International Journal of Scientific Engineering and Science
Volume 2, Issue 4, pp. 1-7, 2018. ISSN (Online): 2456-7361

Research on Stock Market Rumor Spread Model Based on Investor Sentiment

Xiangyang Gao
Faculty of Science, Jiangsu University, Jiangsu, 212013, Zhenjiang, China

Abstract**—***Based on the study of investor sentiment in stock market, a SI1I2RS time delay stock market rumor transmission model with investor sentiment is established on scale-free network. First, combined with the theory of propagation dynamics, we get the basic reproduction number R0 of stock market rumors. Then we study and discuss the stability of the equilibrium point. Finally, we simulate the simulation to verify the conclusion. The results show that increasing the proportion of conservative investors and the time lag of immunization, reducing the contact between investors and enhancing the liquidity of investors in the stock market can help to suppress the spread of stock market rumors.*

Keywords— *Stock market rumor transmission ；Scale-free network ； Basic reproductive numbers ；Numerical simulations.*

I. INTRODUCTION

With the digital process of the information age, the means of obtaining information and exchanging information by stock investors have also changed significantly. The speed and depth of information dissemination have been greatly developed. However, it is precisely because of the rapid development of the network that many malicious information is spread among investors. Malicious information has great harm to the stock market, and it has greatly damaged the stability of the stock market. Therefore, it is necessary to control the rumor in the investor network so as to avoid panic caused by the violent volatility of the stock market.

The spread of rumor in the stock market is similar to the communication mechanism of social network rumor. First, Daley and Kendall[1] point out the difference between infectious disease transmission and rumor spreading. Based on the SIR epidemic model, a classical DK rumor propagation model is proposed. Then Maki and Thomsen [2] modified the DK model as a MT model. Based on these early models, many researchers have studied the rumor propagation and the topological properties of the related social networks [3-7]. On the basis of the network topology, scientists began to seriously consider the role of rumours in human behavior and the spread of different mechanisms of [8-14]. For example, the forgetting mechanism, the memory mechanism, the trust mechanism and so on. In recent years, scientists have found that investor sentiment in stock market has a great impact on the spread of stock market rumors. Investors are more likely to spread rumors based on their emotional changes. Wang[15] et al. Studied investor sentiment theory from the perspective of asset pricing and behavioral finance. It shows that the research of investor sentiment theory should not only focus on the theory itself, but also on the application and its research on financial regulation. Chu[16] based on the weak relationship theory, the investor information screening model is proposed in the investor network. Zhang[17] and so on have analyzed the influence mechanism of investor sentiment on stock returns. The results show that investor sentiment is a systemic factor affecting stock prices, and stock prices fluctuate with investor sentiment. Wang and other [18] proposed a 2SI2R model that two rumours spread in groups, and found that more attractive rumor could inhibit the spread of another kind of rumor. Huo[19] introduces the dynamic model of rumor propagation, called I2SR, which considers the activities of nodes and divides the communicators into active transmission rate and low initiative transmission rate. Based on the LaSalle invariance, the global stability principle of the internal equilibrium point of the model is proved. Li[20] has established a class of SIRS propagation models with time delay characteristics in the scale-free network environment, which shows that increasing time delay is helpful to control the spread of disease. In literature [21-24], a time delay propagation model on scale-free networks is established, and the dynamic state of the model under the degree of degree irrelevance is analyzed. When the basic regeneration number is $R_0 < 1$, only the model has no propagation equilibrium point. When $R_0 > 1$, the non-spreading equilibrium point is unstable, and there is a unique propagation equilibrium point and it is continuous.

The above model has made great contributions to the spread of rumors on social networks, but the social network is different from the stock investor network, associated with a lot of rumors in the network spread and investor sentiment, because these information or rumors are related to their vital interests, mood changes directly affect the spread of rumors. In the stock investor network, investors can be divided into two categories: one is an adventurous investor and the other is a conservative investor. The difference between them is that risk-taking investors are more aggressive because of their high risk. They are more likely to be influenced by stock market rumors and spread rumors easily.

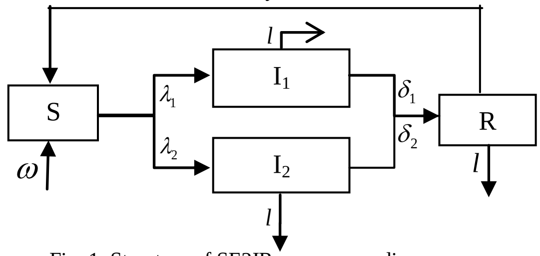

Fig. 1. Structure of SE2IR rumor spreading process.

The more stable sentiment of the conservative investors is less affected by the stock market rumor, and the probability of spreading rumors is relatively small.

On this basis, this paper proposes a SI_1I_2RS stock market rumor spreading model with time delay. According to the emotional changes of investors' exposure to stock market rumors, we consider the influence of two kinds of investors in stock market on stock market rumor spread. Based on mean field theory, we set up corresponding dynamic equations of stock market rumor propagation with time lag. And the simulation is carried out on the scale-free network to verify the correctness of the conclusion.

II. THE SI$_1$I$_2$RS MODEL

Suppose there is a scale-free network consisting of N stock investors. The nodes represent investors and edges represent the direct connection between investors and investors. The network is considered as an undirected network $G=(V,E)$, where V is a set of all nodes and E is a set of all edges. The specific process of the spread of stock market rumors with time delay considering investor sentiment is shown in Figure 1.

The population is subdivided into four types due to different investor sentiment: ignorant(S), spreader1(I1), spreader2(I2), stifler(R). Ignorants stand for the people who never heard or seen rumor. Stifler are those whose retweet is outdated or those who have no interest to spreading it. Owing to differences in the sentiment, we use spreader1 to stand for the conservative spreader, who hears rumors, and they spreading rumors with lower probability. Spreader2 stands for the Risky spreader, they choose to spreading rumors with larger probability. The stock market rumor propagation rules are as follows: when conservative ignorant and spreader of contact, because of emotional infection and become more radical, with the probability of λ_1 into conservative spreader; similarly, when the risk ignorant and spreader of contact, will be due to emotional impact, with a probability of λ_2 transformation for the risk spreader. The two type of spreader may also learn the correct information from various sources, which will make their emotion stable, and to stop the spread of rumors, they will be transformed into stifler by probability of δ_1 and δ_2 respectively, stifler after a time of τ, due to the large amount of information in the network, or the rumors are not interested in factors such as the emotion attributed to stable, so as to forget the previous rumors once again become ignorant.

Basic assumptions and parameters are expressed as follows:

1. Suppose the investor's entry-exit rate are the same, that is $\omega = l$, and at the same time, new investors are supposed to be ignorant investors.
2. q represents the proportion of conservative investors.
3. λ_1 and λ_2 are the transmission rates of conservative and risky investors, namely, the probability of being contagious if the ignorant investor is contagious with the propagating investor.
4. δ_1 and δ_2 are the immune rates of the conservative and risk investors, namely, the probability of transforming from the state of transmission to the state of the immune system.
5. τ represents the immune period, and the immune person will lose immunity again and become an ignorant in the $t-\tau$ moment. According to the mean field theory, the following kinetic equations are obtained:

$$
\begin{cases}
\dfrac{dS_k(t)}{dt} = \omega - kS_k(t)\Theta(t) - lS_k(t) + \delta_1 I_{1,k}(t-\tau)e^{-l\tau} + \delta_2 I_{2,k}(t-\tau)e^{-l\tau} \\
\dfrac{dI_{1,k}(t)}{dt} = qkS_k(t)\Theta(t) - \delta_1 I_{1,k}(t) - lI_{1,k}(t) \\
\dfrac{dI_{2,k}(t)}{dt} = (1-q)kS_k(t)\Theta(t) - \delta_2 I_{2,k}(t) - lI_{2,k}(t) \\
\dfrac{dR_k(t)}{dt} = \delta_1 I_{1,k}(t) + \delta_2 I_{2,k}(t) - lR_k(t) - \delta_1 I_{1,k}(t-\tau)e^{-l\tau} - \delta_2 I_{2,k}(t-\tau)e^{-l\tau}
\end{cases} \tag{1}
$$

In this model, $S(t)$, $I_1(t)$, $I_2(t)$ and $R(t)$ represent the densities of ignorant,spreader1, spreader2 and stifler in the network at time t, respectively; Let $S_k(t)$, $I_{1,k}(t)$, $I_{2,k}(t)$ and $R_k(t)$ be the relative density of ignorant, spreader1, spreader2 and stifler nodes with degree k at time t, respectively, where $k=1,2......,n$.

So we can get:

$$S(t)=\sum_k P(k)S_k(t),\ I_1(t)=\sum_k P(k)I_{1,k}(t),$$
$$I_2(t)=\sum_k P(k)I_{2,k}(t),\ R(t)=\sum_k P(k)R_k(t).$$

Where $P(k)$ represents the degree distribution function of the node, and

$$0 \le S_k(t), I_{1,k}(t), I_{2,k}(t), R_k(t) \le 1,$$
$$0 \le S(t), I_1(t), I_2(t), R(t) \le 1.$$

Since there are only four nodes in the network, the following normalization conditions are satisfied:

$$S_k(t)+I_{1,k}(t)+I_{2,k}(t)+R_k(t)=1 \tag{2}$$

$$S(t)+I_1(t)+I_2(t)+R(t)=1 \tag{3}$$

$\Theta(t)$ represents the probability that the ignorant and communicators are in contact with each other in the stock market, And satisfied:

$$
\begin{aligned}
\Theta(t) &= \sum_{i=1}^{n} p(k'/k)[\lambda_1 I_{1,k'}(t) + \lambda_2 I_{2,k'}(t)] \\
&= \frac{1}{\bar{k}} \sum_{k'=1}^{n} k' P(k')[\lambda_1 I_{1,k'}(t) + \lambda_2 I_{2,k'}(t)]
\end{aligned} \tag{4}
$$

Where $P(k)$ and $\bar{k}$ represent the probability and average degree of degree k, respectively, and $\bar{k} = \sum_{k'=1}^{n} k' p(k')$.

International Journal of Scientific Engineering and Science
Volume 2, Issue 4, pp. 1-7, 2018. ISSN (Online): 2456-7361

III. THEATRICAL ANALYSIS

Let S_k^{∞} represent the proportion of ignorant classes in nodes with a steady-state degree of k. Therefore, $I_{1,k}^{\infty}$, $I_{2,k}^{\infty}$ and R_k^{∞} are the proportions of conservative spreader, risky spreader, and stifler in the nodes with steady-state degree k, respectively.

In this section we will discuss the existence of the spreading equilibrium of system (1). The following theorems hold.

Theorem 1:Let

$$R_0 = \frac{\lambda_1 q(\delta_2 + l) + \lambda_2 (1-q)(\delta_1 + l)]}{(\delta_2 + l)(\delta_1 + l)} \frac{\overline{k}^2}{\overline{k}}$$

For system (1) there is always a non-spreading equilibrium $E_0 = (1,0,0,0,1,0,0,0,\cdots,1,0,0,0)$.

And when $R_0 > 1$, the system (1) had the existence of a unique spreading equilibrium point E_1,where

$$E_1 = (S_1^{\infty}, I_{1,1}^{\infty}, I_{2,1}^{\infty}, R_1^{\infty}, S_2^{\infty}, I_{1,2}^{\infty}, \\ I_{2,2}^{\infty}, R_2^{\infty}, \cdots, S_n^{\infty}, I_{1,n}^{\infty}, I_{2,n}^{\infty}, R_n^{\infty})$$

Proof: It is easy to get the no-spreading equilibrium $E_0(1,0,0,0,1,0,0,0,\cdots,1,0,0,0)$ for system (1). Then let the right side of each equation of the system (1) is equal to 0, and any equilibrium solution $(S_k^{\infty}, I_{1,k}^{\infty}, I_{2,k}^{\infty}, R_k^{\infty})$ can be solved. Satisfy:

$$S_k^{\infty} = \omega(\delta_2 + l)(\delta_1 + l) / \{(k\Theta^{\infty} + l)(\delta_2 + l) \\ (\delta_1 + l) - e^{-l\tau}[\delta_1 qk\Theta^{\infty}(\delta_2 + l) + \delta_2(1-q) \\ k\Theta^{\infty}(\delta_1 + l)]\} \tag{5}$$

$$I_{1,k}^{\infty} = \frac{qk\Theta^{\infty}}{(\delta_1 + l)} S_k^{\infty} \tag{6}$$

$$I_{2,k}^{\infty} = \frac{(1-q)k\Theta^{\infty}}{(\delta_2 + l)} S_k^{\infty} \tag{7}$$

$$R_k^{\infty} = (1 - e^{-l\tau}) \frac{k\Theta^{\infty} S_k^{\infty}}{l} [\frac{q\delta_1}{(\delta_1 + l)} + \frac{(1-q)\delta_2}{(\delta_2 + l)}] \tag{8}$$

In the above formula, Θ^{∞} satisfy:

$$\Theta^{\infty} = \frac{1}{\overline{k}} \sum_{k'=1}^{n} k' P(k')(\lambda_1 I_{1,k'}^{\infty} + \lambda_2 I_{2,k'}^{\infty}) \tag{9}$$

The equations (6) and (7) are substituted into (9); Get the following self-consistent equation:

$$\Theta^{\infty} = \frac{1}{\overline{k}} \sum_{k'=1}^{n} k' P(k')[\lambda_1 \omega qk\Theta^{\infty}(\delta_2 + l) \\ + \lambda_2 \omega(1-q)k\Theta^{\infty}(\delta_1 + l)] / \{(k\Theta^{\infty} + l) \\ (\delta_2 + l)(\delta_1 + l) - e^{-l\tau}[\delta_1 qk\Theta^{\infty}(\delta_2 + l) \\ + \delta_2(1-q)k\Theta^{\infty}(\delta_1 + l)]\} \\ \equiv f(\Theta^{\infty}) \tag{10}$$

Obviously, we can see that: $\Theta^{\infty} = 0$ is a ordinary solution to equation (10). At this time, system (1) has a unique equilibrium point, that is, no-spreading equilibrium point $E_0(1,0,0,0,1,0,0,0,\cdots,1,0,0,0)$. If we want to make the equation have a nontrivial solution b, it must satisfy:

$$\frac{df(\Theta^{\infty})}{d\Theta^{\infty}}\Big|_{\Theta^{\infty}=0} > 1 \tag{11}$$

That is

$$\frac{d[\frac{1}{\overline{k}} \sum_{k'=1}^{n} k' P(k')(\lambda_1 I_{1,k'}^{\infty} + \lambda_2 I_{2,k'}^{\infty})]}{d\Theta^{\infty}}\Big|_{\Theta^{\infty}=0} > 1 \tag{12}$$

So the basic reproductive number can be obtained

$$R_0 = \frac{\lambda_1 q(\delta_2 + l) + \lambda_2 (1-q)(\delta_1 + l)}{(\delta_2 + l)(\delta_1 + l)} \frac{\overline{k}^2}{\overline{k}}$$

When $R_0 < 1$, we can see that the equation (12) is not established, that is, the equation (10) has only one ordinary solution $\Theta^{\infty} = 0$, that is, the system (1) only has a no-spreading equilibrium point. When $R_0 > 1$, the equation (12) is established. That is, there is a nontrivial solution of equation (11), which is equivalent to when $R_0 > 1$, system (1) has a unique spreading equilibrium point E_1, end of proof.

The investor network is made up of a certain number of investors. The network formed by these investors has a maximum degree of connection k_m. So, in this section, the stability of the equilibrium point on the scale-free network will be discussed.

First there is the following lemma

Lemma 1 (Lajmanovich 1976): Consider the following system

$$\frac{dy}{dt} = Ay + N(y) \tag{13}$$

Where A is an $n \times n$ matrix, $N(y)$ is continuous and differentiable in region $D(D \subset R^n)$, if system (13) satisfies the following five conditions simultaneously:

(1) compact convex set $C \subset D$ is a the positive invariant set of system (13), and $0 \in C$;

(2) $\lim_{y \to 0} \|N(y)\| / \|y\| = 0$;

(3) Existence real number $r > 0$ and a real feature vector ω of a A^T, for all $y \in C$,we have $\omega * y \geq r\|y\|$;

(4) for all $y \in C$,we have $\omega \cdot N(y) \leq 0$;

(5)In the collection of $H = \{y \in C \mid \omega \cdot N(y) = 0\}$, $y = 0$ is one of the biggest positive invariant set of system (13).

We have the following conclusion: Either $y = 0$ in set C is globally asymptotically stable, Or for any $y_0 \in C - \{0\}$, the solution $\phi(t, y_0)$ of system (13) satisfy $\liminf_{t \to \infty} \|\phi(t, y_0)\| \geq m$, the $m > 0$ has nothing to do with the y_0,and the system (13) has a constant solution, $y = k$, $k \in C - \{0\}$.

Theorem 2: Let

$$R_0 = \frac{\lambda_1 q(\delta_2 + l) + \lambda_2 (1-q)(\delta_1 + l)]}{(\delta_2 + l)(\delta_1 + l)} \frac{\overline{k}^2}{\overline{k}}$$

Volume 2, Issue 4, pp. 1-7, 2018. ISSN (Online): 2456-7361

(1) When $R_0 < 1$, the stock rumor-free equilibrium E_0 of system (1) is globally asymptotically stable.

(2) When $R_0 > 1$, the equilibrium E_0 of system (1) is unstable. At this time, the system (1) has a unique equilibrium point, and investors in the stock market rumors on the network is uniformly persistent, have a $\varepsilon > 0$, to satisfy the initial conditions of $S_k(0) > 0$, $I_{1,k}(0) > 0$, $I_{2,k}(0) > 0$, $R_k(0) > 0$. we have $\lim_{t\to\infty}\inf\{I_{1,k}, I_{2,k}\}_{k=1}^{k_m} > \varepsilon$.

Proof: First, we discuss the global stability of the stock rumor-free equilibrium E_0 of system (1) .

Define *Liapunov* function $L(t)$ as

$$L(t) = \frac{1}{\bar{k}}\sum_{k'=1}^{n} k' p(k')[\lambda_1 I_{1,k'}(t) + \lambda_2 I_{2,k'}(t)]$$

Calculate the derivative of $L(t)$ along the system (1)

$$\begin{aligned}
L'(t) &= \frac{1}{\bar{k}}\sum_{k=1}^{n} kp(k)[\lambda_1 \frac{dI_{1,k}(t)}{dt} + \lambda_2 \frac{dI_{2,k}(t)}{dt}] \\
&= \frac{1}{\bar{k}}\sum_{k=1}^{n} kp(k)\{\lambda_1[qkS_k(t)\Theta(t) - (\delta_1 + l)I_{1,k}(t)] \\
&\quad +\lambda_2[(1-q)kS_k(t)\Theta(t) - (\delta_2 + l)I_{2,k}(t)]\} \\
&\le \frac{1}{\bar{k}}\sum_{k=1}^{n} kp(k)\{\lambda_1[qk\Theta(t) - (\delta_1 + l)I_{1,k}(t)] \\
&\quad +\lambda_2[(1-q)k\Theta(t) - (\delta_2 + l)I_{2,k}(t)]\} \\
&= \frac{1}{\bar{k}}\sum_{k=1}^{n} k^2 p(k)\{(\delta_1 + l)[\frac{\lambda_1 q}{(\delta_1 + l)}\Theta(t) - I_{1,k}(t)] \\
&\quad +(\delta_2 + l)[\frac{\lambda_2(1-q)}{(\delta_2 + l)}\Theta(t) - I_{2,k}(t)]\} \\
&\le \frac{1}{\bar{k}}\sum_{k=1}^{n} k^2 p(k)\{(\delta_2 + l)[\frac{\lambda_1 q}{(\delta_1 + l)}\Theta(t) \\
&\quad +\frac{\lambda_2(1-q)}{(\delta_2 + l)}\Theta(t) - \lambda_1 I_{1,k}(t) - \lambda_2 I_{2,k}(t)]\} \\
&= (\delta_2 + l)\{[\frac{\lambda_1 q}{(\delta_1 + l)} + \frac{\lambda_2(1-q)}{(\delta_2 + l)}]\frac{\bar{k}^2}{\bar{k}}\Theta(t) \\
&\quad -\frac{1}{\bar{k}}\sum_{k=1}^{n} kp(k)[\lambda_1 I_{1,k}(t) + \lambda_2 I_{2,k}(t)]\} \\
&= (\delta_2 + l)\left[[\frac{\lambda_1 q}{(\delta_1 + l)} + \frac{\lambda_2(1-q)}{(\delta_2 + l)}]\frac{\bar{k}^2}{\bar{k}} - 1\right]\Theta(t)
\end{aligned}$$

Notice that $\Theta(t) \ge 0$, so when $R_0 < 1$, that is $[\frac{\lambda_1 q}{(\delta_1 + l)} + \frac{\lambda_2(1-q)}{(\delta_2 + l)}]\frac{\bar{k}^2}{\bar{k}} - 1 < 0$; we can obtain that $L'(t) \le 0$, if and only if $\Theta(t) = 0$, we have $L'(t) = 0$. Therefore, according to LaSalle Invariance principle (Hale 1969): when $R_0 < 1$, the stock rumor-free equilibrium of system (1) is globally asymptotically stable.

Second, when $R_0 > 1$, the equilibrium E_0 of system (1) is unstable. At this time, the system (1) has a unique equilibrium point, and investors in the stock market rumors on the network is uniformly persistent.

From equation (2), system (1) can be rewritten as

$$\begin{aligned}
\frac{dI_{1,k}(t)}{dt} &= qk(1 - I_{1,k}(t) - I_{2,k}(t) - R_k(t)) \\
&\quad *\Theta(t) - \delta_1 I_{1,k}(t) - lI_{1,k}(t) \\
\frac{dI_{2,k}(t)}{dt} &= (1-q)k(1 - I_{1,k}(t) - I_{2,k}(t) \\
&\quad - R_k(t))\Theta(t) - \delta_2 I_{2,k}(t) - lI_{2,k}(t) \\
\frac{dR_k(t)}{dt} &= \delta_1 I_{1,k}(t) + \delta_2 I_{2,k}(t) - lR_k(t) \\
&\quad - \delta_1 I_{1,k}(t-\tau)e^{-l\tau} - \delta_2 I_{2,k}(t-\tau)e^{-l\tau}
\end{aligned} \tag{14}$$

Definition

$$C = \{(S_k, I_{1,k}, I_{2,k}, R_k)_{k=1}^{k_m}, S_k \ge 0, I_{1,k} \ge 0, I_{2,k} \ge 0,$$
$$R_k \ge 0, S_k + I_{1,k} + I_{2,k} + R_k = 1, k = 1, 2, \cdots, k_m\}.$$

The following will discuss the dynamics of system (14) in compact convex set *C*. for the convenience of the calculation:

$$q(k) = \frac{kp(k)}{\bar{k}} \tag{15}$$

Therefore, $q(k)$ is a function of k, $k = 1, 2, \cdots, k_m$. the Jacobi matrix of the non-spreading of equilibrium point of system(14) is a $3k_m \times 3k_m$ matrix, as shown below:

$$J = \begin{bmatrix} A & \cdots & B \\ \vdots & \ddots & \vdots \\ C & \cdots & D \end{bmatrix}_{3k_m \times 3k_m}$$

Where

$$A = \begin{bmatrix} q\lambda_1 g(1) - \delta_1 - l & q\lambda_2 g(1) & 0 \\ (1-q)\lambda_1 g(1) & (1-q)\lambda_2 g(1) - \delta_2 - l & 0 \\ \delta_1(1 - e^{-l\tau}) & \delta_2(1 - e^{-l\tau}) & -l \end{bmatrix}$$

$$B = \begin{bmatrix} q\lambda_1 g(k_m) & q\lambda_2 g(k_m) & 0 \\ (1-q)\lambda_1 g(k_m) & (1-q)\lambda_2 g(k_m) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} qk_m\lambda_1 g(1) & qk_m\lambda_2 g(1) & 0 \\ (1-q)k_m\lambda_1 g(1) & (1-q)k_m\lambda_2 g(1) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} qk_m\lambda_1 g(k_m) - \delta_1 - l & qk_m\lambda_2 g(k_m) & 0 \\ (1-q)k_m\lambda_1 g(k_m) & (1-q)k_m\lambda_2 g(k_m) - \delta_2 - l & 0 \\ \delta_1(1 - e^{-l\tau}) & \delta_2(1 - e^{-l\tau}) & -l \end{bmatrix}$$

By mathematical induction, the matrix J in the characteristic polynomial equation for the equilibrium point E_0 is:

$$[\mu + (\delta_1 + l)(\delta_2 + l)(1 - R_0)](\mu + l)^{k_m}$$
$$*[\mu + (\delta_1 + l)(\delta_2 + l)]^{2k_m - 1} = 0$$

So we can get the eigenvalue of the matrix J is:

$$u_1 = \mu_2 = \cdots = \mu_{k_m} = -l$$

$$\mu_{k_m+1} = \mu_{k_m+2} = \cdots = \mu_{3k_m-1} = -(\delta_1 + l)(\delta_2 + l)$$

International Journal of Scientific Engineering and Science
Volume 2, Issue 4, pp. 1-7, 2018. ISSN (Online): 2456-7361

$\mu_{3k_m} = (\delta_1 + l)(\delta_2 + l)(R_0 - 1)$

We known from above $\varphi, l, \delta > 0$, that is, $\mu_1, \mu_2, \cdots \mu_{3k_m-1}$ both are less than 0.if and only if $R_0 > 1$, $\mu_{3k_m} > 0$ and E_0 of system（1）is unstable. At this time, J has a unique positive eigenvalue, and there is a unique equilibrium point for rumors. Conversely, the eigenvalues of J are negative. According to the Perron-Frobenius theorem, the maximum value of all the eigenvalues of the Jacobi matrix J is positive if and only if $R_0 > 1$, and then theorem 2 is proved by Lemma 1.

IV. NUMERICAL SIMULATION

In order to verify the correctness of the above-mentioned mean-field theoretical analysis, the spreading characteristics of the proposed SI_1I_2RS model with time-delay on BA scale-free networks are discussed through numerical simulation. For a scale-free network, the degree distribution is $p(k) = Ck^{-\gamma}$, where $2 \le \gamma \le 3$, here $\gamma = 2.5$, and C satisfies $\sum_{k=1}^{n} p(k) = 1$. It is assumed that the generated scale-free network node has maximum degree $k_{\max} = 100$ and minimum degree $k_{\min} = 2$.

Numerical simulation parameters are as follows:

$\omega = l = 0.01$, $\lambda_1 = 0.08$, $\lambda_2 = 0.5$, $\delta_1 = 0.25$, $\delta_2 = 0.15$, $q = 0.7$, $\tau = 10$.

After calculation $R_0 \approx 6.43 > 1$. The stock market rumor must be spread.

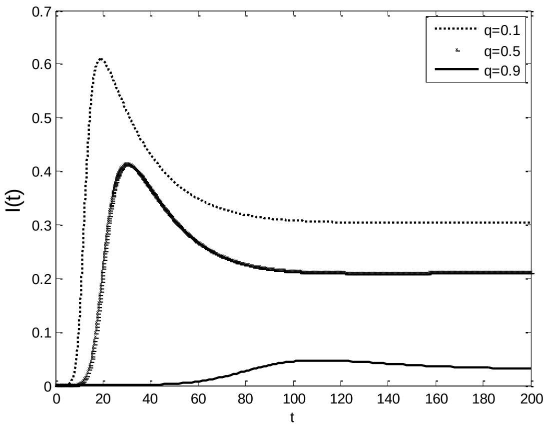

Fig. 2. The change of the total spreader density under the different proportion of conservative investors with time variation.

Figure 2 shows the changes in the density of the total spreading investors $I(t) = I_1(t) + I_2(t)$ under the proportion of different conservative investors q. From Figure 2, we can see that the higher the proportion of conservative investors in stock market is, the lower the total spreading investors is, which indicates that increasing conservative investors will help stabilize stock market and curb rumors spreading in stock market. At the same time, it also provides a way to stabilize the stock market for the government or stock market management institutions: strengthen investor education, and make investors who enter the stock market as possible as possible to develop conservative investors.

Figure 3 shows the effect of spreading rate and immune rate on the basic reproductive number R_0. From Fig. 3(a), it can be seen that the basic reproduction number R_0 is proportional to both λ_1 and λ_2. When the ignorant investor comes into contact with the spreading investor, in order to control the spreading of the rumors, it is necessary to reduce the value of the λ_1 and λ_2. Therefore, it is necessary to the government or regulatory agencies will strengthen the management of the market and take corresponding measures to reduce the spreading of market rumors. Figure 3(b) shows that the basic reproductive number is inversely proportional to the immunization rates δ_1 and δ_2. increase the immune rate can effectively inhibit the spread of market rumors. And it can be found that the immune ratio of the adventurous communicators has a greater impact on R_0, mainly because the adventurous communicators are more aggressive and play a major role in the spread of market rumors. Therefore, strengthening the management of adventurous investors helps to curb the spread of rumors in the stock market.

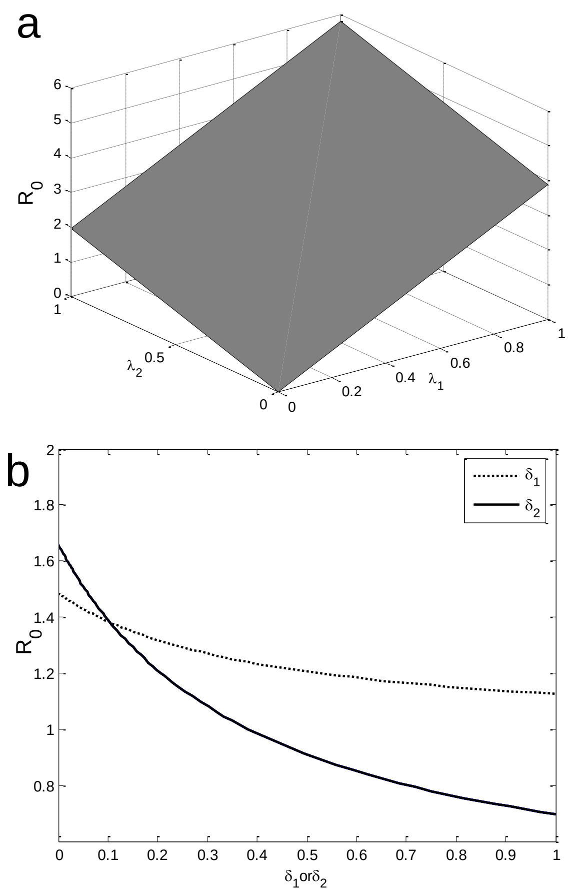

Fig. 3. Effect of the rate of spread λ_1, λ_2 and the immunization rate δ_1, δ_2 on R_0.

International Journal of Scientific Engineering and Science
Volume 2, Issue 4, pp. 1-7, 2018. ISSN (Online): 2456-7361

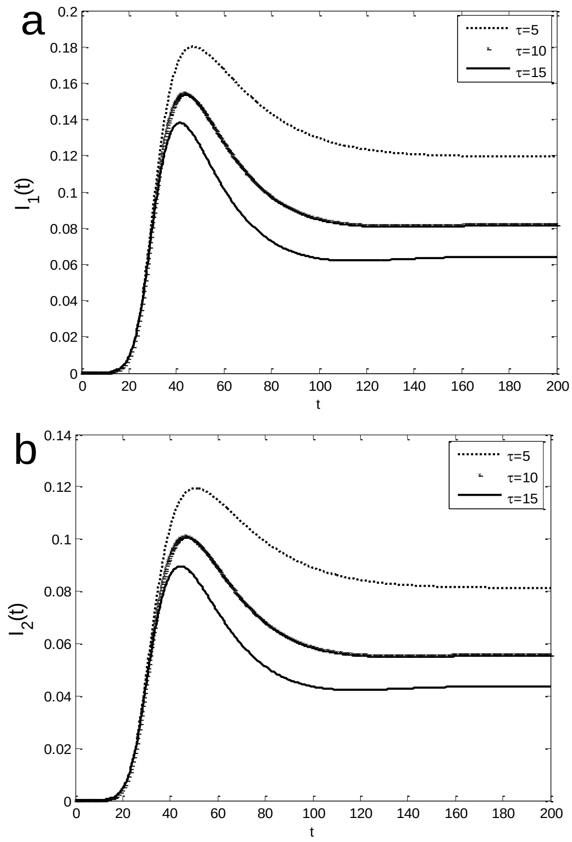

Fig. 4. $I_1(t)$ and $I_2(t)$ density changes with time t under different time lag τ .

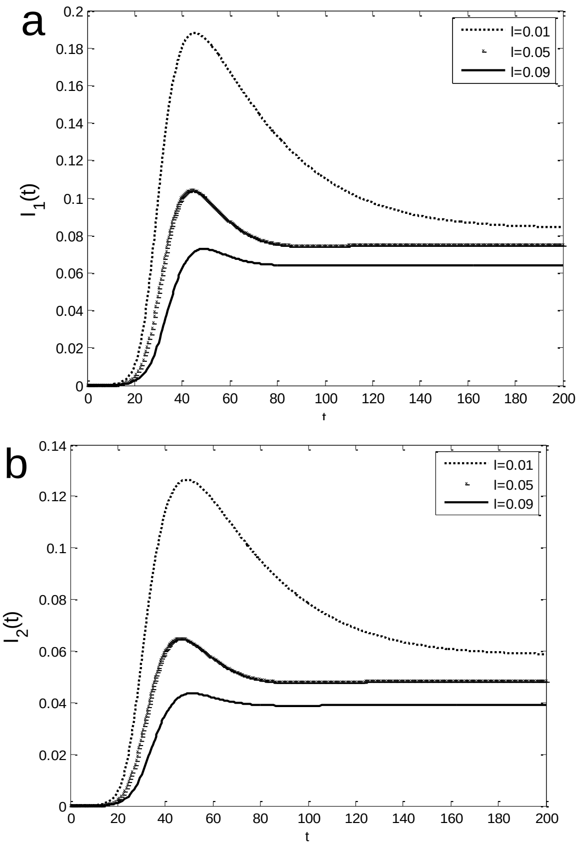

Fig. 5. $I_1(t)$ and $I_2(t)$ densities change with time t at different entry-exit rate l .

Figure 4 shows the variation of the density of spreading nodes $I_1(t)$ and $I_2(t)$ at different time delays τ with time t, respectively. From Fig. 4(a) and (b), it can be seen that as the τ value of the time lag increases, the density peaks of the propagation nodes $I_1(t)$ and $I_2(t)$ and the density at the steady state all decrease. This indicates that the time lag increases can reduce the spread of rumors in the stock market and the density of steady state communicators helps control the spread of rumors in the stock market.

Figure 5 shows the density of spreading nodes $I_1(t)$ and $I_2(t)$ changing with time t at different entry-exit rate l . From Fig. 5(a) and (b), it can be seen that when the value of the entry-exit rate l is larger, the density peaks of the spreading-like class nodes $I_1(t)$ and $I_2(t)$ and the density at the steady state are smaller, thus the entry-exit rate l is increased can effectively restrain the spread of market rumor. In the actual stock market, the entry-exit rate represents the vitality of the market, indicating that increasing the vitality of the market can reduce the spread of market rumor.

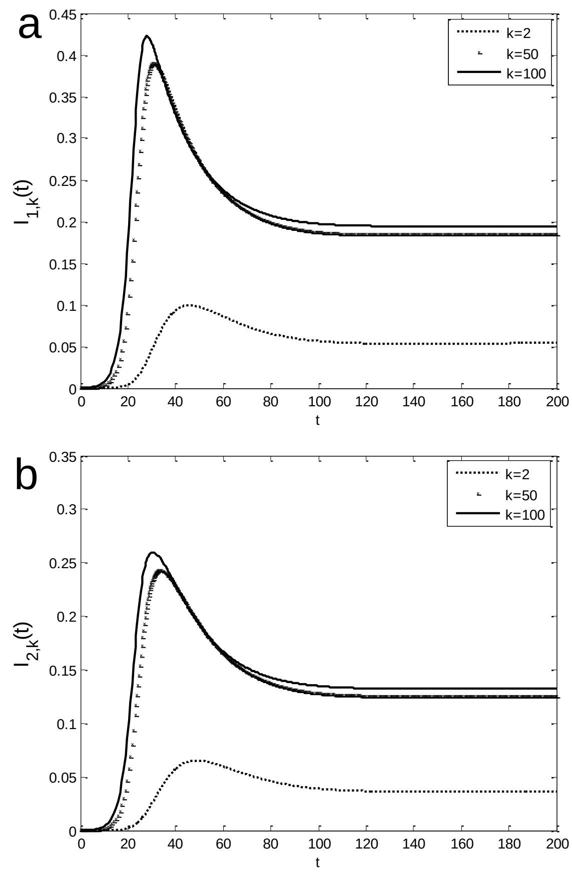

Fig. 6. The change of the density of $I_1(t)$ (a)and $I_2(t)$ (b) with time t under different value of k .

Figure 6 shows the variation of the density of the spreading-like nodes $I_1(t)$ and $I_2(t)$ with time t as shown in the following figure. It can be seen from Fig. 5(a) and (b) that the larger the value of k , the greater the peak density of the spreading-like nodes $I_1(t)$ and $I_2(t)$, the faster the time to reach the peak density, and the steady-state density of the spreading-like nodes it also increases. The k value represents the average number of nodes that are contacted and represents the investor's network in the investor network. In other words, investors in a broad network of investors are more likely than investors with a narrow network to spread rumors in the stock market faster. Therefore, during the spread of rumors in the stock market, reducing the interconnection between investors can effectively curb the spread of rumors in the stock market

V. CONCLUSION

In this paper, a stock market rumor spreading model SI_1I_2RS with time delay on scale-free network is established, and the basic reproduction number R_0 of stock market rumor spreading is obtained. At the same time, the stability of the system's no-spreading equilibrium point and the spreading equilibrium point is also analyzed. It shows that if $R_0 < 1$, stock market rumor will not continue to spread, it will disappear after a period of dissemination. When $R_0 > 1$, there is a unique spreading equilibrium point, and stock market rumors will continue to spreading. And through the simulation of Matlab software, the following conclusions are drawn.

(1) when the stock market rumors spread, market managers should take timely measures to stabilize investors' mood to strength the immunity of the communicators, especially the radical communicators.

(2) the government or relevant management institutions should educate the investors to increase the proportion of the conservative investors as much as possible and help to suppress the spread of the stock market rumors.

(3) to increase the liquidity of investors in the stock market as much as possible is of great significance to the stability of the stock market.

(4) investors with wider connections in the investor network are more likely to have a higher risk of infection than those with narrow connections. Therefore, during the spread of stock market rumors, we should minimize contact with investors.

(5) the longer the time lag of the immunization stage, the stronger the inhibition effect on the stock market rumor. Therefore, the stock market managers should try to increase the time lag, which is beneficial to control the spread of rumor.

ACKNOWLEDGMENT

This work is supported by the National Nature Science Foundation of China (71271103), The authors are grateful to the reviewers for their helpful comments and valuable suggestions.

REFERENCES

[1] D. J. Daley, D. G. Kendall. Epidemics and Rumours [J]. Nature, 1964, 204 (4963):1118-1118.

[2] D. Maki, M. Thomson. Mathematical Models and Applications. [M], Prentice-Hall, Engle- wood Cliffs, 1973.

[3] D. H. Zanette. Dynamics of rumor propagation on small-world networks [J]. Phys.Rev.E.2002, 65(4):041908.

[4] Y. Moreno, M. Nekovee, A. F. Pacheco. Dynamics of rumor spreading in complex networks [J]. Phys. Rev.E.2004, 69(6):066130.

[5] M. Nekovee, Y. Moreno, G. Bianconi, M. Marsili. Theory of rumor spreading in complex social networks [J], Physica A.2007, 374 (1):457-470.

[6] Z. F. Pan, X. F. Wang, X. Li. Research on rumor propagation simulation on scale-free networks with variable clustering coefficients. Journal of system simulation, 2006, 18(8):2346-2348.

[7] J. Arino, F. Brauer, P. V. D. Driessche, J. Watmough, J. H. Wu. A final size relation for epidemic models [J], Math. Biosci. Eng. 2007, 4(2):159-175.

[8] L. J. Zhao, Q. Wang, J. J. Cheng, Y. C. Chen, J. J. Wang, W. Huang. Rumor spreading model with consideration of forgetting mechanism: A case of online blogging Live Journal [J]. Physica A. 2011, 390(13):2619-2625.

[9] L. J. Zhao, J. J. Wang, Y. C. Chen, Q. Wang, J. J. Cheng, H. X. Cui. SIHR rumor spreading model in social networks [J]. Physica A.2012, 391 (7):2444-2453.

[10] Y. Q. Wang, X. Y. Yang, Y. L. Han, X. A. Wang. Rumor spreading model with trust mechanism in complex social networks [J]. Commun. Theor. Phys. 2013, 59(4):510-516.

[11] L. J. Zhao, W. L. Xie, H. O. Gao, X. Y. Qiu, X. L. Wang, S. H. Zhang. A rumor spreading model with variable forgetting rate [J]. Physica A. 2013, 392(23):6146-6154.

[12] L. J. Zhao, X. Y. Qiu, X. L. Wang, J. J. Wang. Rumor spreading model considering forgetting and remembering mechanisms in inhomogene-ous networks [J]. PhysicaA. 2013, 392(4): 987-994.

[13] J. J. Wang, L. J. Zhao, R. B. Huang. SIRaRu rumor spreading model in complex networks [J]. Physica A. 2014, 398(15):43-55.

[14] K.Afassinou.Analysis of the impact of edu- cation rate on the rumor spreading mecha- nism[J].Physica A.2014,414(10):43-52.

[15] B. Wang. The innovation and development of financial research methods from the perspective of investor sentiment theory. Business economy research. 2013(32):61-64.

[16] C. Chu. Theoretical model and empirical analysis of investor network information dissemination and emotional diffusion. Dongbei University of Finance and Economics. 2015.

[17] Q. Zhang, S. E. Yang. Noise trading, investor sentiment and stock returns. Theory and practice of system engineering. 2009, 29(3):40-47.

[18] J. J. Wang, L. J. Zhao, R. B. Huang. 2SI2R rumor spreading model in homogeneous networks [J]. Physica A. 2014, 413(11):153-161.

[19] L. A. Huo, L. Wang, N. X. Song, C. Y. Ma, B. He. Rumor spreading model considering the activity of spreaders in the homogeneous network [J]. Physica A.2017, 468:855–865.

[20] T. Li, Y. M. Wang, Z. H. Guan, H. L. Zhang. Research on SIRS propagation model with time delay on scale-free networks [J]. Journal of Huazhong University of Science and Technology. 2013, 41(4):80-82.

[21] T. Li. SIRS rumor propagation model with time delay on scale-free networks [J]. Journal of the Academy of Armored Forces Engineering. 2016, 30(2): 104-107.

[22] H. B. Wang, Z. Jin, J. P. Zhang, F. Q. Zhang. Stability analysis of SIRS model with time delay on scale-free networks [J]. The practice and understanding of Mathematics. 2015, 45(3):308-314.

[23] X. H. Bi, Z. C. Jiang, M. H. Xia. Stability analysis of a class of two time delay SEIRS models [J]. Journal of Natural Science of Heilongjiang University. 2010, 27(4):482-485.

[24] C. S. Deng, Q. M. Liu. Research on computer virus propagation model with time delay on scale-free networks [J]. Journal of Hebei Normal University. 2014, 39(2):111-115.

[25] A. Lajmanovich, J. A. Yorke. A deterministic model for gonorrhea in a nonhomogeneous population [J]. Math, Biosci. 1976, 28(3–4):221-236.

[26] J. K. Hale, Ordinary Differential Equations, Pure and Applied Mathematics, Wiley- Interscience [M], New York, NY, USA, 1969.