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A Numerical Study on Two-Dimensional Fins with Non-Constant Heat Flux

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Abstract— *In this study, the COMSOL Multiphysics mathematics toolbox was employed in order to analyze the steady state heat transfer of extended surfaces. The simulation results for a two-dimensional problem were validated with the Fourier series analytical solution derived for the case in the literature. The results of COMSOL were shown to have a good agreement with analytical Fourier series approach. Moreover, using the simulation model, a comparison was made between the fins with a convective and isolated tip. The results demonstrated that the fin with a convective tip has about 5% higher rate of heat transfer.*

Keywords— *COMSOL, Fin, Simulation, Steady state heat transfer, Variable heat transfer coefficient.*

I. INTRODUCTION

Fins or extended surfaces are used to increase heat transfer rate from a surface to the environment [1, 2]. Depending on applications, they have been used in different areas like integrated circuits (ICs), engines, turbines, heat exchangers, electronic equipment, and biological wastewater treatment systems [3-5]. There are various geometries of fins available depending on the application and production procedure. In particular, straight fins are very common because of simple geometry and lower production cost.

In the literature, fin problems have been considered with a constant heat transfer coefficient through the surface of the fin [6]. Nonetheless, in reality, this assumption is not valid because heat transfer rate can be different along the entire length of the fin [7]. For instance, heat transfer rate was reduced by almost 40% when a linearly increasing heat transfer coefficient was selected for a finite difference analysis method [8]. Leading edge, boundary layer, and turbulence are the factors that play key roles in variations of heat transfer coefficient along the fin surface. Hence, conducting studies on the heat transfer of the fins with a variable heat transfer coefficient is important from the practical perspective.

Heat transfer analysis of fins has been done through experimental, computational, and analytical methods. Han et al. considered a power series to model a one-dimensional fin with a rectangular profile and constant heat transfer coefficient, while Chen et al. treated the same problem with an exponential distribution of heat transfer [9, 10]. Additionally, a study was conducted on an annular triangular fin with a linearly increasing heat transfer coefficient from the base to the tip. These studies, however, are only useful for special experimental cases and cannot provide accurate solutions for other cases. The two-dimensional (2D) study is an appropriate method in order to analyze the efficiency and heat transfer of straight fins in reality. Heggs et al. showed using Biot number, Bi, one-dimensional heat transfer can be used instead of 2D analysis if the transverse Bi is much less than one [11]. Heat flux was reported much lower (80%) when a 2D analysis implemented [12]. Therefore, the 2D analysis is very important for both practical and theoretical problems. Ma et al. presented analytical solutions for 2D heat transfer analysis of fins with a variable heat transfer coefficient [13]. However, they showed that the classical solution represented in [6] is computationally more efficient than their method and also can be used for a 2D problem with a variable heat transfer coefficient along the fin.

In this study, a numerical approach was employed by the COMSOL mathematical toolbox to simulate a 2D fin with a variable heat transfer and an adiabatic tip in steady state condition and then the results verified with the analytical approach presented in Ma et al.

II. ANALYTICAL SOLUTION FOR A FIN WITH AN ADIABATIC TIP

Consider a fin as presented in Fig 1. According to Fig. 1, a rectangular fin with dimensions of 2B × L dissipates heat only through convection with a variable heat transfer coefficient, h(x). The surrounding temperature is T_∞, the tip has a heat transfer coefficient of $h_L=0$ (adiabatic tip), and T is the temperature of the base. The 2D heat transfer equation for a homogenous material under steady state condition and no heat generation is presented by

$$\frac{\partial^2\phi(x,y)}{\partial x^2}+\frac{\partial^2\phi(x,y)}{\partial y^2}=0, \tag{1}$$

where $\phi(x,y)$ is a dimensionless temperature:

$$\phi(x,y)=\frac{T-T_\infty}{T_b-T_\infty} \tag{2}$$

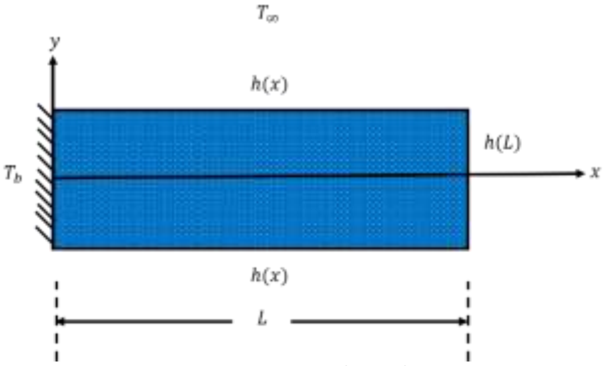

Fig. 1. Schematic of a 2D fin.

In order to solve (1), boundary conditions must be defined in boundaries. The boundary conditions for the heat transfer problem are:

$$\phi = 1, \quad x = 0, 0 \le y \le B; \tag{3}$$

$$-\frac{\partial \phi}{\partial x} = 0, \quad x = L, 0 \le y \le B; \tag{4}$$

$$\frac{\partial \phi}{\partial y} = 0, \quad y = 0, 0 \le x \le L; \text{ and} \tag{5}$$

$$-\frac{\partial \phi}{\partial y} = \frac{h_c}{k}\phi, \quad y = B, 0 \le x \le L, \tag{6}$$

where k and hc are the thermal conductivity of the fin material and the constant heat transfer coefficient, respectively.

The solution to (1) for a fin with an adiabatic tip with the following boundary conditions is represented in (7):

$$\phi(x,y) = \sum_{i=1}^{\infty} \frac{2H \cosh \beta_i (L-x) cos\beta_i y}{[B({\beta_i}^2+H^2)+H] cosh\beta_i L cos\beta_i B} \tag{7}$$

In the (7), H is presented by

$$H = \beta_i \tan \beta_i B = \frac{h_c}{k} \tag{8}$$

where β_i are the roots of the transcendental equation.

Equation (7) can be used either with a constant or variable heat transfer coefficient along the x-direction as long as the Dirichlet boundary condition is used, i.e. h(x) should have a finite constant value throughout the boundary. In addition, the Dirichlet boundary condition cannot apply to unbounded numbers of minimums and maximums for h(x) on the (0, L) interval [13].

Ma et al. considered a cubic fin as shown in Fig. 2. The fin has a thermal conductivity of 100 W m^{-1} K^{-1}. The heat transfer coefficient of 50 and 100 W m^{-2} K are selected for the first 0.01 m and the remaining 0.03 m, respectively. In the next part of the present study, a numerical simulation is carried out and the results are compared to the analytical results in order to validate the accuracy of the numerical solution.

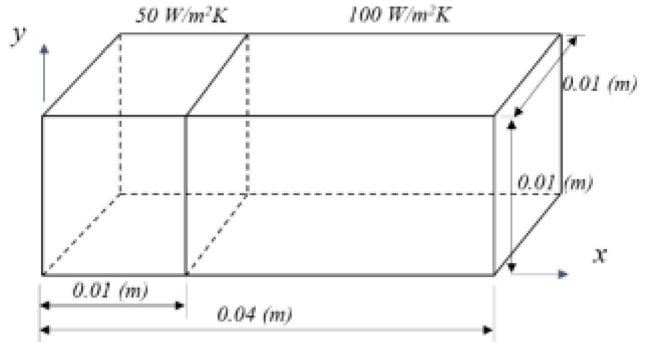

Fig. 2. A fin with a variable heat transfer coefficient.

III. NUMERICAL SOLUTION IN COMSOL

Numerous studies have been done in order to analyze the solutions of steady state and transient responses in fins with different shape [14-18]. Scholars have used various finite element software programs and approaches such as ABAQUS, ANSYS, FLUENT, and COMSOL in areas of mechanical engineering, bioengineering, and chemical engineering, since the finite element method (FEM) technique facilitates design development and can be used to reduce the time-consuming and costly experimental trial-and-error procedures in order to obtain an accurate solution [19-47]. Moreover, FEM techniques used in the computational software show they can provide a very close approximation to complex mathematical and physical problems. In particular, COMSOL Multiphysics is a strong FEM software program that has an advantage of having a useful partial differential equation (PDE) solver, helpful for obtaining solutions to complex problems in different fields like mechanical engineering, physics, and chemical engineering.

A 2D model of a fin with a variable heat transfer coefficient presented in the previous section was simulated in COMSOL, using the heat equation available in the PDE toolbox of the software. The steady state temperature distribution contour of the fin with a variable heat transfer coefficient is illustrated in Fig. 3.

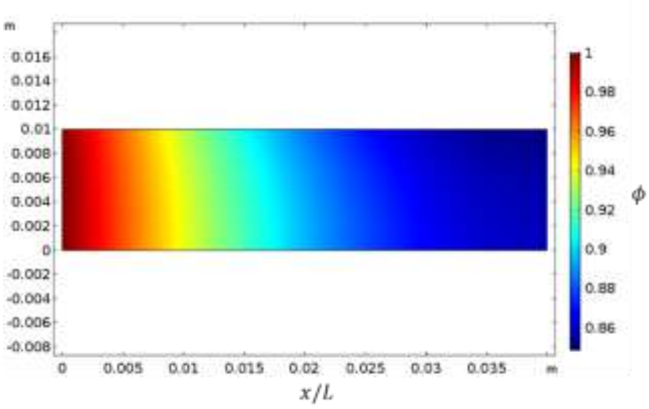

Fig. 3. Steady state temperature distribution of a fin with non-constant heat transfer coefficient.

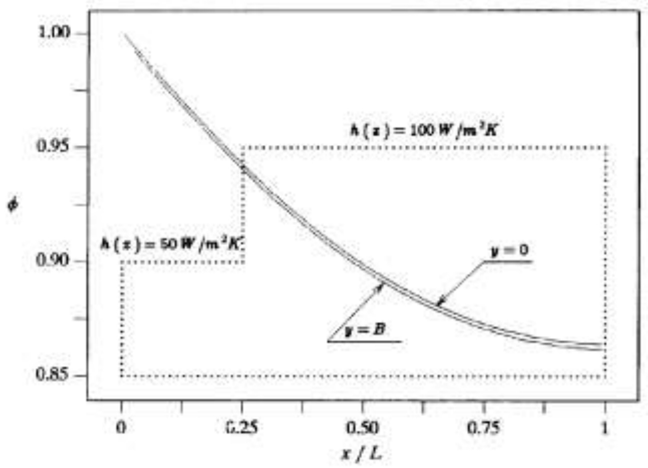

Fig. 4. Analytical solution for the presented fin with a variable heat transfer [13].

Each numerical solution needs to be validated with a different experimental or analytical approach. In this study, the solution obtained by the COMSOL classical PDE solver was validated by comparing the results to the analytical solution in Ma et al. study [13]. Fig. 4 shows the analytical solution of the fin with boundary conditions and heat transfer coefficients according to the problem introduced in Fig. 2. For validation purposes, the temperature distribution at y=0 and y=B along the fin length is plotted using COMSOL. Fig. 5 shows the results of the 2D simulation in COMSOL. As it can be seen, the results of temperature distribution are in a good agreement with Ma et al. with less than 5% error.

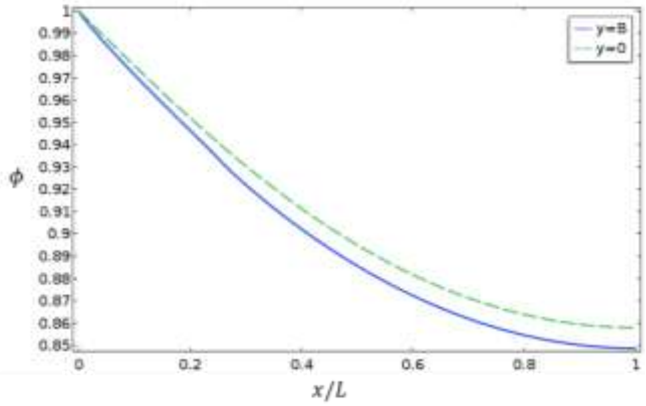

Fig. 5. Solution of the problem using COMSOL.

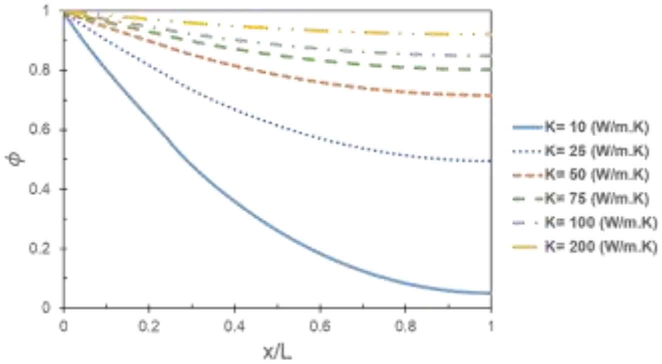

Fig. 6. Effect of thermal conductivity coefficient on the temperature distribution of the fin surface.

In this part, the results of simulations are used in order to provide a better understanding of the problem. Fig. 6 shows the effect of thermal conductivity on the heat transfer of a fin with a variable heat transfer coefficient. As can be seen, increasing the thermal conductivity results in increasing the heat transfer rate in the entire fin that leads to enhancing the temperature of the tip because of a faster heat conduction in the fin. Fig. 7 illustrates the effect of heat transfer coefficient on the temperature distribution of the fin. For each case, the first heat transfer coefficient corresponds to the first 0.01 m and the next 0.03 m, respectively. As increasing the heat transfer coefficient takes place, a significant decrease in the temperature distribution is observed due to a greater heat loss to the environment. Furthermore, Fig. 8 demonstrates the results of fins with a convective and isolated tip. In the fin with a convective tip, increasing the total heat transfer rate has enhanced the heat loss because at any certain distance from the base, the convective fin has about 5% less temperature.

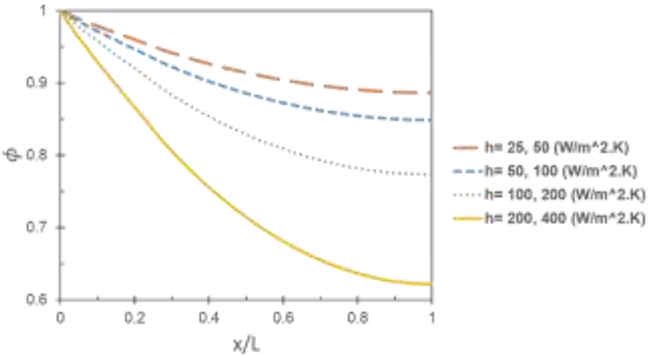

Fig. 7. Effect of heat transfer coefficient on the temperature distribution of the fin.

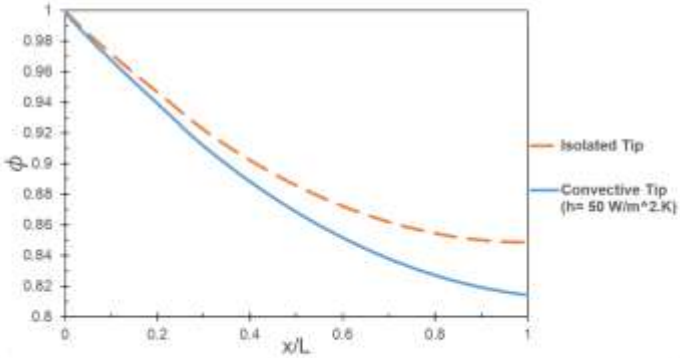

Fig. 8. The temperature distribution of fins with isolated and convective tip.

IV. CONCLUSION

In the presented study, a numerical approach was used to simulate heat transfer in a fin with a variable heat transfer coefficient. In the literature, there are analytical solutions derived for the heat transfer in fins with a variable or constant heat transfer coefficient. Nonetheless, these analytical techniques sometimes are not numerically efficient or hard to solve for different boundary conditions. The partial differential equation (PDE) solver from the COMSOL mathematics toolbox was used to solve the steady state heat transfer in a two-dimensional fin with a variable heat transfer coefficient. The temperature distribution plot obtained by COMSOL had a high accuracy with less than 5 % deviation compared to the analytical solution presented in the literature. In addition, using the simulation model, a comparison made between fins with isolated and convective tip that showed about 5 % decrease in the temperature distribution of the fin with a convective tip.

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