Decoupling and Model Reduction for the Binary Distillation Column Linear System

Araf Jebar¹, Nasir Ahmed Alawad²

¹Department of Computer Engineering, Almustansiriya University, Baghdad, IRAQ
²Department of Computer Engineering, Almustansiriya University, Baghdad, IRAQ
E-mail: nasir.awad @ umustansiriya.edu.iq

Abstract— The issue of model reduction is considered for the binary distillation column linear framework. For a given stable distillation column linear system framework, the goal is to discover the development of a decreased order model, which approximates the first framework well in the strong presentation. The improvement of a proper low request dynamic procedure model of a distillation column is the most significant essential for the plan of an effective control idea. Right now present a second order direct procedure model for a distillation column that can be dealt with completely diagnostically. It gives a consistent state precise portrayal of the segment for any control design in the time area and jams the physical centrality everything being equal. In light of the low computational exertion and the consistency of the displaying data it is a powerful device for control structure investigation and controller plan. Two binary distillation column can be utilized for demonstrating reduction, the Wood-Berry (WB) and Tyreus Stabilizer(TS), where until today are the most mainstream and significant partition models in the oil enterprises for purging of conclusive items. The proposed methods reduces the dimension of the original system based on two techniques are optimization and MATLAB toolbox options. The results show the effectiveness of these techniques.

Keywords— Distillation column, Decoupling methods, Model reduction Techniques, MATLAB toolbox.

I. INTRODUCTION

Structure of multiple-input multi-output (MIMO) control frameworks have gotten a lot of consideration in the control literature. Despite the fact that the structure and tuning of single loop PID controllers have been broadly investigated, they can't be straightforwardly applied to configuration decentralized control frameworks because of the presence of collaborations among control loops [1,2]. Numerous strategies had been proposed to stretch out SISO PID tuning rules to decentralized control by remunerating the impacts of loop collaborations. A typical path is to initially structure singular controller for each control loop by overlooking all cooperation’s, and afterward detune each loop by a detuning factor [3]. Nonetheless, for high-dimensional procedures, effective open-loop process (EOP) is complex, and the controllers must be traditionalist for the inescapable demonstrating mistakes experienced in formulation [4]. In the controller configuration procedure of the two-input/two-output framework, the decoupler matrix technique is utilized to deteriorate a multi-loop control framework into a lot of proportionate autonomous single loops. At that point, an intricate proportional model is gotten, and its order ought to be decreased for every individual loop, Maclaurin arrangement strategy is utilized to diminish the order for the decoupling model [5]. To beat the trouble of the (EOP) and streamlined to initially arrange first order process delay transfer (FOPDT) model, one of the model decrease strategies is coefficient matching method is used [6]. The advancement of suitable low order dynamic procedure models of a plant is the most significant essential for the design of a successful control concept. Further, the model decrease issue has gotten impressive consideration in the ongoing years, and numerous significant outcomes have been accounted for, which include different techniques, for example, the Hankel norm approximation method [7] and balanced truncation method [8]. Recently, the linear matrix inequalities (LMIs) strategy has additionally been utilized to manage the model decrease issue for various frameworks, for example, such as fuzzy systems [9,10], uncertain stochastic systems [11], singular systems [12], and port-controlled Hamiltonian frameworks [13]. Numerous techniques can be utilized to decrease order, for example, polynomial approximation [14], the Gaussian frequency domain approach [15], and least squares algorithm [16]. Model reduction can be described as searching a low-order model with a small time delay to approximate the full order model. In this paper, the model reduction problem is studied for the distillation columns linear system. This paper aims to find a reduced-order model such that the associated model reduction error very small and meets an optimization bound constraint and also using some MATLAB toolbox functions. A practical examples (WB) and (TS) of the distillation column linear system are given to demonstrate the effectiveness of the proposed model reduction method.

The remaining part of this paper is organized as follows: in Section 2, the distillation column system is introduced with process transfer matrix parameters. In Section 3, the decoupling control design for two models (WB) and (TS) are considered. In section 4, the two model reduction methods for distillation column systems by using the Particle Swarm Optimization (PSO) algorithm approach and MATLAB functions are presented. In Section 5, discussions, simulation investigations and comparisons of the models (WB and TS) reduction by the proposed methods are presented to illustrate the superior robustness achieved by using these methods. Finally, conclusions are drawn in Section 6.

II. DISTILLATION COLUMN

Distillation is one of the most important unit operations in chemical engineering. The aim of a distillation column is to
separate a mixture of components into two or more products of different compositions [17]. The physical principle of separation in distillation is the difference in the volatility of the components.

The separation takes place in a vertical column where heat is added to a reboiler at the bottom and removed from condenser at the top. A simple continuous binary tray distillation column for separating a feed stream into two fractions, an overhead distillate product and a bottoms product is shown in Figure 1, where XD(s) and XB(s) are the overhead and bottom compositions respectively, while R(s) is the reflux flow rate and S(s) is the steam flow rate to the reboiler.

In the present work, two distillation column models are taken for case study. The first example is Wood and Berry (WB) column and second example is based on Tyreus Stabilizer system.

Fig. 1. Shows Distillation column

A- Wood and Berry (WB) system

The first 2 x 2 MIMO process is presented by Wood and Berry [18,19,20]. The study was performed on a 9 inch diameter, 8 tray column equipped with a total condenser and a basket type reboiler. The required control action for the manipulative variables in the composition loops, reflux and steam flow, were cascaded to the set points of the appropriate flow controllers.

The process transfer function matrix of the distillation process is given by:-

\[
G(s) = \begin{bmatrix}
12.8e^{-5} & -13.9e^{-5} \\
16.7s+1 & 20s+1 \\
-6.6s-7e & -19.4e^{-3} \\
10.9s+1 & 14.4s+1
\end{bmatrix}
\]  

(1)

B- Tyreus Stabilizer(TS) system

The stabilizer model was developed by Tyreus [4,21]. The process transfer function matrix of the distillation process is given by:-

\[
G(s) = \begin{bmatrix}
-0.1153(10s+1)e^{-0.1s} & 0.2429e^{-2s} \\
(4s+1)^3 & (33s+1)^2 \\
-0.0887e^{-0.17s} & 0.2429e^{-0.17s} \\
(43s+1)(22s+1) & (44s+1)(20s+1)
\end{bmatrix}
\]

(2)

This model contains second and third order with time delay, which is more complex compared with wood and berry model.

III. DECOUPLING

Decoupling is used to reduce the control loop interactions. The theory of decoupling control for MIMO processes has been well-established. Several decoupling schemes were developed during past 30 years for two-input two-output (TITO) systems and have been well-summarized in many literature and process control textbooks (22,23,24,25). The design of decoupling controller matrix is very important. There are many types of decoupling, for an MIMO system controlled by a centralized controller should be decoupled first. Generally there are three types of dynamic decoupling algorithms which have been widely studied and applied in industrial processes i.e., ideal decoupling, simplified decoupling and inverted decoupling Each of these three decouplers has its own properties and limitations [26]. In this work, and due to the advantages of simplified decoupling, so that can be used.

A- Simplified Decoupling

This decouling control design, called “simplified decoupling” by Luyben [23], is widely used in the literature. It consists in selecting the decoupler as follows:

\[
D(s) = \begin{bmatrix}
1 & -\frac{g_{11}}{g_{12}} \\
-\frac{g_{12}}{g_{22}} & 1
\end{bmatrix}
\]

(3)

The resulting transfer matrix \(T(s)\) is then:

\[
T(s) = \begin{bmatrix}
g_{11}(s) - \frac{g_{12}g_{21}(s)}{g_{22}(s)} & 0 \\
0 & g_{22}(s) - \frac{g_{12}g_{21}(s)}{g_{22}(s)}
\end{bmatrix}
\]

(4)

This choice makes the realization of the decoupler easy, but the diagonal transfer matrix \(T(s)\) obtained is complex since its elements are the sum of transfer functions. Controller tuning can therefore be difficult. It is then often suggested to approximate each sum by a simpler transfer function to facilitate controller tuning.

B- Decoupling of WB System

Taking WB column, equ(1), and using equ(3), the decoupler is:

\[
D(s) = \begin{bmatrix}
1 & \frac{1475959.5e^{-20}}{167s+1} \\
0.33(142s+1)e^{-44} & 1
\end{bmatrix}
\]

(5)

Then, according to equation (4), the diagonal matrix of WB column is given by

\[
\begin{align}
\bar{G}_{11}(s) &= \frac{12.8e^{-5}}{16.7s+1} + \frac{6.237(14.36s+1)e^{-73}}{228.69s^2+31.89s+1} \\
\bar{G}_{22}(s) &= \frac{-19.4e^{-35}}{14.4s+1} + \frac{9.745(16.7s+1)e^{-95}}{228.69s^2+31.89s+1}
\end{align}
\]

(6)

(7)

To simplify the equations in (6) and (7), pade approximation is used to remove the nonlinear term in equations by using the following equation [27]:

\[
e^{\theta s} \approx \frac{1-s}{1+s}
\]

(8)

where \(\theta\) is the delay term.
The first term of equation (6) becomes:
\[-12.8 s^2 + 25.6 \]
\[16.7 s^2 + 34.4 s + 2 \]
And the second term of equation (6) becomes:
\[-99.56 s^2 + 19.35 s + 1.782 \]
\[228.7 s^2 + 97.33 s^2 + 10.11 s + 0.3857 \]

By the summing the equations (9) and (10) the result is:
\[G_{11WB} = \]
\[
\begin{bmatrix}
39.15 s^2 + 589.4 s + 2971.7 s^2 + 547.1 s^2 + 30.05 s + 0.5714
\end{bmatrix}
\]
Else also the first term of equation (7) becomes:
\[14.4 s^2 + 10.6 s + 0.6657 \]
And the second term of the equation (7) becomes:
\[152.7 s^2 + 25.42 s + 2.165 \]
By the summing the equations (12) and (13) the result is:
\[G_{22WB} = \]
\[
\begin{bmatrix}
392.5 s^2 + 536.5 s^2 + 114.6 s^2 + 74.47 s + 0.1481
\end{bmatrix}
\]

C. Decoupling of TS System

Taking TS column, equ(2), and using equ(3), the decoupler is:
\[D(s) = \]
\[
\begin{bmatrix}
1 & 2000 e^{-10(s+1)} [64 s^2 + 64 s + 1]
1
\end{bmatrix}
\]

Then, according to equation (4), the diagonal matrix of TS column is given by:
\[G_{11}(s) = -0.1153 (10 s + 1) e^{-0.15 s} \]
\[+ \frac{0.0887 (380 s^2 + 64 s + 1)}{(4 s + 1)^2} \]
\[+ \frac{103.01 94 s^4 + 1331 21 s^3 + 632 6 s^2 + 131 s + 1}{0.242 g e^{-0.7 s}} \]
\[G_{22}(s) = \]
\[
\begin{bmatrix}
44 s + 11 s^2 + 20 \end{bmatrix}
\]
\[+ \frac{103.0194 s^4 + 2361 40 s^3 + 196 74 s^2 + 763 5 s^2 + 141 s + 1}{0.1868 (25 s^4 + 26 s^2 + 96 s^2 + 16 s + 1)} e^{-14.55 s} \]

And also to simplify the equations in (16) and (17), pade approximation is used to remove the nonlinear term in equations by using the equation (8), so pade approximation for first term of equation (16) is:
\[1.153 s^2 - 22.94 s + 2306 \]
\[64 s^2 + 1338 s^2 + 972 s^2 + 241 s + 20 \]
And the second term of equation (16) is:
\[-785.6 s^4 + 536.5 s^2 + 0.8 s + 1 \]

By the summing the equations (18) and (19) the result is:
\[G_{11T} = \]
\[
\begin{bmatrix}
1.835 s^2 + 11.3 s^2 + 1.3 s^2 + 0.135 s + 0.0513 s^2 + 0.1 \times 10^{-7}
\end{bmatrix}
\]

Also the first term of equation (17) is:
\[0.242 g e^{-0.7 s} + 2.588 \]
\[880 s^3 + 1042 s^2 + 1042 s^2 + 1734 s + 117.6 \]
And the second term of the equation (17) is:
\[-17.3 s^4 - 1.2 s^4 - 1.31 s^4 + 0.513 s^4 + 4.235 e^{-0.0237} \]

By the summing the equations (21) and (22) the result is:
\[G_{22T} = \]

It is seen that, they obtained equations (11,14,20,23) are of high order and these made complex and difficult to design and analysis, so model reduction techniques is used to reduce the order of these equations.

IV. MODEL REDUCTION TECHNIQUES

Model reduction is a technique widely used in part of dynamic analysis and design of structures. Flexible structures are described by partial differential equations, and a common practice is to represent their equation of motion via linear ordinary differential equations using the finite element discretization technique [28]. Typically a model with a large number of degrees of freedom causes numerical difficulties in dynamic analysis. In structural controller design the complexity and performance of model-based controller depends on the order of the structural models. Thus a reduced-order system overcomes the above problems if it acquires the essential properties of the full-order model. Model order reduction may be necessary when the model in question is the model of a plant for which it is intended to perform a controller design. The approaches to model reduction belong to one of three main groups of methods [29]:-

The first group of methods

Attempts to retain the important eigenvalues of the system and then obtaining the remaining parameters of the lower-order model in such a way that its response to certain input is a close approximation to that of the original system. The earliest methods of model reduction, for example, the aggregation method, as well as some of the latest methods, such as balanced matrix method belongs to this category. The main advantages of this group of methods is that the reduced – order methods thus obtained can be used for ‘near optimal’ control of the original system in a straightforward manner.

The second group of methods

is based on obtaining a model of specified order such that its impulse – or step-response (or alternatively its frequency response) matches that of the original system in an optimum manner, with no restriction on the location of the eigenvalues. The choice of the criterion for optimality can lead to various interesting methods, which can also be computationally intensive. These methods can also be related to various approaches to system identification, where we obtain a model of given order that will match the given response of the original high-order system in an optimal manner.

The third group of methods

is based on matching some other properties of the response of the system. These include time moments and Markov parameters, and can be considered as special cases of Pade approximation. An attractive feature of these methods is that they require much less computation.

A number of techniques for deriving the reduced order models, such as parameter – optimisation methods, eigenvalue
truncation and Component Cost Analysis (CCA) methods are available and have been described in the literature [30].

Recently, the ‘balanced matrix method’ has attracted a great deal of attention [31,32,33]. In this method, the controllability and observability grammians are utilised to transform the system so that it is internally balanced. Then it is possible to determine and delete the least controllable and observable (and therefore the least significant) states for obtaining the reduced-order model.

In this paper two techniques of model reduction are used, the first is optimization technique, which (PSO) algorithm can be selected and the second is by using matlab toolbox, which is ‘Modred- mdc’ and ‘-del’ options.

A- Optimization methods

An evolutionary algorithm (EA) is a generic population-based meta-heuristic optimization algorithm. An EA uses some mechanisms inspired by biological evolution: reproduction, mutation, recombination, natural selection and survival of the fittest. Candidate solutions to the optimization problem play the role of individuals in a population, and the cost function determines the environment within which the solutions live[34]. Genetic Algorithms and Particle Swarm Optimization are two famous Evolutionary Algorithms. Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) methods have proved to be excellent optimization tools in the past few years[35]. The use of such search-based optimization algorithms in Model Reduction ensures that all the Model Reduction objectives are realized with minimal computational effort.

Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a biologically inspired algorithm and it is a population based stochastic nature. Each individual is referred to a particle and can move in the search space in all the directions in such a way that it reaches optimal solution. Each particle in PSO has memory and capable of storing the best position they visited. The position corresponding to the best fitness is known as pbest and capable of storing the best position they visit. The velocity and position of the particle are dynamically varying according to its own and also depends on the other particles. [36]

Algorithm for PSO [36]

Step 1: Initialize the parameters of PSO and termination criterion (generations).
Step 2: Select the relevant parameters and initial seed values of PSO as reduced numerator and denominator coefficients.
Step 3: Generate initial population with random positions and velocities in the problem space.
Step 4: Evaluate the desired optimization fitness function for each particle.
\[ J = \sum_{t=0}^{n} \left[ y(t) - y_k(t) \right] \]  
(24)
Step 5: Identify the particle that has the best fitness Value. The value of its fitness function is identified as gbest.
Step 6: Update the particle position and velocity

\[ v_{jg}^{(t+1)} = w \cdot v_{jg}^{(t)} + c_1 \cdot r_1 \cdot (pbest_j - x_{jg}^{(t)}) + c_2 \cdot r_2 \cdot (gbest - x_{jg}^{(t)}) \]  
(25)
\[ x_{jg}^{(t+1)} = x_{jg}^{(t)} + v_{jg}^{(t+1)} \]  
(26)

Step 7: Repeat step 3 and step 4 until the termination criterion is met.
Step 8: After stopping criterion the \( x_{jg}^{(t)} \) and \( v_{jg}^{(t)} \) , are holding the information of best found solution

A- Model reduction by MATLAB toolbox

The Matlab provide function to perform model reduction techniques one of this function is ‘Modred- mdc’ and ‘-del’ options.

The MATLAB control system toolbox has a function ‘modred’ (MODel REDUction), which can be used for reducing models while retaining system overall dc gain. The ‘mdc’ or matched dc gain option for the function ‘modred’ reduces defined states by setting the derivatives of the states to be eliminated to be zero, then solving for the remaining states. The method essentially sets up the eliminated states to be infinitely fast and is analogous to Guyan reduction in that low frequency effects of the eliminated states are included in the remaining states. The other option for ‘modred’ is ‘del’ option, which simply eliminates the defined states, typically associated with higher frequency modes. MATLAB provides function ‘balreal’ for balanced reduction. To reduce the order, first compute a balanced state-space realization. And Then Eliminate last diagonal entries of the balanced gramians which are relatively small. Eliminate these least-contributing states with modred using both matched DC gain and direct deletion methods.

V. RESULTS AND DISCUSSION

In this section, for wood and berry is applying (PSO) technique for model reduction and for Tyreus Stabilizer system is applying MATLAB toolbox options.

A- The PSO algorithm for wood and berry system

The steps algorithm of PSO are using and the program for reducing model order of WB was enforced in MATLAB software package. For this case study, generation count limit=200, population size=50, problem dimension=5,
mutation probability=0.06, number of elites=2, after exploitation improvement program, the reduced transfer function for applying PSO algorithm to equation (11) is:

\[ G_{11WB} = \frac{0.8498s + 0.0501}{s^2 + 0.9989s + 0.077} \]  

(27)

The step responses of the reduced order model and the original system are compared with ref [5] in Fig.2.

For transient specifications, the original and reduced systems are compared as shown in table I, with Qibing Jin method [5].

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>( G_{11WB} ) (original)</th>
<th>( G_{11rWB} ) (proposed)</th>
<th>( G_{11rWB} ) (Qibing Jin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ts )</td>
<td>48.4</td>
<td>45.8</td>
<td>41.5</td>
</tr>
<tr>
<td>( Tr )</td>
<td>27.5</td>
<td>25.8</td>
<td>23.1</td>
</tr>
<tr>
<td>( Mp )</td>
<td>0.273%</td>
<td>0.272%</td>
<td>0.273%</td>
</tr>
<tr>
<td>( Ess )</td>
<td>6.43</td>
<td>6.43</td>
<td>6.43</td>
</tr>
</tbody>
</table>

And for equation (14) is:

\[ G_{22WB} = \frac{0.7324s - 1}{0.7811s^2 + 1.047s + 0.1049} \]  

(28)

The step responses of the reduced order model and the original system are compared with ref [5] in Fig.3.

For transient specifications, the original and reduced systems are compared as shown in table II, with Qibing Jin method [5].

As seen from figs(2,3) and tables(I,II), the reduction models by the proposed method of Wb are very close to the original model specially in settling and rise time.

B- Model reduction by MATLAB toolbox for Tyreus system

The result of applying matlab toolbox option to equation (20) is:

\[ G_{11T} = \frac{0.0346s^2 - 0.03167s - 0.00022}{s^3 + 0.2268s + 0.004183} \]  

(29)

The step responses of the reduced order model and the original system are compared in Fig.4.

For transient specifications, the original and reduced systems are compared as shown in table III.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>( G_{11T} ) (original)</th>
<th>( G_{11rT} ) (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ts )</td>
<td>198</td>
<td>205</td>
</tr>
<tr>
<td>( Tr )</td>
<td>2.46</td>
<td>2.43</td>
</tr>
<tr>
<td>( Mp )</td>
<td>144</td>
<td>139</td>
</tr>
<tr>
<td>( Ess )</td>
<td>-0.0541</td>
<td>-0.054</td>
</tr>
</tbody>
</table>

The result of applying matlab toolbox option to equation (23) is:

\[ G_{22T} = \frac{0.50112ss + 0.000135s + 0.0003175}{s^2 + 0.0333s + 0.0003175} \]  

(30)
The step responses of the reduced order model and the original system are compared in Fig. 5.

For transient specifications, the original and reduced systems are compared as shown in Table V.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>$G_{22T}(\text{original})$</th>
<th>$G_{22T}(\text{proposed})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>276</td>
<td>276</td>
</tr>
<tr>
<td>$T_r$</td>
<td>167</td>
<td>169</td>
</tr>
<tr>
<td>$M_p$</td>
<td>0.025%</td>
<td>0.025%</td>
</tr>
<tr>
<td>$E_{ss}$</td>
<td>0.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>

As seen from figs (4,5) and tables (III,V), the reduction models by the proposed method of $T_s$ are very close to the original model in all parameters, especially for $G_{22T}(\text{original})$.

VI. CONCLUSIONS

Distillation columns provide a very challenging example within the field of process dynamics and process control. In this paper, first, a linearized dynamic model of a binary distillation column formulation are described and developed the modeling and simulation of different model reduction methods of a distillation column. The necessary condition to reduce the model of a distillation column while keeping the important control characteristics is achieved.

Reducing by PSO algorithm strategy appeared the best reduction strategy for the column, when compared with MATLAB toolbox functions; especially for fifth order full model (WB). While this method gives unstable reduced system for TS model.

The main Conclusion is that, the MATLAB toolbox functions is a better reduction method, specially for ninth full order of TS, because we have more data patterns available than the method PSO, but needs firstly padé approximation to remove the exponential nonlinear term in TS model.

ACKNOWLEDGMENT

The authors would like to thank Mustansiriyah University (Www.Uomustansiriyah.Edu.Iq) Baghdad – Iraq for its support in the present work.

REFERENCES


