K_7u_3^2+K_8u_4^2\right)\end{array}\right]dt$$

e. Existence of an optimal control

The existence of optimal control pairs is proved in the below theorem.

Theorem 1. There exist an optimal control $u^*=\left(u_1^*,u_2^*,u_3^*,u_4^*\right)\in U$ such that;

$$J\left(u_1^*,u_2^*,u_3^*,u_4^*\right)=\min J\left(u_1,u_2,u_3,u_4\right):\left(u_1,u_2,u_3,u_4\right)\in U$$

Subject to the control system

Proof: The state and the control variables of the system (2.10) are non-negative value. The control set U is closed and convex. The integrand of the objective cost function J of expressed by (2.10) is a convex function of (u_1,u_2,u_3,u_4) on the control set U. Therefore, there exist positive numbers C_1, C_2 and a constant $p>1$ such that;

$$J\left(u_1,u_2,u_3,u_4\right)\geq C_1\left(|u_1|^2+|u_2|^2+|u_3|^2+|u_4|^2\right)^{\frac{p}{2}}-C_2$$

The Lipschitz property of the state system with respect to the state variables is satisfied since the state variables are bounded. This completes the existence of an optimal control.

f. Hamiltonian and Optimality System

By using the principle of Pontryagins Maximum Principle [9], we obtained a Hamiltonian (H) defined as:

$$H(S,E,I_D,I_U,I_S,R,t)=L(E,I_D,I_U,u_1,u_2,u_3,u_4)+$$
$$\lambda_1\frac{dS}{dt}+\lambda_2\frac{dE}{dt}+\lambda_3\frac{dI_D}{dt}+\lambda_4\frac{dI_U}{dt}+\lambda_5\frac{dI_S}{dt}+\lambda_6\frac{dR}{dt}$$

Where

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$$L(E, I_D, I_U, I_S, u_1, u_2, u_3, u_4) = K_1E + K_2I_D + K_3I_U + K_4I_S + \frac{1}{2}K_5u_1^2 + \frac{1}{2}K_6u_2^2 + \frac{1}{2}K_7u_3^2 + \frac{1}{2}K_8u_4^2$$ and

$\lambda_i, i = 1,2,3,4,\ldots 6$ are the adjoint variable to be determined by Pontryagin's maximal principle and also using [10] for existence of the optimal control pairs.

Theorem 2. Given an optimal control $u_1^*, u_2^*, u_3^*, u_4^*$ and solutions $S^*, E^*, I_D^*, I_U^*, I_S^*, R^*$ of the corresponding state system (2.10) that minimizes $J(u_1, u_2, u_3, u_4)$ over U. Then, there exists adjoint variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ such that;

$$\frac{d\lambda_2}{dt} = \lambda_2(\kappa + u_3\sigma_1 + \mu) - \lambda_3\omega\kappa - \lambda_4(1-\omega)\kappa - \lambda_5u_3\sigma_1 - K_1$$

$$\frac{d\lambda_3}{dt} = \frac{(\lambda_1 - \lambda_3)(1-u_1-u_4)\beta\eta_D S}{N} + \frac{(\lambda_3 - \lambda_2)(1-u_2-u_4)\varepsilon\beta\eta_D S}{N} + (\lambda_3 + \lambda_5)\sigma_2u_3 + \lambda_3(\mu+\delta) - K_2$$

$$\frac{d\lambda_4}{dt} = \frac{(\lambda_1 - \lambda_3)(1-u_1-u_4)\beta\eta_U S}{N} + \frac{(\lambda_3 - \lambda_2)(1-u_2-u_4)\varepsilon\beta\eta_u S}{N} + (\lambda_4 + \lambda_6)\gamma_1 + \lambda_4(\mu+\delta) - K_3$$

$$\frac{d\lambda_5}{dt} = \frac{(\lambda_1 - \lambda_3)(1-u_1-u_4)\beta\eta_S S}{N} + \frac{(\lambda_3 - \lambda_2)(1-u_2-u_4)\varepsilon\beta\eta_S S}{N} + (\lambda_5 + \lambda_6)\gamma_2 + \lambda_5(\mu+\delta) - K_4$$

$$\frac{d\lambda_4}{dt} = \lambda_6(\varphi + \mu)$$

With transversality conditions; $\lambda_i(t_f) = 0, \quad i = 1,2,\cdots,6.$

Similarly, obtain the control set $(u_1^*, u_2^*, u_3^*, u_4^*)$ which is characterized by;

$$u_1^* = \max\left\{0, \min\left(1, \frac{(\lambda_3 - \lambda_1)\lambda S^*}{K_5}\right)\right\},$$

$$u_2^* = \max\left\{0, \min\left(1, \frac{(\lambda_2 - \lambda_3)\varepsilon\lambda S^*}{K_6}\right)\right\}$$

$$u_3^* = \max\left\{0, \min\left(1, \frac{(\lambda_2 - \lambda_5)\sigma_1E^* + \lambda_3\sigma_2I_D^*}{K_7}\right)\right\},$$

$$u_4^* = \max\left\{0, \min\left(1, \frac{(\lambda_3 - \lambda_1)\lambda S^* + (\lambda_2 - \lambda_3)\varepsilon\lambda S^*}{K_8}\right)\right\}$$

IV. Numerical Results on Optimal Control Analysis

In this section, the effect of optimal strategy on COVID-19 transmission is investigated by applying some numerical techniques. The optimal strategy is achieved by obtaining a solution for the state system (2.1) and their co-state system. An iterative scheme is explored and used to determine the solution for the optimality system. We begin by solving the state equations with a guess for the controls over the simulated time by applying an in-built forward fourth order Runge-Kutta Method. Also, the co-state equations were at the same time computed by employing a backward fourth order Runge-Kutta Method with the transversality conditions. For numerical simulation, the state system solution is determined based on time with initial conditions

$S(0) = 2069617,\ E(0) = 304064$, $I_D(0) = 44942,$ $I_U(0) = 14323$, $I_S(0) = 35902$ and $R(0) = 44942$ with following parameters:

TABLE 2. Model parameters and values for simulations

Parameters	Values	Parameters	Values
ρ	271.23/days	μ	0.000031
δ	0.7/days	σ_1	0.07143
ω	0.09/days	σ_2	0.04762
κ	0.1429/days	$\eta_D = \eta_U = \eta_S$	0.10
ε	0.7/days	β	0.2
γ_1	0.05	ϕ	0.0051
γ_2	0.1		

The following graphical results were obtained

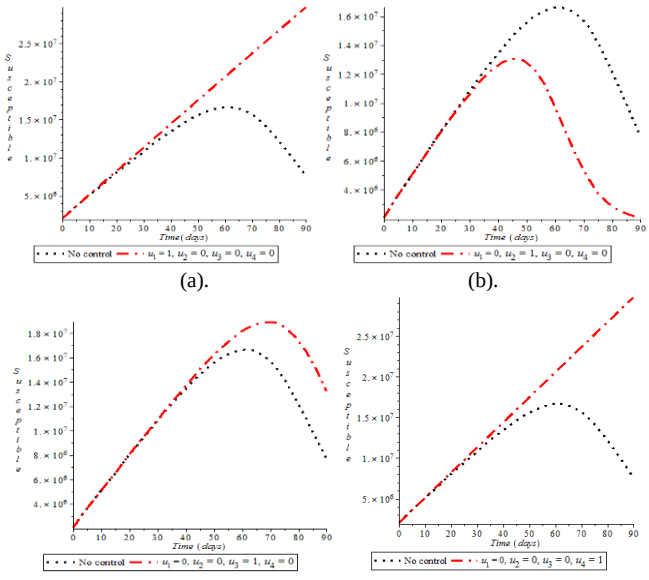

Figure 1. Model simulation showing effect of controls on susceptible population

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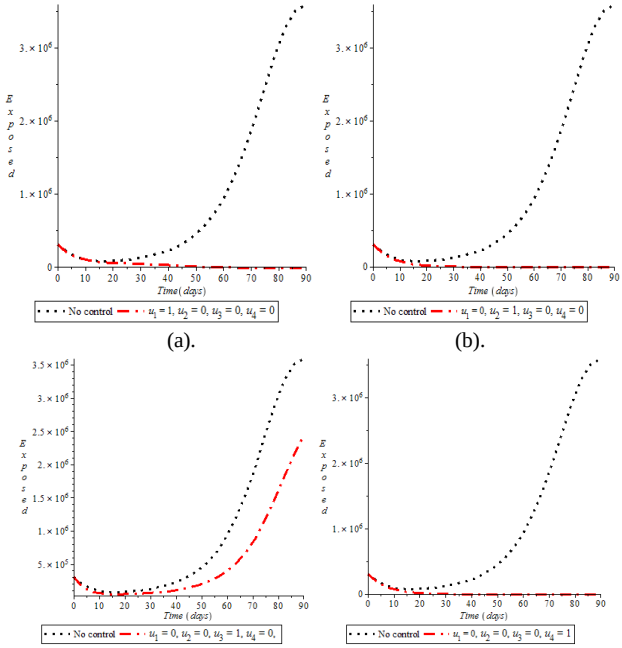

Figure 2. Model simulation showing effect of controls on exposed population

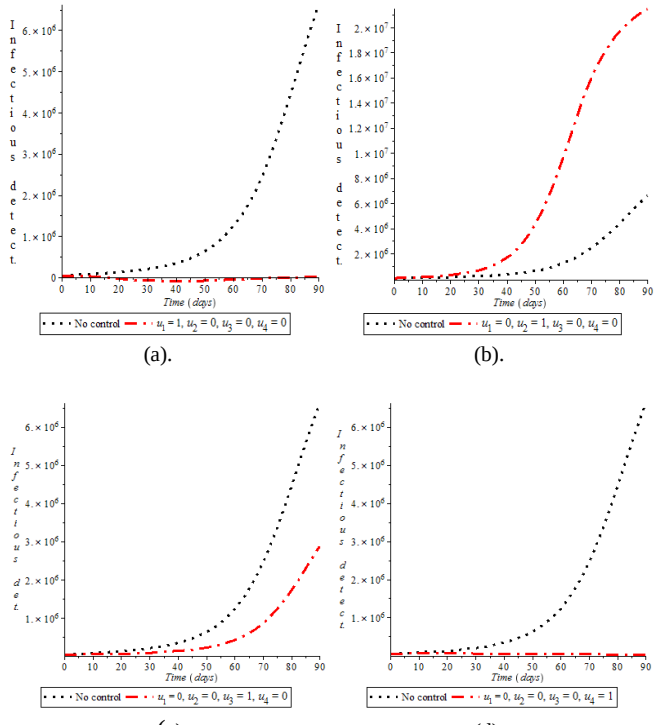

Figure 3. Model simulation showing effect of controls on infectious detected population

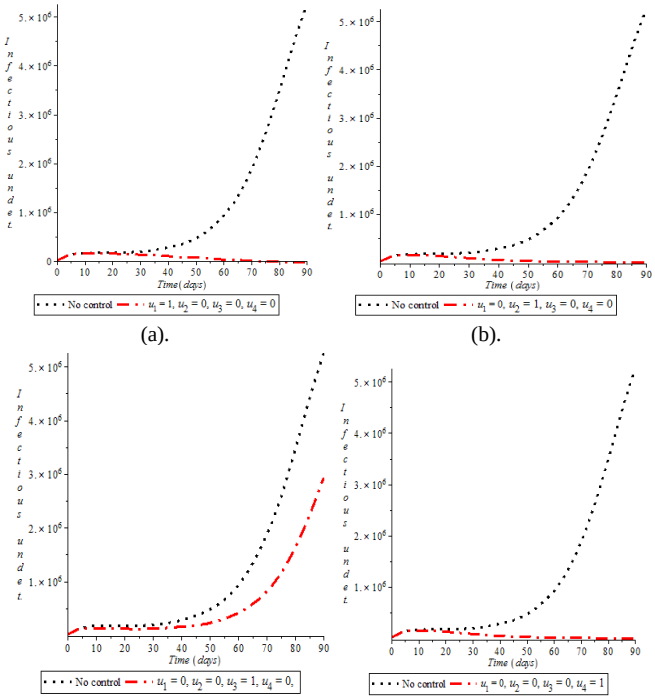

Figure 4. Model simulation showing effect of controls on susceptible population

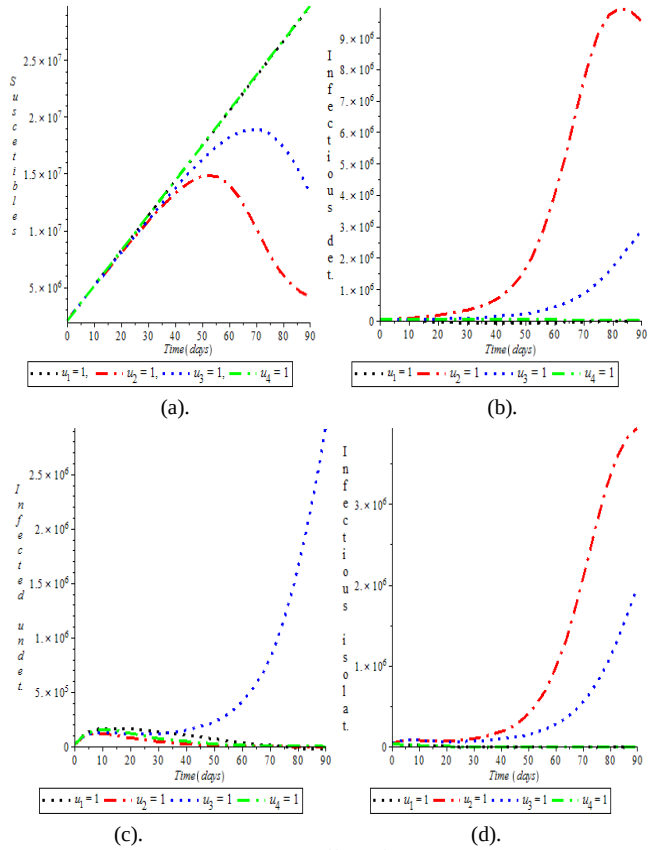

Figure 5. Model simulation showing effect of all controls on each population subdivision

V. Discussion of Results

Figure 1(a) and 1(d) shows that the susceptible population increases due to compliance of social distancing and uses of face mask while in public places. Figure 1(b) and 1(c) rises initially but later declined due to rapid spread of infection. This attribute to treatment and contact tracing have no effect on susceptible population.

Figure 2(a), 2(b) and 2(d) illustrate how the COVID-19 spread is reducing within the expose individual due to effectiveness of social distancing, contact tracing and face masks but in Figure 2(c) the treatment is only effective between 10-36 days for exposed individual. Figure 3 depict the comparative among all the controls on infectious detected population. While figure 3(a) and 3(d) shows that infectious detected population decreases totally and remain steady as the days progresses due to implementation of social distance and use of face masks. This is because those who are infectious which is the major source for contracting the COVID-19 were being controlled. Figure 3(b) reveled sharp population increase despite the implementation of contact tracing. This is because the infectious has been isolated already and contact tracing is no longer effective again. But 3(c) shows a decrease of infectious population up to 45days which is credited to treatment being applied, then it starts going up because recovered individual are joining the infectious population again.

Figure 4(a), 4(b) and 4(d) indicate the effect of social distancing, contact tracing and use of face mask respectively on infectious undetected population. Initially entire population steadily decreases showing the three controls are effective in controlling the disease in infectious undetected. The variation in 4(c) shows treatment is not necessary because their infectious status is unknown, treatment can only applied to infectious detected.

Finally figure 5 shows the effect of social distance, contact tracing, treatment and use of face masks on each population subdivision. Population increases as a result of social distancing and use of face masks unlike contact tracing and treatment that doesn't have effect on Susceptible as depicted in 5(a). Figure 5(b) and 5(d) show that infectious detected and infectious isolated decrease to zero over some weeks as a result of social distance and face masks implementation but treatment only has full impact on infectious detected compare to 5(c) which kept increasing as a result of infectious undetected. But, social distance contact tracing and uses of face masks has positive impact on COVID-19 infectious undetected population.

VI. Conclusion

The result of the simulation of our model reveals that observation of social distancing and uses of face masks should be strictly comply by susceptible, exposed and infectious detected individual in order to combat COVID-19. Finally, contact tracing should be force on both exposed and infectious undetected individual with disease. Treatment is only effective at early stage of infectious detected that is between 1-20 days of detection, anything above that treatment is no longer effective.

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