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Associated Bitopological Spaces of (i,j)- g*b –Closed Sets

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Abstract— *The concept of gb-closed sets for general topology was introduced and studied analytically by M.Ganster & M. Steiner and which motivated the author to coin the concept of (i,j)-gb-closed as well as (i,j)-g*b –closed sets in bitopological spaces.*

*The exploration of these classes of (i,j)-gb-closed as well as (i,j)-g*b –closed sets provide the relation with the class of ij-b-closed sets in the manner that the class of (i,j)-g*b –closed sets properly exists in between the class of j-b-closed sets and the class of (i,j)-gb-closed sets.*

Of course, being the main object of the present paper, two new bitopological (i,j)-T$b_{1/2}$ & (i,j)-*T$b_{1/2}$ are introduced as an application of (i,j)-g*b-closed sets.*

*No doubt in order to serve the purpose, the basic properties of (i,j)-g*b-closed as well as (i,j)-gb-closed sets are investigated and analysed in this paper.*

Keywords— *(i,j)-g*b-closed sets, (i,j)-T*$b_{1/2}$ & (i,j)-*T$b_{1/2}$ spaces.*

I. Introduction

A triple (X, τ_1, τ_2) consisting of a non-empty set X together with a pair of topologies τ_1 & τ_2 on X, is defined to be a bitopological space by J.C.Kelley as introduced through the famous mathematical paper entitled as " Bitopological spaces" and published in proceedings of London Mathematical Society, 13(71-89),1963[1]. Such a space, equipped with two arbitrary topologies, is beyond any doubt an original and fundamental work.

As a recall, in 1963, a semi-open set A was defined as the subset A of a topological space (X, τ) iff there exists O $\in$ τ such that O $\subseteq$ A $\subseteq$ Cl(O) and denoted by s-open or equivalently iff A$\subseteq$ cl(int(A)).

In 1970, generalized closed sets were introduced and studied by N. Levine. He conceptualized a generalized closed set as a subset A of a topological space (X, τ) for which A $\subseteq$ O $\in$ τ $\Rightarrow$ cl(A)$\subseteq$ O and denoted by g-closed.

In 1978, an α-set was introduced as a subset A of a topological space (X, τ) for which A $\subset$ int (cl(int(A))) and denoted by α – open set.

Then in 1981, a pre-open set was investigated as a subset A of a topological space (X, τ) in manner that A$\subset$ int(cl(A)) and denoted by p- open.

In 1983, M.E. Abd El – Monsef etc introduced a β-open set as subset A of a topological space(X, τ) in the manner that A$\subset$ cl(int(cl(A))) and D. Andrijewic named β-open sets as semi- pre open sets.

C. Kuratowaski mentioned that a subset A of a topological space(X, τ) is said to be regular open (resp. regular closed) or an open domain(resp. closed domain) if it is the interior of its closure(resp. if it is the closure of its interior) i.e. A = int (cl(A)).(resp. A = cl(int(A)).

D. Andrijevic investigated a new class of generalized open sets in a topological space, the so called b-open sets, which is contained in the class of semi-pre-open sets and contains all semi-open sets and pre-open sets. Also, such a class of b-open sets generates the same topology as the class of pre-open sets.

Thus, a subset A of a topological space(X, τ) is known to be a b-open or γ -open set iff A $\subset$ cl(int((A))$\cup$ int(cl(A)).

The compliments of the above mentioned open/closed sets are their respective closed/open sets. The smallest $\varkappa$-closed set containing A is called the $\varkappa$ -closure of A and denoted by $\varkappa$ cl(A) where as the largest $\varkappa$ -open set contained in A is called $\varkappa$ -interior of A and denoted by $\varkappa$ int(A) where $\varkappa$ = α, semi(s),pre(p), regular (r) ,b & β.

Extensive research on generalizing of closedness was done in the recent years and in the year 2007, M.Ganster and M.Steneir presented the concept & the study of generalized b-closed (briefly, gb-closed) sets in general topology.

In 1985, T.Fukutake projected the concept of g-closed sets for bitopogical spaces through the mathematical paper " On Generalized Closed Sets In Bitopological Spaces" published in Bull. Fuscnoka Univ. Ed. Part III, 35(19-28), 1986[2].

In 1997, The concepts of pre-open sets and regular open sets have been extended to bitopological spaces symbolized by (i,j)-p-open and (i,j)-r- open sets respectively as mentioned in the mathematical paper 'On Decomposition Of Pair wise Continuity ' by S. Sampath Kumar, published in Bull. Cal. Math. Soc. 89 (1997), pp- 441-446 [8].

The aim of this paper is to extend the concept of gb-closed sets in general topology for bitopology. So, we introduce (i,j)-g*b-closed sets and (i,j)-gb-closed sets and their fundamental interesting properties.

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Again, the bitopological spaces (i,j)- T^{*} $b_{1/2}$ and (i,j) - $^{*}T$ $b_{1/2}$ associated with (i,j)-g*b- closed sets are natural out come as an application. Such spaces are analysed to some extent in this paper.

The following abbreviations are adopted throughout this paper.

(a) All through this paper, the spaces X and Y, always represent non-empty bitopological spaces(or X,τ_1, τ_2) and (Y,σ_1,σ_2)respectively on which no separation axioms are assumed unless explicitly stated.

(b) The interior (resp. closure) of a subset A of a bitopological space (X,τ_1, τ_2) w.r.t. topology τ_i (i = 1,2) is denoted by τ_i –int(A)(resp. τ_i- cl(A)).

(c) The set of all closed (resp. open) sets w.r.to the topology τ_i is denoted by i-C(X)(resp. i-O(X)).And by (i,j)- we mean the pair of topologies (τ_i , τ_j) & (i,j) ∈ {1,2} such that $i \neq$j.

(d) As usual the family of all $\mathcal{K}$-open ($\mathcal{K}$= α,pre,semi,b,β,r and g) sets is denoted by the symbol i-$\mathcal{L}$O(X) ($\mathcal{L}$ =α, P,S,B,β,RG) accordingly.

The following definitions are cited for the reference of all the self –explanatory arguments of the content projected in this paper.

Definition (1.1): A subset A of a bitopological space (X,τ_1, τ_2) is called

(a) (i,j)-g-closed [2&9] if τ_j-cl(A)⊆U whenever A ⊆U and U ∈ i-O(X).

(b) (i,j)-g*-closed [3] if τ_j-cl(A)⊆U whenever A ⊆U and U ∈ i-GO(X).

(c) (i,j)-rg-closed [4] if τ_j-cl(A)⊆U whenever A ⊆U and U ∈ i-RO(X).

(d) (i,j)-w-closed [5] if τ_j-cl(A)⊆U whenever A ⊆U and U ∈ i-SO(X).

(e) (i,j)-wg-closed [6] if τ_j-cl(τ_i-int A)⊆U whenever A ⊆U and U ∈ i-O(X).

(f) (i,j)-p-closed [8&10] if U ⊆A and U ∈ j-O(X) ⇒ ∃ F ∈i-C(X) such that U ⊆F ⊆A.

(g) (i,j)-r-closed [8&9] if A = τ_i-cl(τ_j-int (A)).

The family of all (i,j)-g-closed (resp. (i,j)g*-closed, (i,j)-rg-closed, (i,j)-w-closed,(i,j)-wg-closed,(i,j)-p-closed and (i,j)-r-closed) subsets of a bitopological space(X, τ_1,τ_2) is denoted by D(i,j)(resp. D*(i,j), D_r(i,j),C(i,j),W(i,j),P(i,j)&R(i,j))[11].

Definition (1.2):

(a) A subset A of a bitopological space (X, τ_1,τ_2) is said to be(i,j)-$T_{1/2}$ space[2].(resp. (i,j)-$Tb_{1/2}$ space) if every (i,j)-g-closed(resp. (i,j)-gb-closed) set is τ_j-closed (resp. τ_j-b-closed).

(b) A bitopological space (X, τ_1,τ_2) is said to be strongly pairwise $T_{1/2}$ (resp. strongly pairwise $Tb_{1/2}$ space [2]) if it is both (i,j)-$T_{1/2}$ space and (j,i)- $T_{1/2}$ space(resp. (i,j)-$Tb_{1/2}$ space and (j,i)- $Tb_{1/2}$ space), where i,j∈{1,2} & i≠j.

Definition (1.3): A mapping f :((X, τ_1,τ_2)→ (Y,σ_1,σ_2) is called

(a) $\tau_j - \sigma_k$ continuous [7] if $f^{-1}(V)$ ∈ τ_j -for every V ∈ σ_k

(b) L(i,j) - σ_k continuous if the inverse image of every σ_k closed set is (i,j)-t-closed where L = D,D*, D_r,C,W, P& R and t = g, g*,rg, w,wg, p & r accordingly,([4],[5],[6] &[7]).

- Any other notion & symbol, not defined here, may be found in the appropriate reference.

II. (I,J)-G*B- CLOSED &(I,J)-GB-CLOSED SETS

This section carries the introduction & discussion of the concepts of (i,j)-g*b-closed sets as well as (i,j)-gb-closed sets in a bitopological space. Also, it includes the related fundamental properties with suitable examples.

Definition (2.1):

A subset A of a bitopological space (X, τ_1,τ_2) is said to be

(a) an (i,j)-g*b-closed set if τ_j-bcl(A)⊆U whenever A ⊆U and U ∈ i-GO(X).

(b) an (i,j)-gb-closed set if τ_j-bcl(A)⊆U whenever A ⊆U and U ∈ i-O(X).

The family of all (i,j)-g*b-closed sets(resp.(i,j) gb-closed sets) is denoted by D_b*(i,j)(resp. D_b(i,j)).

Proposition (2.2): In a bitopological space (X, τ_1,τ_2)

(a) Every τ_j-closed set is (i,j)-g*b-closed.

(b) Every (i,j)g*b-closed set is (i,j)-gb-closed.

(c) Every (i,j)-g-closed set is (i,j)-gb-closed.

(d) Every(i,j)-g*-closed set is (i,j)-g*b-closed.

(e) Every (i,j)-g*-closed set is (i,j)-g-closed.

(f) Every (i,j)-g-closed set is (i,j)-rg-closed.

Proof:

(a) Suppose that a τ_j-closed subset of a bitopological (X, τ_1,τ_2) is A which is to be shewn to be an (i,j)-g*b-closed set.

Let U be any i-g-open set in (X, τ_i) such that A ⊆ U. Since, every τ_j -closed set is a τ_j–b-closed set, hence A is a τ_j–bclosed. i.e. A = τ_j - bcl(A). Consequently, it follows that A ⊆ U & U ∈ i-GO(X) ⇒ τ_j - bcl(A) ⊆ U. This means that A is an (i,j)-g*b-closed set.

(b) Let an i-open set U and an (i,j) -g*b-closed set A in a space(X, τ_1,τ_2) be such that A ⊆ U . Now , (i,j) -g*b-closedness of A provides that τ_j - bcl(A) $\subseteq$ V whenever A $\subseteq$ V & V is an i-g-open set ………………………..(I)

Since, every i-open set is an i-g-open set , hence, (I) turns to be τ_j - bcl(A) $\subseteq$ U whenever A $\subseteq$ U & U is i-g-open. But U $\in$ i-O(X) ,so τ_j - bcl(A) $\subseteq$ U whenever A $\subseteq$ U & U$\in$ i-O(X) holds good. Consequently, A is an (i,j)-gb-closed set.

(c) Let A be an (i,j)-g-closed set in a bitopological (X, τ_1,τ_2). Let V $\in$ i-O(X) be such that A $\subseteq$ V. Then, (i,j)- g-closedness of A provides that τ_j - cl(A) $\subseteq$ V whenever A $\subseteq$ V & V $\in$ i-O(X). Since, τ_j -closed set is a τ_j –b-closed set, hence, τ_j - bcl(A) $\subseteq$ τ_j -cl(A) . Combining all these facts, we have, τ_j - bcl(A) $\subseteq$ V whenever A $\subseteq$ V & V$\in$ i-O(X) holds good. Consequently, A is an (i,j)-gb-closed set.

(d) Let A be an (i,j)-g*-closed set in a bitopological space(X, τ_1,τ_2). Let V $\in$ i-GO(X) be such that A $\subseteq$ V. Then, (i,j)-g*-closedness of A provides that τ_j - cl(A) $\subseteq$ V whenever A $\subseteq$ V & V $\in$ i-GO(X). Since, τ_j -closed set is a τ_j –b-closed set, hence, τ_j - bcl(A) $\subseteq$ τ_j -cl(A) . Combining all these facts, we have, τ_j - bcl(A) $\subseteq$ V whenever A $\subseteq$ V & V$\in$ i-GO(X) which means that A is an (i,j)-g*b-closed set.

(e) Let A be an (i,j)-g*-closed set in a bitopological (X, τ_1,τ_2). Let U $\in$ i-O(X) be such that A $\subseteq$ U. Then, (i,j)-g*-closedness of A provides that τ_j - cl(A) $\subseteq$ U_1 whenever A $\subseteq$ U_1 & U_1 $\in$ i-GO(X). …….(I)

Since, every i-open set is an i-g-open set , hence, (I) turns to be τ_j - cl(A) $\subseteq$ U whenever A $\subseteq$ U& U $\in$ i-O(X). Consequently, A is (i,j)-g-closed.

(f) Let A be an (i,j)-g-closed set in a bitopological (X, τ_1,τ_2). Let U $\in$ i-RO(X) be such that A $\subseteq$ U. Then, (i,j)- g-closedness of A provides that τ_j - cl(A) $\subseteq$ U_1 whenever A $\subseteq$ U_1 & U_1 $\in$ i-O(X). …….(I)

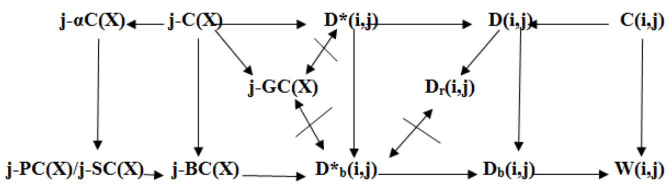
Since, every i-regular open set is an i-open set , hence, (I) turns to be τ_j - cl(A) $\subseteq$ U whenever A $\subseteq$ U& U $\in$ i-RO(X). Consequently, A is (i,j)-rg-closed.

Observation (2.3): (A) Of course, the class of (i,j) -g*b-closed sets properly fits between the class of j-b-closed sets and the class of(i,j) -gb-closed sets in a bitopological (X, τ_1,τ_2) and i,j $\in$ {1,2}, i$\neq$ j.Also, j-BC(X) $\subseteq$ D^*_b(i,j) $\subseteq$ D_b(i,j).

(I) In the space (X, τ_1,τ_2) where X = {a,b,c}, τ_1 = {φ,X,{c},{a,c}} and τ_2={φ, X,{a}}, the subset {b}and {a,b} are (1,2)-g*b-closed set but neither τ_2 -closed nor τ_2 –b-closed.

(II) In the space (X, τ_1,τ_2) where X = {a,b,c}, τ_1 = {φ,X,{a}} and τ_2={φ,X,{a},{a,b}}, the subset {c} is (1,2)-gb-closed but not (1,2) – g*b-closed .

(B) The following diagram summarized the relationship of (i,j)g*b-closed & (i,j)-gb-closed sets with other existing different closed sets:



Implication diagram

A →B (resp. A ⟷B) represents A implies B but not conversely (resp. A & B are independent.)

The following few examples stand as a support of the arrows/ independent relations in the above implication diagram.

Example (2.4):

(I) If X = {a,b,c}, τ_1 = {φ,X,{a}} and τ_2={φ,X,{a},{a,b}}, then

(a) the subsets {b}, {c},{a,b},{a,c} are (1,2)-g*b-closed but not (1,2) – g*-closed.

(b) the subset{c} is (1,2)-g-closed but not (1,2)-g*b-closed.

(II) If X = {a,b,c}, τ_1 = {φ,X,{a},{a,b}} and τ_2={φ,X,{a},{a,b},{b}}, then

(a) the subset {b} is a (1,2)-rg-closed set but not (1,2) – g*b-closed set.

(b) the subset{b} is a (1,2)-wg-closed set but not (1,2)-g*b-closed.

(III) (a) If X = {a,b,c,d,e}, τ_1 = {φ,X,{a},{b},{a,b},{a,c},{a,b,c}} and τ_2={φ,X,{a},{b},{c},{a,c},{b,c},{a,b,c}}, then the subsets{c,d} is (1,2)-w-closed but not (1,2) – g*b-closed.

(b) If X = {a,b,c}, τ_1 = {φ,X,{a},{b},{a,b}} and τ_2={φ,X,{a},{a,b}}, then the subset{a,c} is (1,2)-g*b-closed but not (1,2)-w-closed. These means that (1,2) –w-closedness & (1,2) –g*b-closedness are independent.

(IV) (a) If X = {a,b,c}, τ_1 = {φ,X,{b},{b,c}} and τ_2={φ,X,{a,b}}, then the subsets {a} is (1,2)-g*b-closed but not τ_2 -g-closed.

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(b) If $X = \{a,b,c\}, \tau_1 = \{\varphi, X, \{a\}, \{a,b\}\}$ and $\tau_2 = \{\varphi, X, \{a\}, \{b\}, \{a,b\}\}$, the subset $\{c\}$ is τ_2-g-closed but not (1,2)-g*b-closed. This means that τ_2-g-closedness & (1,2)-g*b-closedness are independent.

Proposition (2.5):
The intersection of two (i,j)- g*b-closed sets is an (i,j)-g*b-closed set, but their union need not be an (i,j)- g*b-closed set.

Proof: suppose that A and B are any two two (i,j)- g*b-closed sets in a bitopological space (X, τ_1, τ_2). Let U $\in$ i-GO(X)be such that A $\subseteq$ U& B $\subseteq$ U so that A $\cap$ B $\subseteq$ U , then the , (i,j) -g*b-closedness of A & B provides that τ_j - bcl(A) $\subseteq$ U & τ_j - bcl(B) $\subseteq$ U . So τ_j - bcl(A) $\cap$ τ_j - bcl(B) $\subseteq$ U. Again, Since, τ_j - bcl(A$\cap$B) $\subseteq$ τ_j - bcl(A) $\cap$ τ_j - bcl(B) , hence, τ_j - bcl(A$\cap$B) $\subseteq$U.

Hence, it establishes that τ_j - bcl(A$\cap$B) $\subseteq$U whenever A$\cap$B $\subseteq$U & U $\in$ i-GO(X) i.e. A $\cap$ B is an (i,j)- g*b-closed set.

But the union of two (i,j)- g*b-closed sets is not necessarily an (i,j)-g*b-closed set as illustrated by the following example:

In the Let $X = \{a,b,c\}, \tau_1 = \{\varphi, X\}$ and $\tau_2 = \{\varphi, X, \{a\}, \{a,b\}\}$, then for the bitopological space (X, τ_1, τ_2) the subsets {a}&{b} are (1,2)-g*b-closed but {a}$\cup$ {b} = {a,b} is not (i,j) – g*b-closed where i= 1& j = 2 .

Corollary (2.6): The intersection of two (i,j)- gb-closed sets is an (i,j)-gb-closed set whereas their union is not necessarily an (i,j)- gb-closed set.

Because, $D^*_b(i,j) \subseteq D_b(i,j)$ & A,B$\in$ $D^*_b(i,j) \Rightarrow$ A,B$\in$ $D_b(i,j)$

Again, A,B$\in$ $D^*_b(i,j) \Rightarrow$ A$\cap$B $\in$ $D^*_b(i,j) \Rightarrow$ A$\cap$B $\in$ $D_b(i,j)$.

So, A&B$\in$ $D_b(i,j) \Rightarrow$ A$\cap$B $\in$ $D_b(i,j)$.

Proposition (2.7): In a bitopological space (X, τ_1, τ_2), i-GO(X) $\subseteq$ j-BC(X) if and only if every subset of X is an (i,j)-g*b-closed set.

Proof: Let (X, τ_1, τ_2), be a bitopological space in which i-GO(X) $\subseteq$ j-BC(X). Let A be a subset of X such that A $\subseteq$ U & U $\in$ i-GO(X).Then A $\subseteq$ U implies that τ_j - bcl(A) $\subseteq$ τ_j - bcl (U) as U $\in$ j-BC(X) , Thus we, have τ_j - bcl(A) $\subseteq$U whenever A $\subseteq$U & U $\in$ i-GO(X) i.e. A is an (i,j)- g*b-closed set.

Conversely, suppose that every subset of X is an ij-g*b-closed. Let U $\in$ i-GO(X). Since, U is ij-g*b-closed, hence, τ_j - bcl(A) $\subseteq$U whenever A $\subseteq$U & U $\in$ i-GO(X). Now, τ_j - bcl(U) $\subseteq$U $\Longrightarrow$ U = τ_j - bcl(U) i.e. U $\in$ j-BC(X) i.e. i-GO(X) $\subseteq$ j-BC(X).

Proposition (2.8): If A is an ij-g*b-closed set in a bitopological space(X, τ_1, τ_2), and B is any set such that A $\subseteq$B$\subseteq$j-bcl(A), then B is also an ij-g*b-closed set.

Proof: In a bitopological space(X, τ_1, τ_2), let B $\subseteq$U where U $\in$ i-GO(X). Since, A is an (i,j)-g*b-closed set, hence, τ_j - bcl(A) $\subseteq$U whenever A $\subseteq$U & U $\in$ i-GO(X).

Again, B$\subseteq$ τ_j - bcl(A) $\Longrightarrow$ τ_j - bcl(B)$\subseteq$ τ_j - bcl(τ_j - bcl(A)

$\Longrightarrow$ τ_j - bcl(B)$\subseteq$ τ_j - bcl(A)

$\Longrightarrow$ τ_j - bcl(B)$\subseteq$ U.

So, we have, τ_j - bcl(B)$\subseteq$ U whenever A $\subseteq$U & U $\in$ i-GO(X)., which claims that B is an ij-g*b-closed set.

Proposition (2.9): If in bitopological space (X, τ_1, τ_2), a subset A of X is i-g-open as well as ij-g*b-closed , then A is j-b-closed.

Proof : Let a subset A of a bitopological space (X, τ_1, τ_2) be such that A$\in$ i-GO(X)&A$\in$ $D^*_b(i,j)$.

Now, the (i,j)-g*b –closedness of A gives that τ_j - bcl(A) $\subseteq$U whenever A $\subseteq$U & U $\in$ i-GO(X) ………………(I)

Since, A$\in$ i-GO(X), hence, (I) also holds good as τ_j - bcl(A) $\subseteq$U.

Again, τ_j - bcl(A) $\subseteq$A$\Longrightarrow$ A = τ_j - bcl(A) i.e. A is j-b-closed and so A$\in$ j-BC(X).

Proposition (2.10): For each element x of (X, τ_1, τ_2),{x} is either g-closed or (i,j)-g*b-open.

Proof: Let x $\in$X where (X, τ_1, τ_2) is bitopological space.

(a) Suppose that {x} is not g-closed set. This implies that $\{x\}^c$ is not a g-open set.So, the X is the only i-g-open set such that $\{x\}^c$ $\subseteq$ X. Now, τ_j - bcl $(\{x\}^C)$$\subseteq$ τ_j - bcl (X) = X Thus, $\{x\}^c$$\subseteq$ X & X$\in$ i-GO(X) $\Longrightarrow$ τ_j - bcl$(\{x\}^c)$ $\subseteq$ X holds good, which in turns establishes that $\{x\}^c$ is an (i,j)-g*b-closed .

Proposition(2.11): For an (i,j)-g*b-closed subset A of (X, τ_1, τ_2), { τ_j - bcl(A) – A} does not contain any non-empty i-g-closed set.

Proof: Let A be an (i,j)-g*b-closed subset A of a bitopological space (X, τ_1, τ_2). Let F be an i-g-closed set such that F)$\subseteq$ { τ_j - bcl(A) – A} i.e. F)$\subseteq$ { τ_j - bcl(A)$\cap$ A^c}& F $\subseteq$ A^c i.e. A $\subseteq$ F^c. Again F^c is i-g-open set & A $\subseteq$ F^c as well as A is (i,j)-g*b-closed. So, τ_j - bcl(A) $\subseteq$ F^c. This implies that F $\subseteq$ $\{\tau_j$ - bcl(A)$\}^c$. Now, F $\subseteq$ [{ τ_j - bcl(A)$\cap$ A^c}] $\cap$ [{ τ_j - bcl(A)$\}^c$]

i.e. F $\subseteq$ φ & so F = φ.

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Corollary (2.12): In a bitopological space (X, τ_1,τ_2) , an(i,j)-g*b-closed set A is j-b-closed iff { τ_j - bcl(A) – A} is i-g-closed.

Proof: Let A be (i,j)-g*b-closed. If A is j-bclosed, then { τ_j - bcl(A) – A} = φ which is i-g-closed.

Conversely, let { τ_j - bcl(A) – A} be i-g-closed then by proposition (2.10) { τ_j - bcl(A) – A} does not contain any non-empty i-g-closed subset and since{ τ_j - bcl(A) – A} is i-g-closed subset of it self, then { τ_j - bcl(A) – A} = φ. this means that A = { τ_j - bcl(A) and so A is j-b-closed.

Proposition(2.13): A subset A of a bitopological space (X, τ_1,τ_2) is said to be An (i,j)-g*b-open set if F $\subseteq$ τ_j-bint(A) whenever F is i-g-closed and F $\subseteq$ A.

Proof: Let A be an (i,j)-g*b-open set and F be an i-g-closed set such that F $\subseteq$ A in a bitopological space (X, τ_1,τ_2). Then A^c is an an (i,j)-g*b-closed set in contained in the i-g-open set F^c. Hence, τ_j-bcl(A^c) $\subseteq$ F^c. This means that [τ_j-bint(A)]^c$\subseteq$$F^c$ i.e. F $\subseteq$ τ_j-bint(A).

Conversely, if F is i-g-closed with F $\subseteq$ τ_j-bint(A) & F $\subseteq$A, then [τ_j-bint(A)]^c$\subseteq$$F^c$. This means that τ_j-bcl(A^c) $\subseteq$ F^c i.e. A^c is (i,j)- g*b-closed. Consequently, A is a (i,j)-g*b-open set.

III. IJ - T* B$_{1/2}$ AND IJ - *T B$_{1/2}$ SPACES

The bitopological spaces ij - T* b$_{1/2}$ & ij - *T b$_{1/2}$ are introduced in this section, being an application of (i,j)-g*b –closed sets.

Definition (3.1): A bitopological space (X,τ_1, τ_2) is said to be an ij - T * b$_{1/2}$ space if every (i,j)-g*b –closed set is τ_j-b-closed.

Definition (3.2): A bitopological space (X,τ_1, τ_2) is said to be an ij - *T b$_{1/2}$ space if every (i,j)-gb-closed set is (i,j)-g*b-closed .

Proposition (3.3): A bitopological space (X,τ_1, τ_2) is an (i,j)-Tb$_{1/2}$ space if and only if it is both an ij - T * b$_{1/2}$ and ij - *T b$_{1/2}$ spaces.

Proof: Suppose that A bitopological space (X,τ_1, τ_2) is an (i,j)-Tb$_{1/2}$ space, then, every (i,j)-gb-closed set is τ_j-b-closed. . We, however, know that every τ_j-b-closed is (i,j)g*b-closed[prop.(2.2)(a)]. This means that in (X,τ_1, τ_2), every (i,j)-gb-closed set is (i,j)g*b-closed and consequently, (X,τ_1, τ_2) is an ij – T* b$_{1/2}$ space.

Also, an (i,j)-g*b-closed set is (i,j)gb-closed i.e. D^*_b(i,j) $\subseteq$ D_b(i,j) so that for (X,τ_1, τ_2) every (i,j)-g*b-closed set is τ_j-b-closed. Hence, (X,τ_1, τ_2) is an ij - *T b$_{1/2}$ space.

Conversely, let (X,τ_1, τ_2) be both an ij - T * b$_{1/2}$ and ij - *T b$_{1/2}$ spaces.

For the space (X,τ_1, τ_2) to be ij - *T b$_{1/2}$, it provides that every (i,j)-gb-closed set is (i,j)-g*b-closed and for space(X,τ_1, τ_2) to be ij - T * b$_{1/2}$ space if every (i,j)-g*b – closed set is τ_j-b-closed.

So, combining these two conditions, it happens to be the fact that, in the space (X,τ_1, τ_2), every (i,j) –gb-closed set is τ_j-b-closed. Consequently, (X,τ_1, τ_2) is an (i,j)-Tb$_{1/2}$ space

Corollary (3.4): If A bitopological space (X,τ_1, τ_2) is both an ij - T * b$_{1/2}$ and ij - *T b$_{1/2}$ spaces, then it is (i,j)-T$_{1/2}$ space.

Proof: Let A bitopological space (X,τ_1, τ_2) is both an ij - T * b$_{1/2}$ and ij - *T b$_{1/2}$ spaces. Then, clearly, (X,τ_1, τ_2) is (i,j)-Tb$_{1/2}$ space. Since , D(i,j) $\subseteq$ D_b(i,j) & j-c(X) $\subseteq$ j-BC(X), hence, due to concepts of (i,j)-T$_{1/2}$ & (i,j)-Tb$_{1/2}$ spaces, an (i,j)-Tb$_{1/2}$ is also an (i,j)-T$_{1/2}$ space, therefore, a bitopological space (X,τ_1, τ_2) , being both an ij - T * b$_{1/2}$ and ij - *T b$_{1/2}$ spaces, is an (i,j)-T$_{1/2}$ space.

Proposition (3.5): A bitopological space (X,τ_1, τ_2) is an(i,j)- T *b$_{1/2}$ space iff {x} is j-open or i-g-closed for each x $\in$X.

Proof: Let x $\in$X where (X,τ_1, τ_2) is a bitopological space which is also an (i,j) - T * b$_{1/2}$ space.

Suppose that {x} is not i-g-closed. Then,{x} is (i,j) –g*b-open by proposition (2.10). i.e. {x}^{c} is (i,j) –g*b-closed.

Now, (X,τ_1, τ_2) is an(i,j)- T * b$_{1/2}$ space, so {x}^{c} is τ_j-b-closed . This means that {x} is j-open or j-b-open.

Conversely, let F be an (i,j)-g*b-closed set and x $\in$X. By assumption, {x} is either

j-b-open or i-g-closed for any x $\in$ τ_j-bcl(F).

Case I : Suppose that {x} is j-b-open. Since,{x} $\cap$ F $\neq$φ, we have x $\in$F.

Case II: Suppose that {x} is i-g-closed. If x $\notin$F, then {x} $\subseteq${τ_j-bcl(F)-F}, which is a contradiction to proposition (2.10). Therefore, x $\in$F.

Thus, in both the cases, x $\in$ τ_j-bcl(F) $\Longrightarrow$ x $\in$F i.e. τ_j-bcl(F) $\subseteq$F. i.e. F = τ_j-bcl(F) i.e. F is τ_j-b-closed.

Consequently, in the bitopological space (X,τ_1, τ_2) ,every(i,j) –g*b-closed set is τ_j-b-closed. This provides that (X,τ_1, τ_2) is an(i,j)- T * b$_{1/2}$ space.

IV. CONCLUSION

The concepts of (i,j)-g*b –closed & (i,j)-gb-closed sets in a bitopological space (X,τ_1, τ_2) have been introduced and their important properties have also been investigated & analyzed. As applications two new bitopological spaces (i,j)-T*b$_{1/2}$ & (i,j)-*Tb$_{1/2}$ have been explored. The fundamental characteristics of such spaces have also been analyzed and studied.

The content of this paper opens the new horizon for the further study of such spaces and related existing spaces.

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