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Determination of Displacement Ductility Corresponding to Ignoring P-Delta Effects Using Incremental Dynamic Analysis

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Abstract— *P–Delta effects are the result of gravity loads acting through the structure's lateral displacement. Although for Columns with small lateral displacement which undergo low levels of nonlinearity it is perfectly acceptable to neglect the P-Delta effect, for columns experience high levels of nonlinearity it is crucial to accurately capture the P-Delta effects. Finding the threshold of safely ignoring P-Delta effects have been subjected to many studies. Incremental Dynamic Analysis (IDA) is a series of nonlinear dynamic analyses of a structure under varying intensities of ground motion record and provides information on the performance of a structure at various stages. This research implements this technique in determination of permissible displacement ductility for ignoring the P-Delta effects by performing IDA with and without P-Delta effects and comparing the obtained IDA curves. For small earthquake record scale factors, the structural response with and without P-Delta effects are reasonably similar, but as ground motions intensifies two graphs start to diverge which means the P-Delta effects is becoming more significant and no longer can be ignored.*

Keywords— *P-Delta effects, nonlinear time-history analysis, Incremental Dynamic Analysis.*

I. INTRODUCTION

P–Delta effects are the result of gravity loads acting through the structure's lateral displacement [1]. The lateral displacement will enlarge as the gravity loads acting on them, and increased gravity load enlarges the lateral displacement [2]. P-Delta effects can have a detrimental impact on the seismic response of bridges because of a reduction in both the shear capacity and initial stiffness of RC bridge columns [3], [4], [5], [6], [7]. The reduction in the initial stiffness imposes an increase in the natural period of the system, and a likely surge in the design displacement demand. According to the American Concrete Institute (ACI 318-14) [8] P-Delta effects can be mitigated by conducting an elastic second order analysis with reduced stiffness values, or as an alternative, elastic first order analysis with increased design moments using moment magnifiers (ACI 318-14) shall be used to compensate for P-Delta effects. Caltrans SDC [9] provides a methodology to determine whether P-Delta effects can be ignored in design. A main concern regarding the P-Delta effects is the threshold of safely ignoring P-Delta effects. This research intends to incorporate Incremental Dynamic Analysis (IDA) in determination of displacement ductility at which P-Delta effects can safely be ignored. IDA performs a series of nonlinear dynamic analyses of a structure subjected to a set of ground motions of varying intensities and provide information on the performance of a structure at various stages, such as, elastic response, inelastic response, and collapse of the structure [8].

II. BACKGROUND

The Caltrans Seismic Design Criteria (SDC) [9] provides the minimum requirements for seismic design of ordinary bridges. These requirements ensure that the bridge will meet the performance goals of the design. Caltrans SDC controls the P-Delta effects using a conservative limit for lateral displacements due to axial load. This goal is met by limiting the ductility demands on structural components. According to the Caltrans SDC [9] if Eq (1) is satisfied, P-Delta effects can be ignored, and structural components can be designed based on predefined ductility demands

$$P\times\Delta_r\leq 0.2\ M_p^{col} \quad (1)$$

Where, Δ_r is the lateral offset between the point of contra-flexure and the base of the plastic hinge, and M_p^{col} is the idealized plastic moment capacity of a column calculated by M-φ analysis. If (1is satisfied, predefined ductility demands limit the design of structural components. According to Caltrans SDC target displacement ductility for single column and multi-column bents are as follows.

Regarding the P-Delta effects, two issues are typically of special concern. First concern is the threshold of safely ignoring P-Delta effects and more importantly upper design limit for P-Delta effects [11]. The earlier limit-state is determined by limiting the amplification requirements, whereas the latter one is governed by collapse-prevention criteria [11]. Codes tend to control the P-Delta effects through simplistic procedures involving first order structural linear analysis [8] or by imposing a conservative limit for lateral displacement which prevents the P-Delta effects from becoming dominant in the structures response (drift limits) [12]. Another commonly used method in order to make P-Delta effects negligible is limiting the ratio of the P-Delta induced moment to the moments induced by lateral forces (Stability coefficient limits). Finding the threshold for ignoring the P-Delta effect has been the subject of many studies. Variables introduced here have been used by many researchers. Elastic stability index is defined as following.

$$\theta_s = \frac{P}{F_{y0}.H/\Delta_y} \quad (2)$$

Nonlinear stability index is proposed by Paulay[13], and is defined as

$$\theta_\Delta = \theta_e \ \mu \ , \mu = \frac{\Delta_u}{\Delta_y} \tag{3}$$

For SDOF systems, Pauley proposed that P-Delta effects could be neglected if $\theta_\Delta < 0.15$. Bernal [4] and Mahin and Boroschek [3]suggested that if the required strength amplification to achieve a specific ductility was less than 10%, then P- Delta effects could be ignored. Using this criterion, Mahin and Boroschek suggested $\theta_\Delta < 0.20$ as the threshold for ignoring the P-Delta effects. FEMA 450 [12]identifies $\theta e \leq 0.10$ as the design tolerance for P-Delta effects. Priestley et al. [15] contended that to obtain stable structural response without producing significant P-Delta displacement, the stability index θ_Δ should be less than 0.30.

III. METHOD

A. Application of IDA in Finding the Permissible Ductility For Ignoring the P-Delta Effects

The focus of this research is to find the threshold of safely ignoring P-Delta effects. The process to perform incremental dynamic analysis in order to find the displacement ductility at which the P-Delta effects can be ignored is categorized in to multiple steps. (1) Start the process with an initial scale factor for the ground motion. (2) Perform the nonlinear time history analysis with and without P-Delta effects. (3) Compute the maximum displacement ductility with and without P-Delta effects. (4) Compute the ratio of ductility with P-Delta effects to ductility without P-Delta effects. (5) If the termination criteria have not been met, increase the scale factor and return to step (2).Adopting a dynamic algorithm for determination of the step size of ground motion scale factor can reduce the computation intensiveness of development of IDA curves. An effective algorithm should use bigger step size at the beginning of the analysis and reduces the step size by getting closer to the termination criteria.

Figure 1 illustrates a sample of a single record IDA (earthquake record 120111). The properties of the column are further described in later section. As it is illustrated in Figure 1 when earthquake record intensifies the IDA curve with P-Delta effects diverges from the IDA curve without the P-Delta effects.

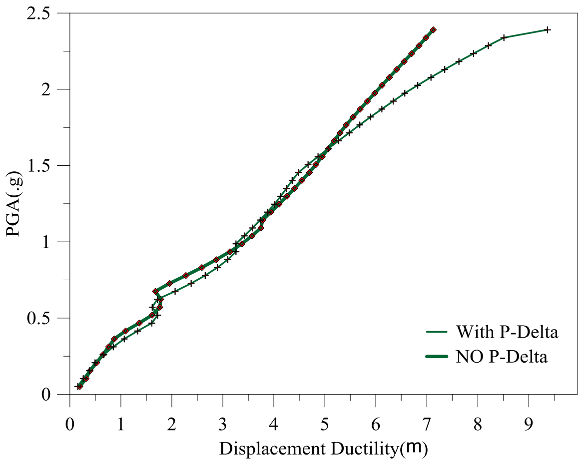

Fig. 1. Single record IDA (EQID=120111, Col Height=40 ft., Axial load=584kips, ρL=2%).

Each point on the IDA curve corresponds to a nonlinear time history analysis. Fig. 1 depicts four different nonlinear time history analysis at different peak ground accelerations. Amplified earthquake records with higher peak ground acceleration make the column to move further into nonlinear range.

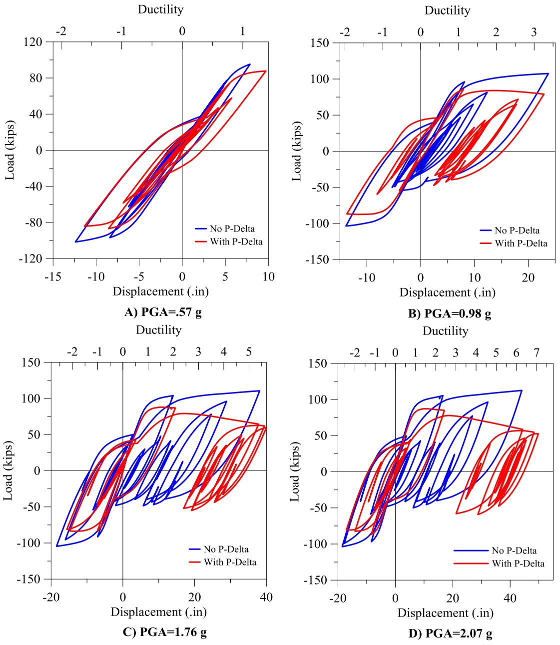

Fig. 1. Nonlinear time-history analysis (Col Height=40 ft., Axial load=584kips, ρL=2%).

B. P-Delta Effects Significance Criteria

For small scale factors for earthquake records the structural response with and without P-Delta effects are reasonably similar. As the applied earthquake record intensifies two graphs deviate from each other, or in other terms the P-Delta effects become significant. (4 is defined as the measure to evaluate the significance of P-Delta effects.

$$\frac{Ductility\ with\ P - Delta\ effects}{Ductility\ without\ P - Delta\ effects} > 1.1 \tag{4}$$

As it is illustrated in Fig. 2 when ductility with P-Delta effects is greater than 1.1 times of the ductility without P-Delta effects it is assumed that it is no longer acceptable to neglect the P-Delta effects.

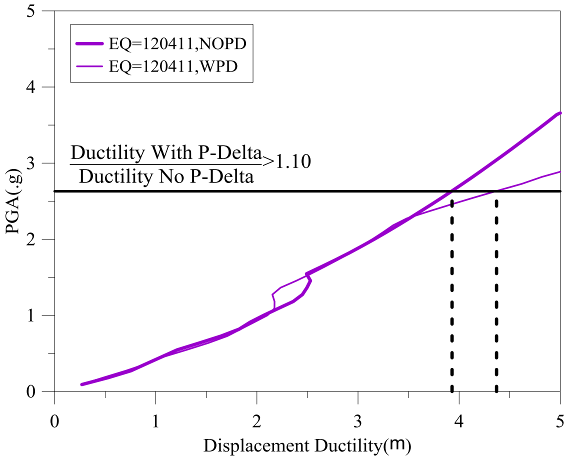

Fig. 2. Significance criterion for P-Delta effects.

C. Finite Element Model

Throughout this research nonlinear pushover and time history analyses were performed using the open source object-oriented nonlinear structural analysis program, Open System for Earthquake Engineering Simulation (OpenSees). OpenSees has been successfully used by other researchers in investigating the nonlinear load-deformation response of RC bridge columns [16], [17]. The circular cross-section was represented by a fiber-based model which was originally developed by Taucer et al. [18] and has been implemented in OpenSees by Scott and Fenves [19]. The cross-section was subdivided in fibers and assigned uniaxial stress-strain laws available in OpenSees to describe the response of the cover and core concrete, and the reinforcing steel.

IV. RESULTS

A. Case Study Results

This section intends to apply the methodology discussed earlier to in detecting the safe threshold for neglecting the P-Delta effects. Table I shows the Column Properties for the column subjected to study in this section.

TABLE I. Column properties.

Concrete Strength, f'c (MPa, ksi)	**37 (5.38)**
yield Strength, fy (MPa, ksi)	**413 (60.0)**
Reinforcement ratio	**2%**
Modulus of elasticity, Es (MPa, ksi)	**2×105(29,00)**
Longitudinal reinforcing steel: yield strain, εy	**0.0015**
Column diameter, L (m, ft)	**1.21 (4)**
Column aspect ratio, CAR	**10**
Cover concrete (cm, in)	**5 (2)**
Axial load (kips)	**584**

B. Pushover Analysis

Pushover analysis is conducted to provide the lateral load and displacement at yielding and ultimate capacity of the column which is used to create the bilinear force-displacement graph. Figure 4 shows the results obtained from Pushover analysis.

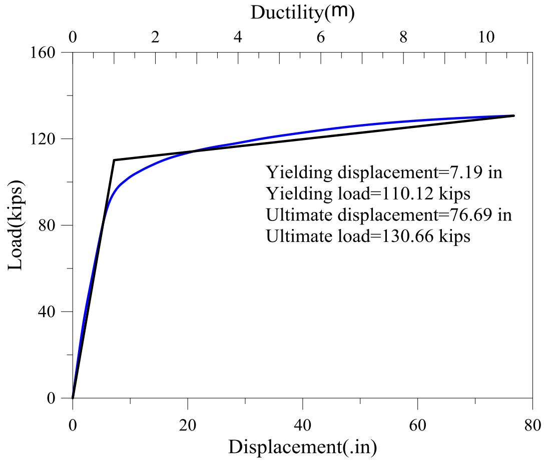

Fig. 4. Load-Deformation obtained from pushover analysis

C. Earthquake Record Selection

Throughout this research ATC Far-Field, ground motion record set is used. The ground motion set is collected from Pacific Earthquake Engineering Research Center (PEER-NGA) database. Table II and table III tabulates the characteristics of the ground motion set. Figure illustrates the response spectrum for the earthquake records (damping ratio of 5%). Following characteristics are common among all these ground motion records.

TABLE II. Ground motion properties.

Distance R	**R > 10 km**
Large Magnitude Events	**M > 6.5**
Equal Weighting of Events	**⩽ 2 records per event**
Strong Ground Shaking	**PGA > 0.2g /PGV > 15 cm/sec**
Source Type	**Both Strike-Slip and Thrust Fault Sources**
Site Conditions	**Rock or Stiff Soil Sites, Vs > 180 m/s**
Record Quality	**Lowest Useable Frequency < 0.25 Hz**

Far-Field earthquake record set specifications are tabulated in table III.

TABLE III. Ground motion records.

EQ ID	**Earthquake**		**PGA (g)**	**EQ ID**	**Earthquake**		PGA (g)
	Year	**Name**			**Year**	**Name**	
12011	**1994**	**Northridge**	**0.52**	**12092**	**1992**	**Landers**	0.42
12012	**1994**	**Northridge**	**0.48**	**12101**	**1989**	**Loma Prieta**	0.53
12041	**1999**	**Duzce, Turkey**	**0.82**	**12102**	**1989**	**Loma Prieta**	0.56
12052	**1999**	**Hector Mine**	**0.34**	**12111**	**1990**	**Manjil, Iran**	0.51
12061	**1979**	**Imperial Valley**	**0.35**	**12121**	**1987**	**Superstition Hills**	0.36
12062	**1979**	**Imperial Valley**	**0.38**	**12122**	**1987**	**Superstition Hills**	0.45
12071	**1995**	**Kobe, Japan**	**0.51**	**12132**	**1992**	**Cape Mendocino**	0.55
12072	**1995**	**Kobe, Japan**	**0.24**	**12141**	**1999**	**Chi-Chi, Taiwan**	0.44
12081	**1999**	**Kocaeli, Turkey**	**0.36**	**12142**	**1999**	**Chi-Chi, Taiwan**	0.51
12082	**1999**	**Kocaeli, Turkey**	**0.22**	**12151**	**1971**	**San Fernando**	0.21
12091	**1992**	**Landers**	**0.24**	**12171**	**1976**	**Friuli, Italy**	0.35

D. Incremental Dynamic Analysis

IDA was performed on the column using mentioned earthquake records, and displacement ductility at which P-Delta effects can be ignored based on equation (4) are presented in table IV. IDA curves obtained from earthquake records 12011 and 12102 in direction 2 did not deviate from each other beyond ductility eight and have been considered as outlier data are presented in red cells, and have been exclude from statistical analysis. Elimination of these two results is a conservative action since reduces the average permissible ductility to ignore the P-Delta effects.

The histogram for obtained results is presented in Fig. 5. Fifty percent of the observations were fall between ductility one and two, followed by twenty one percent between ductility two and three. Lognormal distribution with parameters of (μ=0.73, σ=0.58) best fitted the results as shown in Fig. 5.

TABLE IV. Displacement ductility at which P-Delta effects can be ignored.

EQ ID	**Earthquake**		**EQ ID**	**Earthquake**	
	Direction 1	**Direction 2**		**Direction 1**	**Direction 2**
12011	**1.07**	**8.01**	**12092**	**2.08**	**4.43**
12012	**2.92**	**1.86**	**12101**	**4.53**	**1.04**
12041	**3.80**	**2.57**	**12102**	**1.39**	**8.56**
12052	**1.01**	**2.99**	**12111**	**1.19**	**4.45**
12061	**1.42**	**2.29**	**12121**	**4.26**	**4.53**
12062	**3.00**	**5.73**	**12122**	**1.05**	**1.24**
12071	**1.05**	**1.03**	**12132**	**6.49**	**1.04**
12072	**1.14**	**3.07**	**12141**	**1.07**	**1.57**
12081	**1.25**	**4.70**	**12142**	**1.76**	**2.54**
12082	**2.52**	**1.29**	**12151**	**1.09**	**2.62**
12091	**2.41**	**1.37**	**12171**	**6.18**	**1.78**

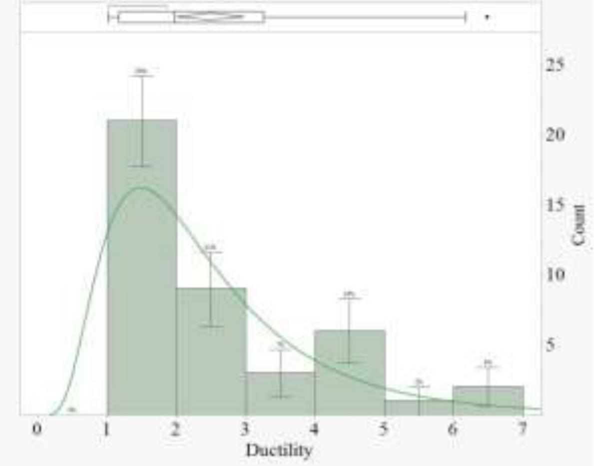

Fig. 5. Histogram of the observed results.

Quantiles of the observed data are presented in table V, and can be used in interpretation of the results. For instance there is a fifty percent probability that this column can reach to ductility 1.97 while P-Delta effects remain negligible.

TABLE V. Statistical analysis results summary.

Quantiles		**Summary Statistics**		**Test mean**	
100% (max)	**6.48**	**Mean**	**2.49**	**Hypothesized value**	**4**
99.5%	**6.48**	**Std. Dev**	**1.55**	**Actual estimator**	**2.49**
97.5%	**6.46**	**Std. Err Mean**	**0.24**	**DF**	**41**
90%	**4.65**	**Upper 95% Mean**	**2.98**	**Std. Dev**	**1.55**
75% (quartile)	**3.23**	**lower 95% Mean**	**2.01**	**t Test**	
50% (median)	**1.97**	**N**	**42**	**Test statistics**	**-6.25**
25% (quartile)	**1.17**			**Prob>|t|**	**<0001***
10%	**1.04**			**Prob>t**	**1.000**
2.5%	**1.01**			**Prob<t**	**<0001***
0.5%	**1.10**				
0%(min)	**1.01**				

Results obtained from test mean are presented in table V. Hypothesized value for the mean was considered as four which is suggested by Caltrans SDC for single column bents supported on fixed foundation. The null hypothesis is rejected since the t statistics is less than 0.05 (means are not the same).

E. Caltrans SDC Recommendations for the Studied Column

According to Caltrans SDC if the ratio of bending moment induced by P-Delta effects to the yielding moment capacity of column is less the twenty percent, then structural components shall be designed based on predefined displacement ductility demands. Table VI presents the results obtained from pushover analysis and checks the Caltrans SDC criteria for ignoring the P-Delta effects.

TABLE VI. Statistical analysis results summary.

Parameter	**Unit**	
Yielding displacement	**in**	**7.19**
Yielding load	**kips**	**110.12**
Yielding Moment	**Kip.in**	**50773**
Ultimate Load	**in**	**130.66**
Ultimate displacement	**in**	**76.69**
Ultimate Ductility	**N/A**	**10.66**
Load at ductility 4	**kips**	**116.49**
P-Delta induced bending moment	**Kip.in**	**16795**
$\frac{P\,\Delta_r}{M_p^{col}}$		**0.33**
Ignore P-Delta		**NOT-OK**

Caltrans SDC requires including the P-Delta effects in the analysis of this column and suggests performing nonlinear time history analysis to further study the structural response of the column. Typically engineering firms to prevent performing time consuming and computationally expensive nonlinear time history analysis resize the column such that it complies with the Caltrans SDC criterion for ignoring the P-Delta effects.

V. CONCLUSION

In this research using incremental dynamic analysis and based on comparing the IDA curves with and without P-Delta effect the displacement ductility at which P-Delta effects can be ignored is computed. In the proposed method after choosing the probability of exceeding for ignoring the P-Delta effects the corresponding ductility level can be obtained from the quantiles table.

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