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Pendant Domination in Some Generalised Graphs

Nayaka S.R.[1], Puttaswamy[1], Purushothama S.[2]
[1]PES College of Engineering, Mandya, India-571401
[2]Maharaja Institute of Technology Mysore, Mandya, India-571438

Abstract— *Let G be any graph. A dominating S set in G is called a pendant dominating set if $\langle S \rangle$ contains at least one pendant vertex. The least cardinality of the pendant dominating set in G is called the pendant domination number of G, denoted by $\gamma_{pe}(G)$. In this paper we study the pendant domination number for crown graph, Banana tree graph, helm graph, cocktail party graph, stacked book graph, octahedral graph and Jahangir graph.*

Keywords— *Dominating set, Pendant dominating set, Pendant domination number.*

I. INTRODUCTION

Let G be any graph. The concept of paired domination is an interesting concept introduced by Teresa W. Haynes in with the following application in mind. If we think of each vertex v as the possible location for a guard capable of protecting each vertex in its closed neighborhood, then domination requires every vertex to be protected. For total domination, each guard must, in turn, be protected by other guard. But for paired-domination, each guard is assigned another adjacent one, and they are designated as backups for each other. We introduce pendant domination for which at least one guard is assigned a backup. In this paper by graph, we mean a simple, finite and undirected graph without isolated vertices.

Let $G=(V,E)$ be any graph with $V(G)=n$ and $E(G)=m$ edges. Then n, m are respectively called the order and the size of the graph G. For each vertex $v\in V$, the open neighborhood of v is the set $N(v)$ containing all the vertices u adjacent to v and the closed neighborhood of v is the set $N[v]$ containing v and all the vertices u adjacent to v. Let S be any subset of V, then the open neighborhood of S is $N(S)=\bigcup_{v\in S} N(v)$ and the closed neighborhood of S is $N[S]=N(S)\bigcup S$.

The minimum and maximum of the degree among the vertices of G is denoted by $\delta(G)$ and $\Delta(G)$ respectively. A graph G is said to be regular if $\delta(G)=\Delta(G)$. A vertex v of a graph G is called a cut vertex if its removal increases the number of components. A bridge or cut edge of a graph is an edge whose removal increases the number of components. A vertex of degree zero is called an isolated vertex and a vertex of a degree one is called a pendant vertex. An edge incident to a pendant vertex is called a pendant edge. The graph containing no cycle is called a tree.

A subset S of $V(G)$ is a dominating set of G if each vertex $u\in V-S$ is adjacent to a vertex in S. The least cardinality of a dominating set in G is called the domination number of G and is usually denoted by $\gamma(G)$.

The Pendant Domination Number of a Graph

Definition: A dominating set S in G is called a pendant dominating set if $\langle S \rangle$ contains at least one pendant vertex. The minimum cardinality of a pendant dominating set is called the pendent domination number denoted by $\gamma_{pe}(G)$.

The pendant domination parameter is defined for all non-trivial connected graphs of order at least two. Hence, throughout the paper we assume that by a graph we mean a connected graph of order at least two

We recall the following results required for our study

(i) Let G be a complete graph. Then $\gamma_{pe}(G)=2$.
(ii) Let G be a wheel W_n or a star $K_{1,n-1}$. Then $\gamma_{pe}(G)=2$.
(iii) Let C_n be a Cycle with $n\geq 3$ vertices and let P_n be a path with $n\geq 2$ vertices. Then

$$\gamma_{pe}(C_n)=\gamma_{pe}(P_n)=\begin{cases}\frac{n}{3}+1, & if\ n\equiv 0(\mathrm{mod}\,3)\\ \left\lceil\frac{n}{3}\right\rceil, & if\ n\equiv 1(\mathrm{mod}\,3)\\ \left\lceil\frac{n}{3}\right\rceil+1, & if\ n\equiv 2(\mathrm{mod}\,3)\end{cases}$$

Definition: The crown graph S_n for $n\geq 3$ is the graph with vertex set $V=\{u_1,u_2,...,u_n,v_1,v_2,...,v_n\}$ and an edge from $V=\{u_iv_i,:1\leq i,j\leq n; i\neq j\}$. Therefore S_n coincides with the complete bipartite graph S_n with horizontal edges removed. Crown graph S_6 is shown in the Fig. 1.

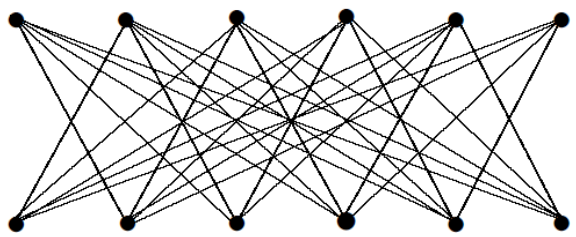

Fig. 1. S_6

Theorem 1: Let G be a crown graph with $2n$ vertices. Then $\gamma_{pe}(G)=\gamma(G)+1$.

Proof: Let G be a crown graph with the vertex set $V=\{u_1,u_2,...,u_n,v_1,v_2,...,v_n\}$. Clearly, the set $V=\{u_1v_1\}$ is a dominating set of G. Choose any one vertex u_i or v_i where $i>1$, then the set $S'=\{u_1,v_1\}\bigcup\{\{u_i\}or\{v_i\}\}$ will be a pendant dominating set of G. Therefore $\gamma_{pe}(G)=\gamma(G)+1$.

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Definition: The helm graph H_n is the graph obtained from a n-wheel graph by adjoining a pendant edge at each node of the cycle. The helm graph H_n has 2n + 1 vertices and 3n edges. Helm graph H_6 is shown in Fig. 2.

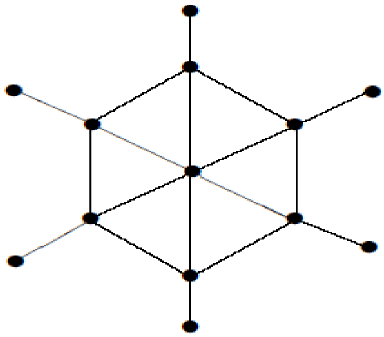

Fig. 2. H_6

Theorem: For any helm graph H_n, then $\gamma_{pe}(H_n) = n$.

Proof: Let (XY) be a bipartition of H_n, with $X = \{v_1, v_2, ..., v_n\}$ and $Y = \{u_1, u_2, ..., u_n\} \cup \{v\}$. Let u_1, u_2 are the adjacent vertices of the graph H_n and S is the set of collection of all leaves of H_n except the leaves of u_1 and u_2, then the set $S' = |S| \cup \{u_1, u_2\}$ will be a pendant dominating set of H_n. Hence $\gamma_{pe}(H_n) = (n-2) + 2 = n$.

Definition: The cocktail party graph $K_{n\times 2}$ is a graph of order 2n, with the vertex set $V = \{u_i v_j \setminus 1 \le i, j \le n\}$ and the edge set $E = \{u_i u_j, v_i v_j, u_i v_j, v_i u_j \setminus 1 \le i \le j \le n\}$. Cocktail graph of order 8 is shown in the Fig. 3.

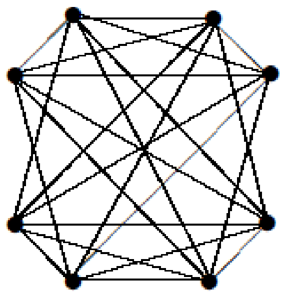

Fig. 3. Cocktail graph of order 8

Theorem: Let G be a cocktail party graph of order 2n, then $\gamma_{pe}(H_n) = 2$.

Proof: Let G be a cocktail party graph and $\{v_1, v_2, ..., v_{2n}\}$ are vertices of G. Let us choose the set $S = \{v_1, v_2\}$, where v_1 and v_2 are two adjacent vertices in G. Then the set S will be a minimal pendant dominating set of G. Hence $\gamma_{pe}(H_n) = 2$.

Definition: A banana tree (n, k) is a graph is obtained by connecting one leaf of each of n copies of a K star graph with a single root vertex that is distinct from all the stars.

Theorem: Let G be a banana tree graph, then $\gamma_{pe}(G) = \gamma(G)$.

Proof: Let G be a banana tree graph, clearly $\gamma(G) = n$. The set $S = \{V\} \cup \{v\}$ is a dominating set of G and S is itself a pendant dominating set of G, where $\{V\}$ is the collection of all centre vertices of n copies of a star graph and v is a vertex in any one copy of a star graph and $\deg(v) > 1$. Therefore $\gamma_{pe}(G) = \gamma(G)$.

Observation: For any firecracker graph $F_{n,k}$ where $n \ge 2, k \ge 3$, then $\gamma_{pe}(G) = (n+1)$

Proposition: Let G be an octahedral graph, then $\gamma_{pe}(G) = \gamma(G)$.

Proof: Let G be an octahedral graph with 6-nodes and 12-edges and is isomorphic to circulant graph. The set $S = \{u, v\}$ is a dominating set of G and S is itself a pendant dominating set if u, v are adjacent vertices. For Otherwise choose any two adjacent vertices of G, and then the set S will be a minimal pendant dominating set of G. Therefore $\gamma_{pe}(G) = \gamma(G)$.

Theorem: For any stacked book graph $B_{n,m}$ where $m \ge 3, n \ge 2$, then $\gamma_{pe}(G) = n$.

Proof: Let $B_{n,m}$ be the stacked book graph with $V(B_{n,m}) = v_1, v_2, \ldots, v_{2n+2}$. Which is obtained by the Cartesian product of $S_{m+1} \otimes P_n$, where S_m is the star graph and P_n is the path graph of order n. Let $\{v_1, v_2, ..., v_n\}$ are vertices of the path and these vertices are dominates all other vertices of $B_{n,m}$. Then the set $S = \{v_1, v_2, ..., v_n\}$ is a dominating set of $B_{n,m}$ and $<S>$ contains a pendant vertex, therefore S will be a minimal pendant dominating set of $B_{n,m}$. Hence $\gamma_{pe}(G) = n$.

Definition: For $m \ge 2$, Jahangir graph $J_{n,m}$ is a graph of order $nm+1$, consisting of a cycle of order nm with one vertex adjacent to exactly m vertices of $C_{n,m}$ at a distance n to each other. Jahangir graph $J_{2,8}$ is shown in Fig. 4.

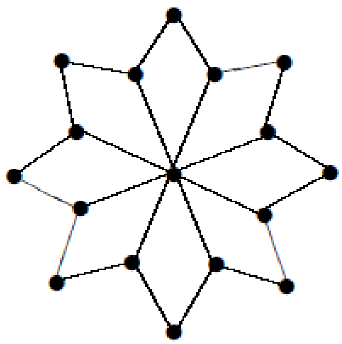

Fig. 4. $J_{2,8}$

Theorem: Let $G \cong J_{n,m}$ be a Jahangir graph with $m, n \ge 3$.

Then
$$\gamma_{pe}(H_n) = \begin{cases} \dfrac{m(n-1)}{3} + 2, & if \;\; n \equiv 1 (\bmod 3) \\ \left\lceil \dfrac{mn}{3} \right\rceil + 1, & if \;\; n \equiv 0 \;\; or \, 2 (\bmod 3) \end{cases}$$

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Proof: Let $G \cong J_{n,m}$ be a Jahangir graph with $m,n \geq 3$ and let $V(G)=\{v_1, v_2, ..., v_{nm}, v_{nm+1}\}$, where v_{nm+1} is the vertex at the center, adjacent to vertices of $C_{n,m}$. First assume $n \equiv 1(\text{mod}3)$ i.e., $n=3k+1$, for some positive integer k.

From the definition, the vertex v_{nm+1} is adjacent to m vertices of $C_{n,m}$ at a distance $3k+1$. Removing the vertex v_{nm+1} and its neighborhood vertices from G, the graph induced by $V(G)-\{N[v]\}$ splits into m components each component isomorphic to P_{3k}. Therefore, the minimum pendant dominating set of G is obtained by taking dominating set from each component together with v_{nm+1} and one of its neighborhood vertex. That is, if $S=\bigcup_{i=1}^{m} S_i$, where S_i denotes γ set of ith component, then $S \cup \{v_{nm+1}, v_1\}$, where v_1 is the vertex adjacent to v_{nm+1}. Then the set S will be a miminal pendant dominating set of G. Hence $\gamma_{pe}(G)=\frac{m(n-1)}{3}+2$.

Next, suppose $n \equiv 2(\text{mod}\,3)$. Here, we may consider two possible cases. First, assume $m \equiv 0(\text{mod}\,3)$. Then $\{v_1, v_m, v_{2m}, v_{3m}, \ldots, v_{nm}\}$ will be a dominating set of cardinality $\frac{nm}{3}+1$. Next, suppose $m \equiv 1(\text{mod}\,3)$. In this case $\{v_1, v_3, v_6, \ldots, v_{nm}\}$ will be a dominating set of size $\frac{nm+4}{3}$ i.e., $\left\lceil \frac{nm}{3} \right\rceil + 1$.

Finally, assume $n \equiv 0(\text{mod}\,3)$. For any integer $m \geq 3$, clearly nm will be a multiple of 3. Further, no dominating set contains the center vertex v_{nm}. Let S be the dominating set of $C_{n,m}$ and $<S>$ contains only isolated vertices. For the purpose of the pendant vertex choose any one vertex in $C_{n,m}$ is adjacent to any one vertex in the dominating set. Hence, $\gamma_{pe}(G)=\gamma(C_{n,m})+1$ i.e., $\gamma_{pe}(G)=\left\lceil \frac{nm}{3} \right\rceil + 1$.

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