Volume 1, Issue 12, pp. 102-105, 2018.

International Journal of Scientific Engineering and Science
ISSN (Online): 2456-7361

The Most Real Evaluation of Efficiency and the Causes that Produce the Inefficiency of the Decision Making Units with Two-Phase Process in DEA

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Abstract— *DEA is a method that serves to the evaluation of the homogeneous decision making units performance that operate in similar conditions by measuring and evaluating the relative technical efficiency. To a better assessment of the magnitude of impact of the specific factors on the efficiency value serves also the examination of the process decomposes into inter-phase processes by the analysis of the efficient value, etc. The intermediate products serve as outputs for the first phase and as inputs for the second phase. The evaluation of efficiencies is done by following two different models: The independent model and the relational model. The independent model is the traditional CCR model considered separately for each phase. Considering the fact that the relational model considers the conditions as related, it is accepted as the most rational model.*

Keywords— *DEA, efficiency, independent model, relational model, two-phase process.*

I. Introduction

The original presentation of DEA theory (Data Envelopment Analysis) was presented in the "Measuring the Efficiency of Decision Making Units" work by Charnes, Cooper and Rhodes (1978). Presently, DEA has become one of the most widely used methods among the operations research methods / management sciences (OR/MS) (Joro & Korhonen, 2015). It became popular in the performance evaluation both in the public and private sectors. "DEA may surely be considered as one of the most recent "Success Stories" in the operations research" – D. Bouyssou). DEA is a model of the mathematical programming, linear programming, that is used when the other approaches face resistance because of the complex nature (often unknown) of the relationship between the many inputs and outputs included in the DMUs. DEA is a non-parametric and "non-statistical" method, as it does not require detailed suppositions of the probability dispersion of the "errors" (i.e. efficient remnants) in the production function. Any deviation from the frontier, in this method, is called inefficient. It offers only one measure of efficiency and it doesn't need much of the predefined weights. The applications keep becoming more sophisticated, but what is important is that DEA shows as a strong and accurate optimizer. In order to best identify the causes that produce the inefficiency of the decision making units, DEA practice uses the study of the process that is decompose into phases where multiple variables (inputs and outputs) and q intermediate "products" are included, Z_{dk}, (d=1,2,…,q; k=1,2,…,n). The Z_{dk}, variables (intermediate products) serve as outputs for the first phase and as inputs for the second phase. The efficiencies evaluated for each phase will respectively be called $E^1_{f_k}$, $E^2_{f_k}$ and the general efficiency will be E_{f_k}. The evaluation of $E^1_{f_k}$ and $E^2_{f_k}$ efficiencies is done by following two different models: The independent model and the relational model. The independent model is the traditional CCR model considered separately for each phase. Considering the fact that the relational model considers the conditions as related, it is accepted as the most rational model, where $E_{f_k} = E^1_{f_k} \times E^2_{f_k}$.

II. Methodology

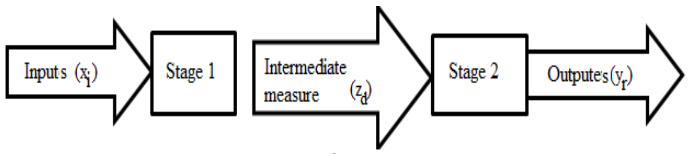

Fig. 1. Two-phase process.

$$\begin{array}{c|ccc|ccc|}
x_{1k} & \rightarrow & z_{1k} & \rightarrow & y_{1k} \\
x_{2k} & \rightarrow & z_{2k} & \rightarrow & y_{2k} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
x_{mk} & \rightarrow & z_{qk} & \rightarrow & y_{sk}
\end{array}$$

Suppose, we have n DMU_s,that each DMU_j (j=1,2,…,m) ham m inputs to the first stage and q outputs from the first stage, z_{dj} (d=1,2,…,q).These q outputs then become the inputs to the second stage and are called intermediate measures. The conventional two-stage DEA study is to measure the overall efficiency and the following models to measure the efficiencies of stage 1, E^1_{fk}, and stage 2, E^2_{fk}, respectively (Independent two-stage Model).

$$E^1_{f_k} = \max \sum_{d=1}^{q} w_d Z_{dk} \Big/ \sum_{i=1}^{m} v_i X_{ik},$$

$$\text{S.t} \quad \sum_{d=1}^{q} w_d Z_{dj} \Big/ \sum_{i=1}^{m} v_i X_{ij} \le 1, \quad (j=1,\dots,n) \qquad [1]$$

$$w_d, v_i \ge \varepsilon \quad (d=1,\dots,q;\ i=1,\dots,m),$$

and

$$E^2_{f_k} = \max \sum_{r=1}^{s} u_r Y_{rk} \Big/ \sum_{d=1}^{q} w_d Z_{dk},$$

$$\text{S.t} \sum_{r=1}^{s} u_r Y_{rj} \Big/ \sum_{d=1}^{q} w_d Z_{dj} \le 1, \ (j=1,\dots,n \qquad [2]$$

$$u_r, w_d \ge \varepsilon, \quad (r = 1,\dots,s;\ d = 1,\dots,q).$$

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Denote u_r^*, v_i^* and w_d^* as the multipliers that DMU_k has selected to calculate its overall efficiency E_{f_k} and sub-process efficiencies $E_{f_k}^1$, $E_{f_k}^2$. Then we have:

$$E_{f_k} = \sum_{r=1}^{s} u_r^* Y_{rk} / \sum_{i=1}^{m} v_i^* X_{ik} \leq 1,$$
$$E_{f_k}^1 = \sum_{d=1}^{q} w_d^* Z_{dk} / \sum_{i=1}^{m} v_i^* X_{ik} \leq 1, \quad (1)$$
$$E_{f_k}^2 = \sum_{r=1}^{s} u_r^* Y_{rk} / \sum_{d=1}^{q} w_d^* Z_{dk} \leq 1,$$

the overall efficiency is: $E_{f_k} = E_{f_k}^1 \times E_{f_k}^2$.

Then, the way to calculate the overall efficiency E_{f_k} is to incorporate the constraints of the two sub-processes:

$$E_{f_k} = \max \sum_{r=1}^{s} u_r Y_{rk}$$
$$\text{S.t} \quad \sum_{i=1}^{m} v_i X_{ik} = 1, \quad [3]$$
$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0 \quad (j = 1,\ldots,n),$$
$$\sum_{d=1}^{q} w_d Z_{dj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0 \quad (j=1,\ldots,n),$$
$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{d=1}^{q} w_d Z_{dj} \leq 0 \quad (j=1,\ldots,n)$$
$$u_r, v_i, w_d \geq \varepsilon \quad (r = 1,\ldots,s;\ i = 1,\ldots,m;\ d = 1,\ldots,q).$$

After the optimal multipliers u_r^*, v_i^* and w_d^* are solved, the efficiencies are:

$$E_{f_k} = \sum_{r=1}^{s} u_r^* Y_{rk} \text{ , } E_{f_k}^1 = \sum_{d=1}^{q} w_d^* Z_{dk} / \sum_{i=1}^{m} v_i^* X_{ik}$$

and $E_{f_k}^2 = \sum_{r=1}^{s} u_r^* Y_{rk} / \sum_{d=1}^{q} w_d^* Z_{dk}$.

Clearly, we have $E_{f_k} = E_{f_k}^1 \times E_{f_k}^2$.

The optimal multipliers solved from [3] may not be unique, then one solution to this problem is to solve the following problem, maintaining the overall efficiency score at E_{f_k} calculated from model [3]:

$$E_{f_k}^1 = \max \sum_{d=1}^{q} w_d Z_{dk}, \quad [4]$$
$$\text{S.t} \quad \sum_{i=1}^{m} v_i X_{ik} = 1,$$
$$\sum_{r=1}^{s} u_r Y_{rk} - E_{f_k} \sum_{i=1}^{m} v_i X_{ik} = 0,$$
$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0 \quad (j = 1,\ldots,n),$$
$$\sum_{d=1}^{q} w_d Z_{dj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0 \quad (j = 1,\ldots,n),$$
$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{d=1}^{q} w_d Z_{dj} \leq 0 \quad (j=1,\ldots,n),$$
$$u_r, v_i, w_d \geq \varepsilon \quad (r = 1,\ldots,s;\ i = 1,\ldots,m;\ d = 1,\ldots,q).$$

So, the efficiency of the second stage is $E_{f_k}^2 = E_{f_k} / E_{f_k}^1$.

III. Applications

The relation between the basic models in combination with the differently expanded and extended theoretical approaches is the application used in this study that made an evaluation of the Albanian economy performance progress dynamics during the period 2007-2014 (here are given only two years with the data of the variables). In this study, the performance of the Albanian economy is evaluated in relation to other 17 European countries: 1. Albania, 2. Macedonia, 3. Serbia, 4. Montenegro, 5. Croatia, 6. Bosnia-Herzegovina, 7. Romania, 8. Bulgaria, 9. Turkey, 10. Hungary, 11. Poland, 12. Czech Republic, 13. Armenia, 14. Kazakhstan, 15. Kosovo, 16. Moldavia, 17. Russia, and 18. Ukraine. In order to measure and evaluate the performance are taken variables:

the inputs :

X_1 – Investments;

X_2 - Imports of goods;

X_3- Stock of public (S.p.dept)

the outputs:

Y_1- GDP (Gross Domestic Product)

Y_2 – Family consume

Y_3 – Export of goods

and the intermediate variables are:

Z_1 – Trada (the entire import-export trade)

Z_2 – GNI index

TABLE I. Table of data per capita (Year 2007).

No.	Countries	Investments	Imports	S.P.dept	Trada	GNI	GDP	Consume	Export
1	Albania	1464.88	1482.92	1924.73	1864.53	7870	3789.57	3041.25	381.61
2	Mcedonia	1013.51	2707.38	956.06	4449.70	9220	4274.15	3305.82	1742.31
3	Serbia	1670.47	2730.62	1834.46	3988.03	10240	5740.74	4306.10	1257.41
4	Montenegro	2118.13	4896.77	1640.69	5966.34	12630	6265.56	5537.46	1069.56
5	Croatia	4232.13	6073.85	5027.16	8999.70	18540	14248.15	8353.38	2925.85
6	Bos.-Herc.	1096.17	2662.05	746.48	3799.16	8260	4320.70	3703.09	1137.10
7	Romania	2702.73	3541.37	1040.12	5580.58	12810	8639.51	5814.87	2039.20
8	Bulgaria	2081.85	4176.43	1035.69	6757.78	12360	6240.10	4242.21	2581.35
9	Turkey	2062.84	2573.07	3710.95	4196.11	13900	9791.55	6981.99	1623.04
10	Hungary	3530.80	9995.60	9075.25	19973.90	18060	14546.99	7881.21	9978.30
11	Polond	2933.77	4572.08	4974.70	8438.84	16320	11829.95	7157.54	3866.76
12	Czech.Rep.	6195.53	12068.18	5092.61	24578.46	24970	19283.27	8933.34	12510.28
13	Armenia	1277.27	1150.19	438.99	1555.78	6830	3304.12	2428.46	405.59
14	Kazakhstan	2236.54	2224.94	397.75	5468.69	15590	8523.76	3618.09	3243.74
15	Kosovo	612.42	1361.08	678.94	1518.59	7350	3069.11	2617.15	157.51
16	Moldavia	444.10	1084.99	302.79	1479.53	3700	1497.10	1406.08	394.54
17	Russia	2303.25	1645.97	783.29	4256.15	16280	10532.33	4974.60	2610.18
18	Ukraine	971.06	1370.81	377.84	2485.59	7890	3220.01	1789.44	1114.78

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TABLE II. Table of data per capita (Year 2014).

No.	Countries	Investments	Imports	S.P.dept	Trada	GNI	GDP	Consume	Export
1	Albania	1034.99	1665.02	3293.27	2438.87	10990	4227.35	3335.87	773.84
2	Mcedonia	1530.06	3229.77	2088.65	5419.65	13260	5023.88	3514.76	2189.87
3	Serbia	999.25	2662.60	4464.58	4580.58	13150	5711.96	4319.73	1917.98
4	Montenegro	1374.47	3506.54	4427.33	4159.41	15250	6797.09	5454.40	652.87
5	Croatia	2259.45	4953.58	11671.63	7965.73	21200	12419.18	7442.88	3012.15
6	Bos.-Herc.	821.66	2652.22	2136.00	4073.78	10500	4469.64	3746.33	1421.56
7	Romania	2328.02	3599.62	4055.28	6826.62	20130	9223.46	5640.28	3227.00
8	Bulgaria	1550.86	4425.01	2075.40	8159.70	17060	7233.05	4553.30	3734.69
9	Turkey	1899.58	2877.93	3453.45	4750.90	19190	9492.57	6539.71	1872.97
10	Hungary	2869.47	9796.91	10681.77	20125.70	23950	12917.81	6499.51	10328.79
11	Polond	2691.04	5421.02	7236.89	10756.67	24380	13208.29	7927.24	5335.65
12	Czech.Rep.	4536.69	13512.11	8326.07	28837.76	29440	17966.79	8730.59	15325.65
13	Armenia	811.33	1355.89	1600.72	1830.07	8500	3700.11	2889.29	474.18
14	Kazakhstan	2607.30	2200.44	1856.54	6434.47	22310	10645.46	5369.14	4234.03
15	Kosovo	856.73	1609.25	682.23	1828.26	9400	3621.69	3150.98	219.01
16	Moldavia	485.57	1377.32	705.39	1983.35	5540	1986.93	1821.76	606.03
17	Russia	2605.95	1973.12	2244.50	5161.63	24260	11672.32	6620.35	3188.51
18	Ukraine	405.30	1103.37	2051.16	2204.08	8580	2961.26	2323.77	1100.71

TABLE III. Evaluation of inefficiencies for the independent model and for the relational model (in %) (Year 2007).

No.	Countries	EK RELAT.	EK1 Relat.	EK2 Relat.	EK1 Indep.	EK2 Indep.	Ek.Indep.
1	Albania	0.55443	0.66025	0.83972	0.66803	0.95988	0.72993
2	Mcedonia	0.81848	1.00000	0.81848	1.00000	0.81848	1.00000
3	Serbia	0.62012	0.76810	0.80733	0.76810	0.84447	0.79228
4	Montenegro	0.53389	0.69567	0.76745	0.71102	0.87286	0.73540
5	Croatia	0.56919	0.64995	0.87575	0.66478	1.00000	0.71737
6	Bos.-Herc.	0.76379	0.90608	0.84296	0.93074	0.90226	1.00000
7	Romania	0.61427	0.69755	0.88061	0.71281	0.93025	0.92237
8	Bulgaria	0.71539	0.90719	0.78858	0.99650	0.78858	0.84449
9	Turkey	0.83691	0.83691	1.00000	0.83691	1.00000	1.00000
10	Hungary	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
11	Polond	0.82412	0.84834	0.97144	0.84834	0.99590	0.89736
12	Czech.Rep.	0.98090	0.98090	1.00000	1.00000	1.00000	1.00000
13	Armenia	0.52783	0.67438	0.78269	0.72522	0.93221	0.87274
14	Kazakhstan	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
15	Kosovo	0.82019	1.00000	0.82019	1.00000	1.00000	1.00000
16	Moldavia	0.70016	0.95848	0.73049	0.95848	0.76188	0.93675
17	Russia	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
18	Ukraine	0.71431	0.95435	0.74847	1.00000	0.75833	0.83487

TABLE IV. Evaluation of inefficiencies for the independent model and for the relational model (in %) (Year 2014).

No.	Countries	EK RELAT.	EK1 Relat.	EK2 Relat.	EK1 Indep	EK2 Indep	Ek Indep.
1	Albania	0.65160	0.72453	0.89934	0.73490	0.92039	0.82136
2	Mcedonia	0.66909	0.81716	0.81879	0.82428	0.81879	0.75764
3	Serbia	0.71862	0.75081	0.95712	0.83302	0.97010	0.82758
4	Montenegro	0.67092	0.67092	1.00000	0.67092	1.00000	0.82618
5	Croatia	0.60726	0.60726	1.00000	0.74139	1.00000	0.83995
6	Bos.-Herc.	0.89433	0.89433	1.00000	0.91611	1.00000	1.00000
7	Romania	0.71369	0.78380	0.91055	0.80323	0.91257	0.78768
8	Bulgaria	0.87322	1.00000	0.87322	1.00000	0.87322	1.00000
9	Turkey	0.82200	0.82200	1.00000	0.82200	1.00000	0.98326
10	Hungary	0.95016	1.00000	0.95016	1.00000	0.95036	1.00000
11	Polond	0.76981	0.77437	0.99411	0.88580	1.00000	0.88324
12	Czech.Rep.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
13	Armenia	0.79399	0.79399	1.00000	0.80095	1.00000	0.93924
14	Kazakhstan	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
15	Kosovo	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
16	Moldavia	0.89398	0.96682	0.92466	0.96682	0.92515	1.00000
17	Russia	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
18	Ukraine	0.92024	1.00000	0.92024	1.00000	0.92024	1.00000

TABLE V. Summarizing results of evaluation of inefficiencies for the independent model and for the relational model (in %) (2007-2014).

		DMUs	2007	2008	2009	2010	2011	2012	2013	2014
Independent two -stage Model	E_{K_1}	Albania	33.20	32.29	39.83	27.07	31.38	14.57	14.77	26.51
		Macedonia	0.00	0.00	1.39	4.15	16.18	17.15	18.21	17.57
		18 countries	12.11	15.83	12.77	12.19	13.90	9.47	9.01	11.11
	E_{K_2}	Albania	4.01	0.00	4.90	10.97	6.91	7.82	6.52	7.96
		Macedonia	18.15	21.38	24.91	22.76	17.81	20.86	18.69	18.12
		18 countries	7.97	7.71	6.64	8.61	5.23	5.83	4.67	3.94
	E_K	Albania	27.01	20.43	34.97	26.03	18.23	17.71	14.88	17.86
		Macedonia	0.00	0.00	5.84	7.76	15.68	20.92	23.19	24.24
		18 countries	9.54	10.00	8.84	5.78	4.18	4.40	3.73	7.41
Relational two-stage Model	E_{K_1}	Albania	33.97	33.66	41.67	27.07	31.73	14.57	14.77	27.55
		Macedonia	0.00	1.49	1.39	6.98	16.70	17.21	18.59	18.28
		18 countries	13.68	17.98	15.06	13.56	15.92	10.05	9.67	13.30
	E_{K_2}	Albania	16.03	10.09	7.87	23.60	15.05	20.64	18.72	10.07
		Macedonia	18.15	21.38	24.91	22.76	17.81	20.86	18.69	18.12
		18 countries	12.92	11.24	7.93	10.74	6.97	7.31	6.29	4.18
	E_K	Albania	44.56	40.35	46.26	44.28	42.01	32.20	30.73	34.84
		Macedonia	18.15	22.56	25.95	28.14	31.53	34.48	33.81	33.09
		18 countries	24.48	27.37	22.14	22.92	22.00	16.62	15.33	16.95

TABLE VI. Position of Albania and Macedonia ranking based on the independent model and on the relational model.

Years	Albania						Macedonia					
	Independent .M			Relational.M			Independent.M			Relational.M		
	E_{K_1}	E_{K_2}	E_K	E_{K_1}	E_{K_2}	E_K	E_{K_1}	E_{K_2}	E_K	E_{K_1}	E_{K_2}	E_K
2007	17	9	17	17	10	16	3.5	15	4.5	3	12	8
2008	14	4.5	15	14	8	14	4.5	15	4	7	13	6
2009	17	11	18	15	11	18	9	18	12	9	18	11
2010	15	12	18	15	18	17	9	18	12	9	17	12
2011	17	13	17	17	15	18	9	17	16	8	17	14
2012	11	13	16	11	17	17	14	18	18	14	18	18
2013	13	13	16	13	18	17	16	18	18	16	17	18
2014	17	14	16	16	16	17	12	18	18	11	18	16

IV. CONCLUSION

Based on the above tables it is evident that the input potentials factor for Albania to generate "income" is much smaller compared specifically to Macedonia and in general to the other 18 countries taken altogether. Among the intermediate variables, except GNI, the values of the inefficiency differences between the two phases show that the level of the import coverage by the export of goods in Albania is much more sensible for the higher values of the inefficiency. This reasoning is reinforced by the position of Albania rankings for each phase in general and for both models, where the ranking position in the first phase is worse than that in the second phase. The worsening role of the Albanian exports is very sensible. Structuring a process divided into several phases is an opportunity to achieve the best identification of the impact of the inefficiency factors.

This application examines relations between two existing DEA approaches (independent model, relational model) that address measuring the performance of two-stage processes, in order to best identify the causes that produce the inefficiency of the decision making units.

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