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A Nonlinear Discrete Optimization with Preconditioning for Heterogeneous Vehicle Routing Problem with Simultaneous Pickup and Delivery

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Abstract—*This paper investigates a further generalization of the Vehicle Routing Problem, in which each customer demand is associated with pickup and delivery occurred simultaneously. The number of customer to be served is so large and spread across the city, therefore the company use several type of vehicle with different capacity. We use mixed integer nonlinear program to get the optimal decision model of the heterogeneous vehicle routing with simultaneous pickup and delivery problem. A neighborhood search approach is developed by adapting preconditioned method. to solve the model.*

Keywords— *Logistic system, Transportation, Pickup and delivery, Modeling, Precondition.*

I. INTRODUCTION

The vehicle routing problem (VRP) is a special terms in a logistic system for determining least-cost vehicle routes, with condition that each vehicle departs from a specified depot, serves and fulfills customers' demand and returns back to the depot at the end of its service. Therefore VRP may consists of a depot, a fleet of vehicles placed in the depot, routes for the vehicles to travel, and a set of customers who need demand to be delivered from the depot. The aims are to determine the optimal set of routes to be served by these vehicles, so that the total cost of used vehicles is minimized, subject to vehicle capacity constraints and other technical constraints [1]. Beside the basic VRP, there is a vast variety of related problems. Toth and [2] and [3] provided interesting comprehensive details of these problems.

The basic VRP can be extended with the occurrence of simultaneous pickup and delivery (VRPSPD). This type of problem that was introduced by [4], not only customers require delivery of goods, but also they need a simultaneous pickup of goods that should be sent to the depot.

Practically, many situations need simultaneously pickup and delivery; for example in soft drink industry not only full bottles should be delivered to the customers, but also empty ones must be picked up and sent back to the depot. The Aqua drinking water, considered in this paper, is another area in which planning for vehicle routes is a kind of VRPSPD. This is a very important issue, especially in countries that companies are obliged for taking the responsibility of their products during the life cycle. Managing the returned goods can also take the form of VRPSPD in some problems.

Several applications of the VRPSPD with time windows VRPSPDTW have been reported in road, maritime, and air transportation environments, to name a few, [5], [6], [7], [8], and [9] in road cargo routing and scheduling; [10], [11], and [12] in sea cargo routing and scheduling; and [13], [14], [15], and [16] in air cargo routing and scheduling. Further applications of the VRPPDTW can be found in transportation of elderly or handicapped people [17], [18], [19], and [20], school bus routing and scheduling [21] and [22], and ride-sharing ([23], and [24]). [25] and [26] offer an interesting review and give a systematic classification of paired sharing systems.

[4] considered a real life problem concerning the distribution of books among libraries by two vehicles. To solve this problem, a two-step procedure was proposed. The first step was to cluster customers into group which depend on their positions. Then in each cluster, a TSP is solved by using branch-and-price technique. [27] introduced a comprehensive classification of different types of vehicle routing problem with pickup and delivery (VRPPD) and separated VRPSPDs from vehicle routing problem with mixed backhaul (VRPMB). They developed four insertion-based heuristics to solve VRPSPD which first generated partial routes for the customers and then inserted the remaining customers into these routes. [28] proposed a memetic algorithm based on genetic algorithm and local search the VRP with selective pickup and delivery. [29] used a reactive tabu search which utilized modified sweep algorithm for generating initial solutions. [30] addressed a VRPSPDTW encountered in home health care logistics. They used genetic algorithm and tabu search for solving the problem. [31] suggested a parallel metaheuristic algorithm consist of random neighborhood ordering and iterated local search using a constructive algorithm to generate initial solution for this problem.

[32] proposed a hybrid algorithm of tabu search and guided local search that used a construction heuristic based on cost saving to get an initial point. A hybrid tabu search algorithm was also proposed by [33] to solve VRPSPD with maximum tour time length. [34] proposed a particle swarm optimization for this problem. [35] developed a metaheuristic algorithm based on adaptive memory which used different aspects of the good solutions to make better ones. [36] proposed a hybrid algorithm of ant colony and local search which made initial solution by the nearest neighborhood heuristic. [37] added time windows and maximum distance traveled

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constraints to the original VRPSPD and solved it by a differential evolution algorithm. [38] presented an Adaptive Large Neighborhood Search (ALNS) heuristic algorithm to handle the problem. Complex aspects involved in the problem are efficiently considered in the proposed algorithm.

Although the VRPSPDTW belongs to the NP- hard, there are some attempts have been made to solve the problem exactly [39]. [40] use a branch-and bound scheme in which lower bounds are computed by column generation. [41] proposed a branch-and-cut algorithm based on a three-index formulation. [42] presented a branch-and-cut algorithm to minimize the total routing cost, based on a two-index formulation. [43] presented a new branch-and-cut-and-price algorithm in which the lower bounds are computed by the column generation algorithm and improved by introducing different valid inequalities to the problem. Based on a set-partitioning formulation improved by additional cuts. [44] developed another branch and price algorithm for the problem considering the assumption of time windows. [45] used branch-price-and-cut algorithms to solve VRPSPDTW with multiple stacks. Another branch-and-price-and-cut approach for solving the problem was proposed by [46]. A dynamic programming approach based on state-space-time network representations was addressed by [47] and [48] to solve the VRPSPDTW.

The service company focused in this paper has various type of vehicle in its operation to deliver goods for customers. Each type of vehicle has different capacities. The variant of VRP which considers mixed fleet of vehicles is called Heterogeneous VRP (HVRP), introduced firstly by [49]. This generalization is important in practical terms, for most of customers demand are served by several type of vehicles [50], [51]. The objective of the HVRP is to find fleet composition and a corresponding routing plan that minimizes the total cost.

This paper concerns with HVRPTW in which occurs simultaneously pickup and delivery faced by Aqua Drinking water company located in Medan City, Indonesia. The fleet of vehicle with several capacity is operated every day from the depot (service center) to serve customers by delivering filled aqua gallons to customers and picking up empty aqua gallons to be taken to the depot. Therefore for simplicity we use term HVRPSPDTW as an abbreviation for the problem.

II. PROBLEM FORMULATION

This paper considers daily scheduling problem of vehicles of Aqua Water company for delivery aqua gallon(s) drinking water to a customer and for pickup empty gallon(s) from the customer.

In terms graph theory, the problem of HVRPSPDTW for servicing customers can be defined as follows. Let $G=(V,A)$ be a directed graph, where V = {0, 1, ... , *n*} is the vertex set and $A = \{(i, j): i, j \in V, i \neq j\}$ is the set of route. For each route $(i, j) \in A$ a distance (or travel) cost c_{ij} is defined. Vertex 0 (i = 0) and vertex n (i = n) are the depot vertex, center of service, where the vehicle fleet is located. Define V_c is the set of customers' vertex. Each vertex $i \in V_c$ has a known fixed daily nonnegative demand q_i, a nonnegative service time s_i, and a service time windows $[a_i,b_i]$ In particular, at depot the demand $q_0 = q_n = 0$, $q_i = -q_{p+1}, (i = 1,..., p)$ and service time $s_0 = s_n = 0$.

As this is a heterogeneous problem, we define a fleet of K vehicles composed by m different type of vehicles, each with capacity Q_m. The number of vehicles available for vehicle type m is n_m. Define K_m as the set of vehicle type m. Each customer is served by exactly one vehicle with the associated type. At the depot (i=0), a time window for vehicles to leave and to return to depot is given by [a_0, b_0]. The arrival time of a vehicle at customer i is denoted by a_i and its departure time is b_i . Vehicle routes are restricted to a maximum duration of H_k , k = 1, ..., K. Each type of vehicle is associated with a fixed cost, f_m. Another cost occurs for travelling through route $(i,j) \in A$, defined as $\alpha_{ij}^m = d_{ij} g_{ij}$, where d_{ij} is the distance travelled between route i to route j and g_{ij} a factor cost for travelling, for m type of vehicle. Further more a fixed waiting cost f_k is incurred for each of vehicle k at each customer node. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer i before a_i and after b_i however, the vehicle can arrive before a_i and wait for service. We impose travel time as a function of departure time b_i ($t_{ij} \geq 0$).

III. MATHEMATICAL MODELING

Firstly we define the decision variables as follows.

x_{0j}^k binary variable equal to 1 if vehicle $k \in K$ to deliver from depot to customer $j \in V_c$

x_{ij}^m binary variable equal to 1 if vehicle of type $m \in K_m$ travel directly from node i to node j

z_0^m binary variable equal to 1 if vehicle of type $m \in K_m$ is available at the depot

l_{ij}^m travel time from node i to node j of vehicle type $m \in K_m$ (non-negative continuous variable)

u_i^k duration of service of vehicle $k \in K$ at customer i (non-negative continuous variable)

In this basic framework the manager of the service company wants to use the available vehicle for each type efficiently, such that the total cost is minimized, and the pick-up and delivery conditions are met.

The total cost consists of traveling cost of all vehicle used and the cost for the duration of service of vehicle.

Minimize

$$\sum_{j\in V_c} c_{0j} \sum_{k\in K} x_{0j}^k + \sum_{(i,j)\in V_c} \sum_{m\in K_m} \alpha_{ij}^m x_{ij}^m + \sum_{k\in K} f_k \sum_{i\in V_c} u_i^k + \sum_{i\in V_c} b_i \sum_{m\in K_m} l_i^m \quad (1)$$

Subject to

$$\sum_{k\in K} x_{0j}^k = 1, \qquad \forall j \in V_c \quad (2)$$

$$\sum_{k\in K}\sum_{j\in V} x_{ij}^{k} = 1, \qquad \forall i \in V_c \tag{3}$$

Constraints (2) and (3)) are to ensure that exactly one vehicle regardless their type enters and departs from every customer and from the central depot.and comes back to the depot.

$$\sum_{i\in V} x_{ij}^{k} - \sum_{i\in V} x_{ji}^{k} = 1; \qquad \forall j \in V_c, \forall k \in K \tag{4}$$

A flow conservation equation is necessarily needed to maintain the continuity of each vehicle route on each period of time. This equation is presented in Constraints (4).

$$x_{ij}^{m} \le z_0^{m}, \qquad (i,j) \in V_c, \forall m \in K_m \tag{5}$$

Constraint (5) represents that each customer is served only by the available and active vehicle of type m.

$$\sum_{j\in V_c} x_{1j}^{k} \le 1; \qquad \forall k \in K \tag{6}$$

$$\sum_{i\in V_c, i>1} x_{i1}^{k} \le 1; \qquad \forall k \in K \tag{7}$$

Constraints (6) and (7) state the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the central depot, not more than one, respectively.

$$\sum_{j\in V} x_{oj}^{k} = 1 - z_0^{k}; \qquad \forall k \in K \tag{8}$$

$$\sum_{j\in V} x_{j0}^{k} = 1 - z_0^{k}; \qquad \forall k \in K \tag{9}$$

Constraints (8) and (9) are to guarantee that if a vehicle leaves its depot center it has to return to the depot.

$$\sum_{i\in V}\sum_{j\in V_c} x_{ij}^{m} \le M(1 - z_0^{m}); \qquad \forall m \in K_m \tag{10}$$

Constraint (10) is to show that it is possible to assign a vehicle to a route segment *i* to *j* if the vehicle is used.

$$\sum_{k\in K}\sum_{j\in V_c} x_{ij}^{k} = 1; \qquad \forall i \in P \tag{11}$$

$$\sum_{j\in V_c} x_{ij}^{k} - \sum_{j\in V_c} x_{p+1,j}^{k} = 0; \qquad \forall i \in P, k \in K \tag{12}$$

Constraints (11) and (912are to make sure that each request from customers are served exactly once and that the pick-up and delivery nodes are visited by the same vehicle.

$$\sum_{i\in V_c} d_i \sum_{j\in V_c} x_{ij}^{m} \le Q_m; \qquad \forall m \in K_m \tag{13}$$

Constraints (13) ensure that each delivery does not exceed the capacity of each type of vehicle

$$x_{ij}^{m}(l_i^{m} + u_i^{m} + s_i + t_{ij} - l_j^{m}) = 0; \qquad \forall m \in K_m, (i,j) \in A \tag{14}$$

Constraints (14) establishes the equilibrium among the arrival time, duration of service, service time and travel time at customers in the routes assigned.

$$l_i^{m} \le a_i \sum_{j\in V_c} x_{ij}^{m}; \qquad \forall m \in K_m, i \in V_c \tag{15}$$

$$a_i \sum_{j\in V_c} x_{ij}^{m} \le l_i^{m} + u_i^{m} \le b_i \sum_{j\in V_c} x_{ij}^{m}; \qquad \forall m \in K_m, i \in V_c \tag{16}$$

Constraints (15) and (16) present time window as the committed scheduled time-dependent for each customer.

$$a_0 \le l_{0j}^{k} \le b_0; \qquad \forall j \in V_c, \forall k \in K \tag{17}$$

Constraint (17) represents that time to leave depot should be within the time windows.

$$\sum_{j\in V_c} x_{oj}^{m} \le n_m; \qquad \forall m \in K_m \tag{18}$$

Constraint (18) guarantees that the number availability of active vehicle does not exceed the number of vehicle available at the central depot of the Aqua company.

$$x_{0j}^{k} \in \{0,1\}; \quad \forall j \in V_c, \forall k \in K \tag{19}$$

$$x_{ij}^{m} \in \{0,1\}; \quad \forall i \in V, \forall j \in V_c, \forall m \in K_m \tag{20}$$

$$l_{ij}^{m} \ge 0; \quad \forall i \in V, \forall j \in V_c, \forall m \in K_m \tag{21}$$

$$u_i^{k} \ge 0; \quad \forall i \in V, \forall k \in K \tag{22}$$

Constraints (19) and (20) are to define the binary variables and Constraints (21) and (22) are to define the non-negative continuous variables.

The model formulated in expressions (1) – (22) is in the form of large scale Mixed Integer Nonlinear Programming (MINLP).

IV. THE ALGORITHM

Stage 0.
Solve the relaxation of model (1) to (15), as a large scale nonlinear programming, using a mixed preconditioning of Newton-Krylov-Schwarz and nonlinear Schwarz methods. The idea is to nonlinearly precondition of nonlinear equations $F(x)=0$ such that the resultant equations $f(x)=0$ are closer to the linear equations.
If all variables in the continuous optimal solution is integer valued, the optimal integer solution to the problem (1) to (15) is found. STOP.
Otherwise go to Stage 1.
Stage 1.
Step 1. Get row *i** the smallest integer infeasibility, such that

$$\delta_{i*} = \min\{f_i, 1 - f_i\}$$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i*}^{T} = e_{i*}^{T} B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i*}^{T}\alpha_j$
With *j* corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased form LB or decreased from UB. If none go to next i^*.

Step 4.

Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\varnothing\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

REFERENCES

[1] M. Fischer, "Vehicle routing," *In Handbooks in OR & MS Vol. 8,* M.O. Ball, T.L. Magnanti, C.L. Monma, G.L. Nemhauser (eds.), Elsevier Science Publisher, pp. 1–33, 1995.

[2] P. Toth, and D. Vigo, "The vehicle routing problem," Philadelphia, PA: SIAM Philadelphia, 2002.

[3] M. Drexl, "Applications of the vehicle routing problem with trailers and transshipment," *Technical Report LM-2011-06,* Johannes Gutenberg University Mainz, 2011.

[4] H. Min, "The multiple vehicle routing problem with simultaneous delivery and pickup points," *Transp. Res. A,* 23(5), pp. 377-386, 1989.

[5] M. L. Fisher, A. Greenfield, R. Jaikumar, and J. Lester, "A computerized vehicle routing application," *Interfaces*, 12, pp. 42–52, 1982.

[6] W Bell, L. Dalberto, M. L. Fisher, A. Greenfield, R. Jaikumar, P. Kedia, R. Mack, and P. Prutzman, "Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer," *Interfaces*, 13, pp. 4-23, 1983.

[7] M.W.P. Savelsbergh and M. Sol, "DRIVE: Dynamic routing of independent vehicles," *Operations Research,* 46, pp. 474-490, 1998.

[8] X. Wang, and A. C. Regan, "Local truckload pickup and delivery with hard time window constraints," *Transportation Research Part B: Methodological*, 36, pp. 97-112, 2002.

[9] E. Zachariadis, C. Tarantilis, and C. Kiranoudis, "The load-dependent vehicle routing problem and its pick-up and delivery extension," *Transportation Research Part B: Methodological*, 71, pp. 158-181, 2015.

[10] H. N. Psaraftis, J. B. Orlin, D. Bienstock, and P. M. Thompson, "Analysis and solution algorithms of sealift routing and scheduling problems: Final report." *Technical Report 1700-85,* MIT, Sloan School of Management, Cambridge, MA, 1985.

[11] M. L. Fisher, and M. B. Rosenwein, "An interactive optimization system for bulk-cargo ship scheduling," *Naval Research Logistic Quarterly*, 35, pp. 27-42, 1989.

[12] M. Christiansen, "Decomposition of a combined inventory routing and time constrained ship routing problem," *Transportation Science*, 33, pp. 3-16, 1999.

[13] R. S. Solanki, and F. Southworth, "An execution planning algorithm for military airlift," *Interfaces,* 21, pp. 121-131, 1991.

[14] M. M. Solomon, A. Chalifour, J. Desrosiers, and J. Boisvert, "An application of vehicle routing methodology to large-scale larvicide control programs," *Interfaces*, 22, pp. 88–99, 1992.

[15] H. K. Rappoport, L. S. Levy, B. L. Golden, and K. Toussaint, "A planning heuristic for military airlift," *Interfaces*, 22, pp. 73-87, 1992.

[16] H. K. Rappoport, L. S. Levy, K. Toussaint, and B. L. Golden, "A transportation problem formulation for the MAC airlift planning problem," *Annals of Operations Research*, 50, pp. 505-523, 1994.

[17] J. Jaw, A. Odoni, H. N. Psaraftis, and N. Wilson, "A heuristic algorithm for the multi-vehicle advance-request dial-a-ride problem with time windows," *Transportation Research Part B: Methodological*, 20, pp. 243-257, 1986.

[18] A. S. Alfa, "Scheduling of vehicles for transportation of elderly," *Transportation Planning and Technology*, 11, pp. 203-212, 1986.

[19] I. Ioachim, J. Desrosiers, Y. Dumas, M. M. Solomon, and D. Villeneuve, "A request clustering algorithm for door-to-door handicapped transportation," *Transportation Science*, 29, pp. 63-78, 1995.

[20] P. Toth, and D. Vigo, "Heuristic algorithms for the handicapped persons transportation problem," *Transportation Science*, 31, pp. 60-71, 1997.

[21] A. Swersey, and W. Ballard, "Scheduling school buses," *Management Science*, 30, pp. 844-853, 1983.

[22] J. Bramel, and D. Simchi-Levi, "A location based heuristic for general routing problems," *Operations Research*, 43, pp. 649-660, 1995.

[23] H. Hosni, J. Naoum-Sawaya, and H. Artail, "The shared-taxi problem: formulation and solution methods," *Transportation Research Part B: Methodological*, 70, pp. 303-318, 2014.

[24] X. Wang, M. Dessouky, and F. Ordonez, "A Pickup and delivery problem for ridesharing considering congestion," *Transportation Letters The International Journal of Transportation Research*, doi: 10.1179/1942787515Y.0000000023, 2015.

[25] M. Furuhata, M. Dessouky, F. Ordóñez, M. Brunet, X. Wang, and S. Koenig, "Ridesharing: The state-of-the-art and future directions," *Transportation Research Part B: Methodological*, 57, pp. 28-46, 2013.

[26] S. Parragh, K. Doerner, and R. Hartl, "A survey on pickup and delivery problems: Part II: Transportation between pickup and delivery locations," *Journal fur Betriebswirtschaft,* 58, pp. 81-117, 2008.

[27] S. Salhi and G. Nagy, "A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling," *J. Oper. Res. Soc.* 50(10), pp. 1034-1042, 1999.

[28] C. K. Ting, X. L. Liao, "The selective pickup and delivery problem: Formulation and a memetic algorithm," *Int. J. Production Economics,* 141, pp. 199-211, 2013.

[29] N. A. Wassan, and G. Nagy, "Vehicle routing problem with deliveries and pickups: Modeling issues and meta-heuristics solution approach," *Int. J. of Transportation,* 2(1), pp. 95-110, 2014.

[30] R. Liu, X. Xie, V. Augusto, and C. Rodriguez, "Heuristic algorithms for a vehicle routing problem with simultaneous delivery and pickup and time windows in home health care," *Journal of Operational Research,* 230(3), pp. 475-486, 2013

[31] A. Subramanian, L. Drummonda,, C. Bentes, L. Ochi, and R. Farias, "A parallel heuristic for the Vehicle Routing Problem with Simultaneous Pickup and Delivery," *Comput. Oper. Res.* 37(11), pp. 1899-1911, 2010.

[32] E. Zachariadis, C. Tarantilis, and C. Kiranoudis,"A hybrid metaheuristic algorithm for the vehicle routing problem with simultaneous delivery and pick-up service," *Expert Syst. Appl.* 36(2), pp. 1070-1081, 2009.

[33] M. K. Fard, and M. R. Akbari, "A hybrid tabu search algorithm for the vehicle routing problem with simultabeous pickup and delivery and maximum tour time length African," *Journal of Business Management,* 7(11), pp. 801-810, 2013.

[34] T. Ai, and V. Kachitvichyanukul, "A particle swarm optimization for the vehicle routing problem with simultaneous pickup and delivery," *Comput. Oper. Res.* 36, pp. 1693-1702, 2009.

[35] E. Zachariadis, C. Tarantilis, and C. Kiranoudis, "An adaptive memory methodology for the vehicle routing problem with simultaneous pick-ups and deliveries," *Eur. J. Oper. Res.* 202(2), pp. 401-411, 2010.

[36] B. Catay, "A new saving-based ant algorithm for the Vehicle Routing Problem with Simultaneous Pickup and Delivery", *Expert Syst. Appl.* 37(10), pp. 6809-6817, 2010.

[37] L. Mingyong, and C. Erbao, "An improved differential evolution algorithm for vehicle routing problem with simultaneous pickups and deliveries and time windows," Eng. Appl. Artif. Intell. 23(2), pp. 188-195, 2010.

[38] V. Ghilas, E. Demir, and T. van Woensel, "An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows and scheduled lines," *(Beta publicatie: working papers; Vol. 483),* Eindhoven: Technische Universiteit Eindhoven, 2015.

[39] R. Baldacci, E. Bartolini, and A. Mingozzi, "An exact algorithm for the pickup and delivery problem with time windows," *Operations Research*, 59(2), pp. 414-426, 2011.

[40] G. Desaulniers, J. Desrosiers, I. Ioachim, M.M. Solomon, F. Soumis, and D. Villeneuve, "A unified framework for deterministic time

constrained vehicle routing and crew scheduling problems," In T.G. Crainic and G. Laporte, editors, Fleet Management and Logistics, pages pp. 57-93. Kluwer, Norwell, MA, 1998.

[41] J.-F. Cordeau, "A branch-and-cut algorithm for the dial-a-ride problem," *Operations Research,* 54, pp. 573–586, 2006.

[42] S. Ropke, J.-F. Cordeau, and G. Laporte, "Models and branch-and-cut algorithms for pickup and delivery problems with time windows," *Networks*, 49(4), pp. 258-272, 2007.

[43] S. Ropke, and J.-F. Cordeau, "Branch and cut and price for the pickup and delivery problem with time windows," *Transportation Science*, 43(3). pp. 267-286, 2009.

[44] G. Gutiérrez-Jarpa, G. Desaulniers, G. Laporte, and V. Marianov, "A branch-and-price algorithm for the Vehicle Routing Problem with Deliveries, Selective Pickups and Time Windows", *Eur. J. Oper. Res.* 206, pp. 341-349, 2010.

[45] M. Cherkesly, G. Desaulniers, S. Irnich, and G. Laporte, "Branch-price-and-cut algorithms for the pickup and delivery problem with time windows and multiple stacks," *European Journal of Operational Research,* 250(3), pp. 782–793, 2016.

[46] Y. Qu, and J. F. Bard, "A branch-and-price-and-cut algorithm for heterogeneous pickup and delivery with configurable vehicle capacity," *Transportation Science,* 49(2), pp. 254-270, 2015.

[47] M. Mahmoudi, and X. Zhou, "Finding Optimal Solutions for Vehicle Routing Problem with Pickup and Delivery Services with Time Windows: A Dynamic Programming Approach Based on State-space-time Network Representations," *Transportation Research Part B 89*, pp. 19-42, 2016.

[48] M. Mahmoudi, J. Chen, and X. Zhou, "Embedding Assignment Routing Constraints through Multi-Dimensional Network Construction for Solving the Multi Vehicle Routing Problem with Pickup and Delivery with Time Windows," arXiv:1607.01728, 2016.

[49] B. Golden, A. Assad, L. Levy, and F. G. Gheysens, "The fleet size and mix vehicle routing problem", *Computers and Operations Research,* 11, pp. 49-66, 1984.

[50] A. Hoff, H. Andersson, M. Christiansen, G. Hasle and A. Lokketangen, "Industrial aspects and literature survey: fleet composition and routing," *Computers & Operations Research,* vol. 37, no. 12, pp. 2041-2061, 2010.

[51] Koc, C.,T. Bektas, O. Jabali, and G. Laporte., "Thirty years of heterogeneous vehicle routing," *European Journal of Operational Research*, 249, pp. 1-21, 2016.