Computer Simulation of Performance Response of a Digital Audio Tape System

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Abstract — A computer simulation of performance response of a digital audio tape (DAT) system. The objective of the paper is to design a discrete compensator that will improve the speed performance of the tape drive of a DAT system so that the heads follow the tracks on the tape. This will ensure fast response and accurate positioning of drive heads. In order to do this, a dynamic model of a typical motor/load system of a digital audio tape is obtained. A discrete time compensator is designed using MATLAB software considering a sampling time of 0.1 seconds. The designed controller is integrated into the motor/load loop. Computer simulation was performed for the system for two separate conditions – when the system is uncompensated and when it is compensated. The process parameter values were varied so that three different results were obtained for each of the condition considered. The result obtained show that the response performance compensated system for input a unit step was largely improved.

Keywords — Compensator, Computer Simulation, DAT, Step Response, Speed.

I. INTRODUCTION

At professional level of quality, Digital Audio Tape (DAT) is a known medium and technology for tape recording of audio in digital. It has rotating heads which are similar to those found in video deck [1]. The DAT technology is somewhat close to that of video records, in which rotating head and helical are used to record data. This makes it impossible for DATs to be physically edited in the cut- and –splice manner analogue tapes, or open-reel digital tapes like Pro Digi or DASH [2].

The standard and technology of DAT permits four sampling modes, which are: 32 kilohertz (kHz) at 12 bits, and 44.1 kHz or 48 kHz at 16 bits [2]. Some recorders seem to operate above the specification such that recording is allowed at 96 kHz and 24 bits [2]. In order to move DAT to hard disk, a real-time conversion is required. This depends largely on the machine tape used [3]. The DAT technology was widely used in 1990s for professional audio recording industry, and it is still used in some extent today [2]. As per its use, the format was designed for audio use. Though based on the ISO Digital Data Storage Standard, it has been adopted for general data storage, storing from 1.3 to 80 gigabyte (GB) on a 60-to-180m tape. This depends on the standard and compression. According to [2], it is sequential-access media and is mainly used for backups. As a result of the higher requirements for capacity and integrity in data backups, a DAT of a computer-grade type was introduced which is called Digital Data Storage (DDS).

Today, DAT format are still being used regularly in films and television recording. This is largely due to the support in some for SMPTE time code synchronization [2]. Lighthart et al [4] presented design of digital audio tape input output chip. In [3], a digital audio tape (DAT) is presented.

This paper has been presented under the following sections: problem formulation, methodology, simulation and discussion, and finally the conclusion of the work. The goal of this paper is to design a compensated system to improve performance response of a typical digital audio tape.

II. PROBLEM FORMULATION

The relative speeds of the drum and tape is electronically controlled by the tape drive such that the heads follow the tracks on the tape [5]. The DAT control system is more complex than that for a CD ROM because more motor have to be accurately controlled. This includes: capstan, take-up and supply reels, drum, and tension control.

A typical DAT is shown in Fig. 1 in which the heads follow the tracks on the tape. It is required to design a speed compensated system for a DAT such that the system must respond rapidly with an overshoot to a step of less than 13% and settling time of less than 2 second with 2 % criterion [5]. The system needs a fast peak time, and such the overdamped condition is not allowed.

III. METHODOLOGY

The mathematical equations of a motor/load system of a digital audio tape (DAT) is obtained and a compensator is designed for the system using Matlab software in this section. The mathematical equations are transformed into transfer functions representing the motor and the load. The motor and
the load system are integrated with the designed compensator to form a closed-loop control system which employs a feedback sensor with a unit gain. Fig. 2 shows the elements for the realization of the design objective.

\[
\text{Fig. 2. Element of the compensated DAT system.}
\]

**A. Mathematical Equation of Motor and Load System**

Fig. 3 shows a typical printer belt drive system for a computer. In Fig.1, \( R \) is the motor circuit resistance, \( L \) is the inductance of motor armature, \( \tau \) is the rotational torque of the motor shaft, \( v_{\text{emf}} \) is the back induced electromotive force (e.m.f), \( J \) is the total inertia of the printer belt drive, \( v_a \) is the applied voltage, \( i(t) \) is the current of the motor circuit, \( \omega \) the angular velocity of the motor shaft.

\[
\text{Fig. 3. A typical motor and load system.}
\]

In this way, it acts as an actuator. Applying Kirchhoff’s voltage law to the DC motor circuit gives:

\[
v_a(t) = Ri(t) + L \frac{di(t)}{dt} + v_{\text{emf}}(t) \tag{1}
\]

The angular positioning of the shaft \( \theta \) and the angular speed of the shaft \( \omega \) are related as follows:

\[
\omega(t) = \frac{d\theta(t)}{dt} \tag{2}
\]

The back e.m.f \( v_a(t) \) is directly proportional to the angular speed of the motor shaft. This is given as:

\[
v_{\text{emf}}(t) = k_m \omega(t) \tag{3}
\]

where \( k_m \) is the motor torque constant. Substituting (3) into (1) gives:

\[
v_a(t) = Ri(t) + L \frac{di(t)}{dt} + k_m \omega(t) \tag{4}
\]

Equation (4) is expressed in Laplace transform as:

\[
V_a(s) = RI(s) + LsI(s) + k_m \omega(s) \tag{5}
\]

The rotational elements of the DC motor shaft are represented by:

\[
\tau(t) = J \frac{d\omega(t)}{dt} + f\omega(t) \tag{6}
\]

The rotational torque of the motor shaft is:

\[
\tau(t) = k_m \omega(t) \tag{7}
\]

The Laplace transform of equating (7) to (6) gives:

\[
k_mI(s) = Js\omega(s) + f\omega(s) \tag{8}
\]

Elimination of \( I(s) \) from (5) and (8) gives:

\[
G(s) = \frac{\omega(s)}{V_a(s)} = \frac{k_m}{LJs^2 + (RJ + L)s + Rf + k_m^2} \tag{9}
\]

Using the parameters of [5] wherein the transfer function is varied because the tape moves from one reel-to-reel gives:

\[
G(s) = \frac{k_m}{(s + p_1)(s + p_2)} \tag{10}
\]

where the nominal values are \( k_m = 4 \), \( p_1 = 1 \), and \( p_2 = 4 \). The range of variation is \( 3 \leq k_m \leq 5 \), \( 0.5 \leq p_1 \leq 1.5 \), and \( 3.5 \leq p_2 \leq 4.5 \). And comparing (9) with (10) shows that:

\[
LJ = 1; \ (RJ + L) = p_1 + p_2; \ 	ext{and} \ Rf + k_m^2 = p_1p_2 \tag{11}
\]

Table I is the definition of the parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance</td>
<td>( L )</td>
<td>H</td>
</tr>
<tr>
<td>Viscous friction</td>
<td>( f )</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>Resistance</td>
<td>( R )</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>Inertia</td>
<td>( J )</td>
<td>Kg/m^2</td>
</tr>
</tbody>
</table>

Considering three nominal conditions, (10) is represented in discrete time form using a sampling time of 0.1 seconds given by:

\[
G_1(z) = \frac{0.01317z + 0.01152}{z^2 - 1.656z + 0.6703} \tag{11}
\]

\[
G_2(z) = \frac{0.01699z + 0.01438}{z^2 - 1.575z + 0.6065} \tag{12}
\]

\[
G_3(z) = \frac{0.02056z + 0.01683}{z^2 - 1.498z + 0.5488} \tag{13}
\]
The motor converts electrical energy it receives from electrical supply system to mechanical energy for driving the load. It is assumed in this context that the field current of the armature-controlled DC motor is constant.

**B. System Configuration and Compensator Design**

The configuration of the motor and load system integrating a compensator is shown in Fig. 4. The system is a single input single output (SISO) system.

![Fig. 4. Performance response loop of compensated motor/load system.](image)

The compensator is designed using MATLAB software. It is given by:

\[ D(z) = \frac{(z - 1)\left[5.98 + 11.3(z - 1)\right] + 0.793}{0.1(z - 1)} \quad (14) \]

The close loop transfer function for the different nominal conditions considered are:

\[ G_1(z) = \frac{1.485z^3 - 0.8824z^2 - 1.108z + 0.7016}{2.485z^3 - 3.538z^2 + 1.218z + 0.0313} \quad (15) \]

\[ G_2(z) = \frac{1.916z^3 - 1.194z^2 - 1.349z + 0.8757}{2.916z^3 - 3.769z^2 + 0.8326z + 0.2692} \quad (16) \]

\[ G_3(z) = \frac{2.318z^3 - 1.509z^2 - 1.538z + 1.025}{3.318z^3 - 4.007z^2 + 0.5094z + 0.476} \quad (17) \]

**IV. SIMULATION AND DISCUSSION**

**A. Simulation Result**

Simulations are performed considering when the system is uncompensated and when it is compensated.

**Uncompensated performance response:**

Fig. 5, 6, and 7 show the simulation results of the different responses for the system for various values of the process parameters \( k_m, p_1, \) and \( p_2 \) when it has not been compensated, considering a unit step input.
**B. Simulation Result Performance Analysis**

<table>
<thead>
<tr>
<th>DAT System</th>
<th>Rise time (s)</th>
<th>Settling time (s)</th>
<th>Overshoot (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G_1(z)</td>
<td>4.47</td>
<td>8.13</td>
<td>0</td>
</tr>
<tr>
<td>G_2(z)</td>
<td>2.31</td>
<td>4.2</td>
<td>0</td>
</tr>
<tr>
<td>G_3(z)</td>
<td>1.59</td>
<td>2.96</td>
<td>0</td>
</tr>
</tbody>
</table>

**C. Discussion**

The response of the system to a unit step input are shown for two separate conditions, that is, uncompensated and compensated states. Three simulation results are presented for the different conditions considering the variation of the process nominal parameter values.

It can be seen in table II that for uncompensated state the step response performance of the system seems to be sluggish and not fast for the DAT system, considering the process with varied parameter values. This can be seen as plotted in Fig. 5, 6, and 7 for G_1(z), G_2(z), and G_3(z).

In Fig. 8, 9, and 10, the response of the process with nominal parameter values to unit step input show that the responses meet the specifications based on percentage overshoot value of less than 13%, and a settling time of less than 2 seconds. This show that the designed compensator performed robustly and optimally. This is based on the fact that irrespective of the varying parameter values, the compensator was able to keep the system at the required stead response characteristics.

**V. Conclusion**

This paper has presented computer simulation of performance response of a digital audio tape (DAT) system. The goal of the paper is to design a compensator that will improve the response of a typical digital audio tape (DAT) system. A dynamic model of a motor and load system is obtained and a compensator is designed and integrated into the loop of the process. The integration of the compensator improved the response performance of the system as shown from the result obtained for the compensated DAT system.

**REFERENCE**


